# UNIVERSITY OF ESWATINI DEPARTMENT OF STATISTICS AND DEMOGRAPHY RE-SIT EXAMINATION PAPER 2020/2021

TITLE OF PAPER: Introduction to Statistics

COURSE CODE: STA 141

TIME ALLOWED: 2 Hours

REQUIREMENTS: Statistical Tables and Calculator

### INSTRUCTIONS

- 1) Answer any three (3) questions
- 2) Show clearly all your working

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

### **Question 1** [5+7+6+2 marks]

Spot prices per barrel of crude oil reached their highest levels in history during June and July of 2008. The following data give the spot prices (in dollars) of a barrel of crude oil for 14 business days from June 30, 2008, through July 18, 2008 (Energy Information Administration, April 15, 2009).

- a. Find the mean and variance for these data.
- b. Construct a frequency distribution table for these data using a class width of 3.00 and the lower boundary of the first class equal to 128.00.
- c. Find the mean and variance of the grouped data of part (b).
- d. Compare your means from parts (a) and (c). If the two means are not equal, explain why they differ.

## Question 2 [2+2+3+2+5+3+3 marks]

- a. In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that
  - i. the student took mathematics or history;
  - ii. the student did not take either of these subjects;
  - iii. the student took history but not mathematics.
- b. State (without proof) the law of total probability.
- c. State and prove the Bayes rule
- d. Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations  $L_1$ ,  $L_2$ ,  $L_3$ , and  $L_4$  will be operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?
- e. If the person in (d) received a speeding ticket on her way to work, what is the probability that she passed through the radar trap located at  $L_2$ ?

# Question 3 [2+4+4+5+5 marks]

a. It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are

inoculated, find the probability that

- i. none contracts the disease;
- ii. fewer than 2 contract the disease;
- iii. more than 3 contract the disease.
- b. A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.
- c. A salesperson for a medical device company has two appointments on a given day. At the first appointment, he believes that he has a 70% chance to make the deal, from which he can earn E1000 commission if successful. On the other hand, he thinks he only has a 40% chance to make the deal at the second appointment, from which, if successful, he can make E1500. What is his expected commission based on his own probability belief? Assume that the appointment results are independent of each other.

### **Question 4** [6+10+4 marks]

The following is a portion of a classic data set called the pilot plot data in Fitting Equations to Data by Daniel and Wood, published in 1971. The response y is the acid content of material produced by titration, whereas the regressor x is the organic acid content produced by extraction and weighing.

- y: 76 62 66 58 88 70 37 82 88 43
- x: 123 55 100 75 159 109 48 138 164 28
  - 1) Plot the data; does it appear that a simple linear regression will be a suitable model?
  - 2) Fit a simple linear regression; estimate a slope and intercept.
  - Graph the regression line on the plot in (a).

# Question 5 [3+3+3+3+2+2+2+2 marks]

- a. The heights of 1000 students are normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights
  - i. less than 160.0 centimeters?
  - ii. between 171.5 and 182.0 centimeters inclusive?
  - iii. equal to 175.0 centimeters?
  - iv. greater than or equal to 188.0 centimeters?
- b. If Z is a standard normal random variable, find the value  $z_0$  such that

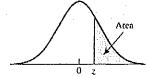
i. 
$$P(Z > z_0) = 0.5$$

ii. 
$$P(Z < z_0) = 0.8643$$

iii. 
$$P(-z_0 < Z < z_0) = 0.90$$

iv. 
$$P(-z_0 < Z < z_0) = 0.99$$

Table 4 Normal Curve Areas Standard normal probability in right-hand tail (for negative values of z, areas are found by symmetry)



2	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.464
1.0	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.424
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.38:
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.34
0.4	.3446	,3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.31
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.27
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.24
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.21
8.0	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.18
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.16
0.1	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.13
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.11
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.09
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	80.
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.06
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.05
1.6	.0548	.0537	.0526	.0516	.0505	0495	.0485	.0475	.0465	.04
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.03
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.02
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.02
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	,01
2.1	0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.01
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	,01
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.00
2.4	.0082	.0800.	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.00
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.00.
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.00.
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.00
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.00
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	,00
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 000.								•	
5.0	.000 000	0 287								

From R. E. Walpole, Introduction to Statistics (New York: Macmillan, 1968).