UNIVERSITY OF ESWATINI

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

FINAL EXAMINATION PAPER 2018

TITLE OF PAPER : NONPARAMETRIC STATISTICS

COURSE CODE : STA 409/ST 409

TIME ALLOWED : 2 HOURS

REQUIREMENTS : STATISTICAL TABLES AND CALCULATOR

INSTRUCTIONS

1. Answer any three (3) questions
2. Show clearly all your working

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

Question 1

A researcher conducts a pilot study to compare two treatments to help obese female teenagers lose weight. She tests each individual in two different treatment conditions. The data in Table 1 provide the number of pounds that each participant lost.

Table 1: Question 1 Data

	Pounds lost	
Participant	Treatment 1	Treatment 2
1	10	18
2	20	12
3	15	16
4	9	7
5	18	21
6	11	17
7	6	13
8	12	14

Use a two-tailed Wilcoxon signed rank test and a two-tailed sign test to determine which treatment resulted in greater weight loss. Use $\alpha = 0.05$. [20 marks]

Question 2

(a) A research study was conducted to see if an active involvement in a hobby had a positive effect on the health of a person who retires after age 65. The data in Table 2 below describe the health (number of doctor visits in 1 year) for participants who are involved in a hobby almost daily and those who are not.

Use a one-tailed Mann-Whitney U-test with $\alpha = 0.05$ to determine whether the hobby tends to reduce the need for doctor visits. Report your findings. [15 marks]

Table 2: Question 2 Data

No hobby group	Hobby group
12	9
15	5
8	10
11	3
9	4
17	2
5	

(b) Calculate and draw the empirical distribution function of the following sample: (-15.4, -8.8, 8.2, 3.4, -7.1, 4.5, -12.7, 5.2, -10.6, -11.2) [5 marks]

Question 3

A researcher conducted a study with $n = 15$ participants to investigate strength gains from exercise. The participants were divided into three groups and given one of three treatments. Participants strength gains were measured and ranked. The rankings are presented in Table 3 below.

Table 3: Question 3 Data

Treatments		
Treatment I	Treatment II	Treatment III
7	13	12
2	1	5
4	7	16
11	8	9
15	3	14

(a) Use an appropriate test, with $\alpha = 0.05$, to determine if one or more of the groups are significantly different. [15 marks]

(b) If a significant difference exists, discuss briefly how to identify which groups are significantly different. [5 marks]

Question 4

(a) The business department at a small college wanted to compare the relative class rank of its MBA graduates with their fifth-year salaries. The data collected by the department are presented in Table 4. Compare the graduates class rank with their fifth-year salaries

Table 4: Question 4 Data

Relative class rank	Fifth-year salary
1	83450
2	67900
3	89000
4	80500
5	91000
6	55440
7	101300
8	50560
9	76050

Use a two-tailed Spearman rank-order correlation with $\alpha = 0.05$ to determine if a relationship exists between the two variables. [15 marks]

(b) The empirical distribution function can be written as $\hat{F}_n(x) = \frac{\sum_{i=1}^{n} I(X \leq x)}{n}$, find the mean and the variance of the EDF. [5 marks]

Question 5

A graduate student performed a pilot study for his dissertation. He wanted to examine the effects of animal companionship on elderly males. He selected 10 male participants from a nursing home. Then he used an ABAB research design, where A represented a week with the absence of a cat and B represented a week with the presence of a cat. At the end of each week, he administered a 20-point survey to measure quality of life satisfaction. The survey results are presented in Table 5.

Table 5: Question 5 Data

Participant	Week 1	Week 2	Week 3	Week 4
1	7	6	8	9
2	9	8	10	7
3	15	18	16	17
4	7	6	8	9
5	7	8	10	11
6	10	14	13	11
7	12	19	11	13
8	7	4	2	5
9	8	7	9	5
9	12	16	14	15

(a) Use an appropriate test to determine if one or more of the groups are significantly different. Since this is pilot study, use $\alpha = 0.10$. [15 marks]

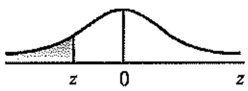
(b) You are given a dataset $\mathcal{D} = \{0, 1, 1, 2, 3, 4, 4, 4, 5\}$. Using techniques from non-parametric density estimation, answer the following questions

  i. Draw a histogram of $\mathcal{D}$ with a bin-width of 1 and bins centered at $\{0, 1, 2, 3, 4, 5\}$. [2 marks]

  ii. Write the formula for the kernel density estimate given an arbitrary kernel K and also give some conditions that K must satisfy to be a kernel. [3 marks]

