UNIVERSITY OF ESWATINI

MAIN EXAMINATION PAPER 2019

TITLE OF PAPER	**:**	**PROBABILITY THEORY II**
COURSE CODE	**:**	**STA 212**
TIME ALLOWED	**:**	**2 HOURS**
INSTRUCTIONS	**:**	**ANSWER ANY THREE QUESTIONS.**
REQUIREMENTS	**:**	**SCIENTIFIC CALCULATOR AND STATISTICAL TABLES.**

Question 1

The continuous random variables X and Y have the joint probability density function

$$\frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}x^{\alpha-1}y^{\beta-1}(1-x-y)^{\gamma-1} \quad ,0<x<1,0<y<1,0<x+y<1$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$ and $\Gamma(\,)$ is the gamma function.

a) Let r and s be non-negative integers. Show that the expected value of X^rY^s is

$$E(X^rY^s) = \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)}\cdot\frac{\Gamma(\beta+s)}{\Gamma(\beta)}\cdot\frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha+\beta+\gamma+r+s)}$$

(8 Marks)

b) Hence determine the expected value and variance of X.

(6 Marks)

c) Find the correlation between X and Y.

(6 Marks)

Question 2

Two tennis players, A and B, are playing a match. Let X be the number of serves faster than 125 Km/h served by A in one of his service games and let Y be the number of these serves returned by B. The following probability model is proposed:

$$P(X=0)=0.4, P(X=1)=0.3, P(X=2)=0.2, and\ P(X=3)=0.1$$

The conditional distribution of Y (given that $X = x > 0$) is binomial with parameters x and 0.4, and $P(Y=0|X=0)=1$. Assume that this model is correct when answering the following questions.

(a) Find the joint probability distribution of X and Y and display it in the form of a two-way table.

(7 Marks)

(b) Find the marginal distribution of Y and evaluate E(Y).

(4 Marks)

(c) Find Cov(X, Y).

(4 Marks)

(d) Use your joint probability distribution table to find the probability distribution of the number of serves faster than 125 km/h that are not returned by B in a game.

(5 Marks)

Question 3

The joint density of X and Y is given by

$$f(x,y) = c(3y - x)e^{-y}; \qquad 0 \leq x \leq 3y, y \geq 0$$

a) Find the value of c making this a valid joint pdf.

(3 Marks)

b) Find the marginal densities of X and Y . Are X and Y independent?

(8 Marks)

c) Find E[Y].

(3 Marks)

d) Find the conditional density of Y given X = x.

(3 Marks)

e) Use the density calculated above to get E[Y|X = x].

(3 Marks)

Question 4

Suppose that X and Y are independent random variables with the same probability density function (*pdf*), f (x).

(a) Write down, without proof, a formula for the pdf of X + Y.

(2 Marks)

(b) Suppose that f (x) = $x/2$ for $0 < x < 2$ (and f (x) = 0 elsewhere). Find the *pdf* of W = X + Y for $0 < w < 2$ and for $2 < w < 4$.

(12 Marks)

(c) Find the pdf of V = $(X - 1)^2$.

(6 Marks)

Question 5

A random vector (X, Y) has joint pdf , given by

$$f(x,y) = 2e^{-x-2y} \quad , x > 0, y > 0$$

a) Calculate E[XY].

(6Marks)

b) Calculate the covariance of X + Y and X − Y .

(14 Marks)