Table IV Standard Normal Distribution Table

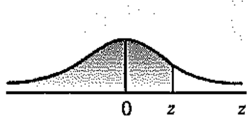
The entries in this table give the cumulative area under the standard normal curve to the left of *z* with the values of *z* equal to 0 or negative.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table IV Standard Normal Distribution Table (continued)

The entries in this table give the cumulative area under the standard normal curve to the left of z with the values of z equal to 0 or positive.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE B.2 The χ^2 Distribution.

df	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01
1	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73
12	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
21	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89

TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics *T*.

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68
26	110	98	84	75
27	119	107	92	83
28	130	116	101	91
29	140	126	110	100
30	151	137	120	109

Source: Adapted from McCornack, R. L. (1965). Extended tables of the Wilcoxon matched pair signed rank statistic. *Journal of the American Statistical Association, 60*, 864–871. Reprinted with permission from *The Journal of the American Statistical Association*.

TABLE B.4 Critical Values for the Mann–Whitney U-Test Statistic.

α	m	n = 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.10	1																				
	2																				
	3		0	1																	
	4		0	1	3																
	5		1	2	4	5															
	6		1	3	5	7	9														
	7		1	4	6	8	11	13													
	8		2	5	7	10	13	16	19												
	9	0	2	5	9	12	15	18	22	25											
	10	0	3	6	10	13	17	21	24	28	32										
	11	0	3	7	11	15	19	23	27	31	36	40									
	12	0	4	8	12	17	21	26	30	35	39	44	49								
	13	0	4	9	13	18	23	28	33	38	43	48	53	58							
	14	0	5	10	15	20	25	31	36	41	47	52	58	63	69						
	15	0	5	10	16	22	27	33	39	45	51	57	63	68	74	80					
	16	0	5	11	17	23	29	36	42	48	54	61	67	74	80	86	93				
	17	0	6	12	18	25	31	38	45	52	58	65	72	79	85	92	99	106			
	18	0	6	13	20	27	34	41	48	55	62	69	77	84	91	98	106	113	120		
	19	1	7	14	21	28	36	43	51	58	66	73	81	89	97	104	112	120	128	135	
	20	1	7	15	22	30	38	46	54	62	70	78	86	94	102	110	119	127	135	143	151
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.05	1																				
	2																				
	3			0																	
	4			0	1																
	5		0	1	2	4															
	6		0	2	3	5	7														
	7		0	2	4	6	8	11													
	8		1	3	5	8	10	13	15												
	9		1	4	6	9	12	15	18	21											
	10		1	4	7	11	14	17	20	24	27										
	11		1	5	8	12	16	19	23	27	31	34									
	12		2	5	9	13	17	21	26	30	34	38	42								
	13		2	6	10	15	19	24	28	33	37	42	47	51							
	14		3	7	11	16	21	26	31	36	41	46	51	56	61						
	15		3	7	12	18	23	28	33	39	44	50	55	61	66	72					
	16		3	8	14	19	25	30	36	42	48	54	60	65	71	77	83				
	17		3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96			
	18		4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109		
	19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	
	20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

(Continued)

TABLE B.4 ***(Continued)***

		n																			
α	*m*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.025	1																				
	2																				
	3																				
	4				0																
	5			0	1	2															
	6			1	2	3	5														
	7			1	3	5	6	8													
	8		0	2	4	6	8	10	13												
	9		0	2	4	7	10	12	15	17											
	10		0	3	5	8	11	14	17	20	23										
	11		0	3	6	9	13	16	19	23	26	30									
	12		1	4	7	11	14	18	22	26	29	33	37								
	13		1	4	8	12	16	20	24	28	33	37	41	45							
	14		1	5	9	13	17	22	26	31	36	40	45	50	55						
	15		1	5	10	14	19	24	29	34	39	44	49	54	59	64					
	16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75				
	17		2	6	11	17	22	28	34	39	45	51	57	63	69	75	81	87			
	18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99		
	19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	
	20		2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.01	1																				
	2																				
	3																				
	4																				
	5				0	1															
	6				1	2	3														
	7			0	1	3	4	6													
	8			0	2	4	6	7	9												
	9			1	3	5	7	9	11	14											
	10			1	3	6	8	11	13	16	19										
	11			1	4	7	9	12	15	18	22	25									
	12			2	5	8	11	14	17	21	24	28	31								
	13		0	2	5	9	12	16	20	23	27	31	35	39							
	14		0	2	6	10	13	17	22	26	30	34	38	43	47						
	15		0	3	7	11	15	19	24	28	33	37	42	47	51	56					
	16		0	3	7	12	16	21	26	31	36	41	46	51	56	61	66				
	17		0	4	8	13	18	23	28	33	38	44	49	55	60	66	71	77			
	18		0	4	9	14	19	24	30	36	41	47	53	59	65	70	76	82	88		
	19		1	4	9	15	20	26	32	38	44	50	56	63	69	75	82	88	94	101	
	20		1	5	10	16	22	28	34	40	47	53	60	67	73	80	87	93	100	107	114

Source: Adapted from Milton, R. C. (1964). An extended table of critical values for the Mann-Whitney (Wilcoxon) two-sample statistic. *Journal of the American Statistical Association*, *59*, 925–934. Reprinted with permission from *The Journal of the American Statistical Association*.

TABLE B.5 Critical Values for the Friedman Test Statistic F_r.

k	N	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.025$	$\alpha \leq 0.01$
3	3	6.000	6.000		
	4	6.000	6.500	8.000	8.000
	5	5.200	6.400	7.600	8.400
	6	5.333	7.000	8.333	9.000
	7	5.429	7.143	7.714	8.857
	8	5.250	6.250	7.750	9.000
	9	5.556	6.222	8.000	8.667
	10	5.000	6.200	7.800	9.600
	11	4.909	6.545	7.818	9.455
	12	5.167	6.500	8.000	9.500
	13	4.769	6.000	7.538	9.385
	14	5.143	6.143	7.429	9.000
	15	4.933	6.400	7.600	8.933
4	2	6.000	6.000		
	3	6.600	7.400	8.200	9.000
	4	6.300	7.800	8.400	9.600
	5	6.360	7.800	8.760	9.960
	6	6.400	7.600	8.800	10.200
	7	6.429	7.800	9.000	10.371
	8	6.300	7.650	9.000	10.500
	9	6.467	7.800	9.133	10.867
	10	6.360	7.800	9.120	10.800
	11	6.382	7.909	9.327	11.073
	12	6.400	7.900	9.200	11.100
	13	6.415	7.985	7.369	11.123
	14	6.343	7.886	9.343	11.143
	15	6.440	8.040	9.400	11.240
5	2	7.200	7.600	8.000	8.000
	3	7.467	8.533	9.600	10.133
	4	7.600	8.800	9.800	11.200
	5	7.680	8.960	10.240	11.680
	6	7.733	9.067	10.400	11.867
	7	7.771	9.143	10.514	12.114
	8	7.800	9.300	10.600	12.300
	9	7.733	9.244	10.667	12.444
	10	7.760	9.280	10.720	12.480
6	2	8.286	9.143	9.429	9.714
	3	8.714	9.857	10.810	11.762
	4	9.000	10.286	11.429	12.714
	5	9.000	10.486	11.743	13.229
	6	9.048	10.571	12.000	13.619
	7	9.122	10.674	12.061	13.857

(Continued)

TABLE B.5 (*Continued*)

k	N	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.025$	$\alpha \leq 0.01$
	8	9.143	10.714	12.214	14.000
	9	9.127	10.778	12.302	14.143
	10	9.143	10.800	12.343	14.229

Source: Adapted from Martin, L., Leblanc, R., & Toan, N. K. (1993). Tables for the Friedman rank test. *The Canadian Journal of Statistics / La Revue Canadienne de Statistique, 21*(1), 39–43. Reprinted with permission from *The Canadian Journal of Statistics*.

TABLE B.6 The Critical Values for the Kruskal–Wallis *H*-Test Statistic.

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
2	2	2	4.571429	–	–
3	1	1	–	–	–
3	2	1	4.285714	–	–
3	2	2	4.464286	4.714286	–
3	3	1	4.571429	5.142857	–
3	3	2	4.555556	5.361111	–
3	3	3	4.622222	5.600000	6.488889
4	2	1	4.500000	–	–
4	2	2	4.458333	5.333333	–
4	3	1	4.055556	5.208333	–
4	3	2	4.511111	5.444444	6.444444
4	3	3	4.700000	5.790909	6.745455
4	4	1	4.166667	4.966667	6.666667
4	4	2	4.554545	5.454545	7.036364
4	4	3	4.545455	5.598485	7.143939
4	4	4	4.653846	5.692308	7.653846
5	2	1	4.200000	5.000000	–
5	2	2	4.373333	5.160000	6.533333
5	3	1	4.017778	4.871111	–
5	3	2	4.650909	5.250909	6.821818
5	3	3	4.533333	5.648485	7.078788
5	4	1	3.987273	4.985455	6.954545
5	4	2	4.540909	5.272727	7.204545
5	4	3	4.548718	5.656410	7.444872
5	4	4	4.668132	5.657143	7.760440
5	5	1	4.109091	5.127273	7.309091
5	5	2	4.623077	5.338462	7.338462
5	5	3	4.545055	5.626374	7.578022
5	5	4	4.522857	5.665714	7.791429
5	5	5	4.560000	5.780000	8.000000

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
6	2	1	4.200000	4.822222	–
6	2	2	4.436364	5.345455	6.654545
6	3	1	3.909091	4.854545	6.581818
6	3	2	4.681818	5.348485	6.969697
6	3	3	4.538462	5.615385	7.192308
6	4	1	4.037879	4.946970	7.083333
6	4	2	4.493590	5.262821	7.339744
6	4	3	4.604396	5.604396	7.467033
6	4	4	4.523810	5.666667	7.795238
6	5	1	4.128205	4.989744	7.182051
6	5	2	4.595604	5.318681	7.375824
6	5	3	4.535238	5.601905	7.590476
6	5	4	4.522500	5.660833	7.935833
6	5	5	4.547059	5.698529	8.027941
6	6	1	4.000000	4.857143	7.065934
6	6	2	4.438095	5.409524	7.466667
6	6	3	4.558333	5.625000	7.725000
6	6	4	4.547794	5.724265	8.000000
6	6	5	4.542484	5.764706	8.118954
6	6	6	4.538012	5.719298	8.222222
7	1	1	4.266667	–	–
7	2	1	4.200000	4.706494	–
7	2	2	4.525974	5.142857	7.000000
7	3	1	4.173160	4.952381	6.649351
7	3	2	4.582418	5.357143	6.838828
7	3	3	4.602826	5.620094	7.227630
7	4	1	4.120879	4.986264	6.986264
7	4	2	4.549451	5.375981	7.304553
7	4	3	4.527211	5.623129	7.498639
7	4	4	4.562500	5.650000	7.814286
7	5	1	4.035165	5.063736	7.060597
7	5	2	4.484898	5.392653	7.449796
7	5	3	4.535238	5.588571	7.697143
7	5	4	4.541597	5.732773	7.931092
7	5	5	4.540056	5.707563	8.100840
7	6	1	4.032653	5.066667	7.254422
7	6	2	4.500000	5.357143	7.490476
7	6	3	4.550420	5.672269	7.756303
7	6	4	4.561625	5.705882	8.016340
7	6	5	4.559733	5.769925	8.156725

(*Continued*)

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
7	6	6	4.530075	5.730075	8.257143
7	7	1	3.985714	4.985714	7.157143
7	7	2	4.490546	5.398109	7.490546
7	7	3	4.590103	5.676937	7.809524
7	7	4	4.558897	5.765664	8.141604
7	7	5	4.545564	5.745564	8.244812
7	7	6	4.568027	5.792517	8.341497
7	7	7	4.593692	5.818182	8.378479
8	1	1	4.418182	–	–
8	2	1	4.011364	4.909091	–
8	2	2	4.586538	5.355769	6.663462
8	3	1	4.009615	4.881410	6.804487
8	3	2	4.450549	5.315934	6.986264
8	3	3	4.504762	5.616667	7.254762
8	4	1	4.038462	5.043956	6.972527
8	4	2	4.500000	5.392857	7.350000
8	4	3	4.529167	5.622917	7.585417
8	4	4	4.560662	5.779412	7.852941
8	5	1	3.967143	4.868571	7.110000
8	5	2	4.466250	5.415000	7.440000
8	5	3	4.514338	5.614338	7.705515
8	5	4	4.549020	5.717647	7.992157
8	5	5	4.555263	5.769298	8.115789
8	6	1	4.014583	5.014583	7.256250
8	6	2	4.441176	5.404412	7.522059
8	6	3	4.573529	5.678105	7.795752
8	6	4	4.562865	5.742690	8.045322
8	6	5	4.550263	5.750263	8.210263
8	6	6	4.598810	5.770238	8.294048
8	7	1	4.045431	5.041229	7.307773
8	7	2	4.450980	5.403361	7.571429
8	7	3	4.555556	5.698413	7.827068
8	7	4	4.548496	5.759211	8.118045
8	7	5	4.550612	5.777449	8.241939
8	7	6	4.552876	5.781231	8.332715
8	7	7	4.573687	5.795031	8.356296
8	8	1	4.044118	5.039216	7.313725
8	8	2	4.508772	5.407895	7.653509
8	8	3	4.555263	5.734211	7.889474
8	8	4	4.578571	5.742857	8.167857
8	8	5	4.572727	5.761039	8.297403

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
8	8	6	4.572134	5.778656	8.366601
8	8	7	4.570652	5.791149	8.418866
8	8	8	4.595000	5.805000	8.465000
9	1	1	4.545455	–	–
9	2	1	3.905983	4.841880	6.346154
9	2	2	4.483516	5.260073	6.897436
9	3	1	4.073260	4.952381	6.886447
9	3	2	4.492063	5.339683	6.990476
9	3	3	4.633333	5.588889	7.355556
9	4	1	3.971429	5.071429	7.171429
9	4	2	4.488889	5.400000	7.363889
9	4	3	4.514706	5.651961	7.613971
9	4	4	4.576253	5.703704	7.909586
9	5	1	4.055556	5.040000	7.148889
9	5	2	4.464706	5.395588	7.447059
9	5	3	4.587364	5.669717	7.733333
9	5	4	4.531384	5.712671	8.024561
9	5	5	4.557193	5.769825	8.169825
9	6	1	3.933824	5.049020	7.247549
9	6	2	4.481481	5.392157	7.494553
9	6	3	4.541910	5.664717	7.822612
9	6	4	4.545614	5.744737	8.108772
9	6	5	4.573651	5.761905	8.230794
9	6	6	4.554113	5.808081	8.307359
9	7	1	4.011204	5.042017	7.270464
9	7	2	4.480089	5.429128	7.636591
9	7	3	4.535338	5.656140	7.860652
9	7	4	4.547732	5.731406	8.131406
9	7	5	4.565492	5.757988	8.287941
9	7	6	4.570864	5.782985	8.353284
9	7	7	4.583851	5.802622	8.403037
9	8	1	3.986355	4.984893	7.394250
9	8	2	4.491667	5.419737	7.642105
9	8	3	4.568651	5.717460	7.927381
9	8	4	4.559163	5.744228	8.203102
9	8	5	4.551252	5.783465	8.318050
9	8	6	4.560688	5.775362	8.408514
9	8	7	4.563770	5.807579	8.450000
9	8	8	4.582821	5.809744	8.494359
9	9	1	4.007018	4.961404	7.333333

(*Continued*)

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
9	9	2	4.460317	5.411111	7.692063
9	9	3	4.565657	5.708514	7.959596
9	9	4	4.550066	5.751647	8.202240
9	9	5	4.587440	5.770048	8.370048
9	9	6	4.555556	5.814444	8.427778
9	9	7	4.567326	5.802198	8.468864
9	9	8	4.570750	5.815052	8.514720
9	9	9	4.582011	5.844797	8.564374
10	1	1	4.653846	4.653846	–
10	2	1	4.114286	4.839560	6.428571
10	2	2	4.434286	5.120000	6.537143
10	3	1	3.996190	5.076190	6.851429
10	3	2	4.470000	5.361667	7.041667
10	3	3	4.529412	5.588235	7.360294
10	4	1	4.042500	5.017500	7.105000
10	4	2	4.462500	5.344853	7.356618
10	4	3	4.587582	5.654248	7.616993
10	4	4	4.564912	5.715789	7.907018
10	5	1	3.988235	4.905882	7.107353
10	5	2	4.454902	5.388235	7.513725
10	5	3	4.552047	5.618713	7.752047
10	5	4	4.556842	5.744211	8.047895
10	5	5	4.574286	5.777143	8.162857
10	6	1	3.967320	5.041830	7.316340
10	6	2	4.479532	5.405848	7.588304
10	6	3	4.542105	5.655789	7.882105
10	6	4	4.550476	5.726190	8.142857
10	6	5	4.554978	5.754978	8.267532
10	6	6	4.575494	5.780237	8.338340
10	7	1	3.981454	4.985965	7.252130
10	7	2	4.491880	5.377444	7.641203
10	7	3	4.545034	5.698095	7.901224
10	7	4	4.550278	5.751206	8.172356
10	7	5	4.567250	5.763862	8.295652
10	7	6	4.563043	5.798758	8.376915
10	7	7	4.562286	5.796571	8.419429
10	8	1	3.963947	5.037632	7.358684
10	8	2	4.482857	5.429286	7.720714
10	8	3	4.533983	5.711688	7.977273
10	8	4	4.550988	5.744466	8.206126
10	9	5	4.556522	5.789130	8.344022

TABLE B.6 ***(Continued)***

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 3$).

n_1	n_2	n_3	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
10	9	6	4.573333	5.793833	8.397833
10	9	7	4.564484	5.810637	8.480967
10	9	8	4.561538	5.829060	8.494017
10	9	1	4.025714	4.988571	7.436508
10	9	2	4.476479	5.446176	7.693795
10	9	3	4.570751	5.700659	7.997628
10	9	4	4.556401	5.757609	8.223430
10	9	5	4.547556	5.792000	8.380222
10	9	6	4.561231	5.813128	8.449436
10	9	7	4.559707	5.817610	8.507475
10	9	8	4.567063	5.833730	8.544489
10	9	9	4.578982	5.830706	8.575698
10	10	1	3.987013	5.054545	7.501299
10	10	2	4.477470	5.449802	7.726482
10	10	3	4.559420	5.687681	8.026087
10	10	4	4.567000	5.776000	8.263000
10	10	5	4.554462	5.793231	8.403692
10	10	6	4.561823	5.796011	8.472934
10	10	7	4.558277	5.820408	8.536508
10	10	8	4.565025	5.837438	8.565887
10	10	9	4.567050	5.837241	8.606130
10	10	10	4.583226	5.855484	8.640000

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 4$.)

n_1	n_2	n_3	n_4	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
2	2	2	1	5.357143	5.678571	–
2	2	2	2	5.666667	6.166667	6.666667
3	2	1	1	4.892857	–	–
3	2	2	1	5.555556	5.833333	–
3	2	2	2	5.644444	6.333333	7.133333
3	3	1	1	5.333333	6.333333	–
3	3	2	1	5.622222	6.244444	7.044444
3	3	2	2	5.745455	6.527273	7.636364
3	3	3	1	5.654545	6.600000	7.400000
3	3	3	2	5.878788	6.727273	8.015152
3	3	3	3	5.974359	6.897436	8.435897
4	2	1	1	5.250000	5.833333	–
4	2	2	1	5.533333	6.133333	7.000000
4	2	2	2	5.754545	6.545455	7.390909
4	3	1	1	5.066667	6.177778	7.066667

(Continued)

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 4$.)

n_1	n_2	n_3	n_4	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
4	3	2	1	5.572727	6.309091	7.454545
4	3	2	2	5.750000	6.621212	7.871212
4	3	3	1	5.666667	6.545455	7.757576
4	3	3	2	5.858974	6.782051	8.320513
4	3	3	3	6.000000	6.967033	8.653846
4	4	1	1	5.181818	5.945455	7.909091
4	4	2	1	5.568182	6.386364	7.909091
4	4	2	2	5.807692	6.730769	8.346154
4	4	3	1	5.660256	6.634615	8.217949
4	4	3	2	5.901099	6.873626	8.620879
4	4	3	3	6.004762	7.038095	8.866667
4	4	4	1	5.653846	6.725275	8.587912
4	4	4	2	5.914286	6.957143	8.871429
4	4	4	3	6.029167	7.129167	9.075000
4	4	4	4	6.088235	7.235294	9.286765
5	1	1	1	5.333333	–	–
5	2	1	1	5.266667	5.960000	–
5	2	2	1	5.541818	6.109091	7.276364
5	2	2	2	5.636364	6.563636	7.772727
5	3	1	1	5.130909	6.003636	7.400000
5	3	2	1	5.518182	6.363636	7.757576
5	3	2	2	5.771795	6.664103	8.202564
5	3	3	1	5.656410	6.641026	8.117949
5	3	3	2	5.865934	6.821978	8.606593
5	3	3	3	6.020952	7.011429	8.840000
5	4	1	1	5.254545	6.040909	7.909091
5	4	2	1	5.580769	6.419231	8.173077
5	4	2	2	5.782418	6.725275	8.472527
5	4	3	1	5.639560	6.681319	8.408791
5	4	3	2	5.901905	6.925714	8.801905
5	4	3	3	6.029167	7.093333	9.029167
5	4	4	1	5.674286	6.760000	8.725714
5	4	4	2	5.947500	6.990000	9.002500
5	4	4	3	6.035294	7.172794	9.220588
5	4	4	4	6.066667	7.262745	9.392157
5	5	1	1	5.153846	6.076923	8.107692
5	5	2	1	5.564835	6.540659	8.327473
5	5	2	2	5.794286	6.777143	8.634286
5	5	3	1	5.662857	6.737143	8.611429
5	5	3	2	5.921667	6.946667	8.946667
5	5	3	3	6.023529	7.117647	9.188235

TABLE B.6 (*Continued*)

(The Critical Values for the Kruskal–Wallis H-Test Statistic, $k = 4$.)

n_1	n_2	n_3	n_4	$\alpha \leq 0.10$	$\alpha \leq 0.05$	$\alpha \leq 0.01$
5	5	4	1	5.670000	6.782500	8.870000
5	5	4	2	5.944853	7.032353	9.156618
5	5	4	3	6.052288	7.217647	9.356863
5	5	4	4	6.070175	7.291228	9.536842
5	5	5	1	5.682353	6.829412	9.052941
5	5	5	2	5.945098	7.074510	9.286275
5	5	5	3	6.043275	7.250292	9.495906
5	5	5	4	6.082105	7.327895	9.669474
5	5	5	5	6.097143	7.377143	9.800000

Source: Adapted from Meyer, J. P., & Seaman, M. A. (2008, March). A comparison of the exact Kruskal-Wallis distribution to asymptotic approximations for N ≤ 105. Paper presented at the annual meeting of the American Educational Research Association, New York. Reprinted with permission of the authors.

TABLE B.7 Critical Values for the Spearman Rank-Order Correlation Coefficient r_s.

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
4	1.000			
5	0.900	1.000	1.000	
6	0.829	0.886	0.943	1.000
7	0.714	0.786	0.893	0.929
8	0.643	0.738	0.833	0.881
9	0.600	0.700	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.536	0.618	0.709	0.755
12	0.503	0.587	0.671	0.727
13	0.484	0.560	0.648	0.703
14	0.464	0.538	0.622	0.675
15	0.443	0.521	0.604	0.654
16	0.429	0.503	0.582	0.635
17	0.414	0.485	0.566	0.615
18	0.401	0.472	0.550	0.600
19	0.391	0.460	0.535	0.584
20	0.380	0.447	0.520	0.570
21	0.370	0.435	0.508	0.556
22	0.361	0.425	0.496	0.544
23	0.353	0.415	0.486	0.532

(*Continued*)

TABLE B.7 ***(Continued)***

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
24	0.344	0.406	0.476	0.321
25	0.337	0.398	0.466	0.511
26	0.331	0.390	0.457	0.501
27	0.324	0.382	0.448	0.491
28	0.317	0.375	0.440	0.483
29	0.312	0.368	0.433	0.475
30	0.306	0.362	0.425	0.467
31	0.301	0.356	0.418	0.459
32	0.296	0.350	0.412	0.452
33	0.291	0.345	0.405	0.446
34	0.287	0.340	0.399	0.439
35	0.283	0.335	0.394	0.433
36	0.279	0.330	0.388	0.427
37	0.275	0.325	0.383	0.421
38	0.271	0.321	0.378	0.415
39	0.267	0.317	0.373	0.410
40	0.264	0.313	0.368	0.405
41	0.261	0.309	0.364	0.400
42	0.257	0.305	0.359	0.395
43	0.254	0.301	0.355	0.391
44	0.251	0.298	0.351	0.386
45	0.248	0.294	0.347	0.382
46	0.246	0.291	0.343	0.378
47	0.243	0.288	0.340	0.374
48	0.240	0.285	0.336	0.370
49	0.238	0.282	0.333	0.366
50	0.235	0.279	0.329	0.363

Source: Adapted from Zar, J. H. (1972). Significance testing of the Spearman rank correlation coefficient. *Journal of the American Statistical Association, 67*, 578–580. Reprinted with permission from The *Journal of the American Statistical Association*.