UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2018/19

COURSE TITLE:	OPERATIONS RESEARCH I
COURSE CODE:	ST 307/ STA 307
TIME ALLOWED:	THREE (3) HOURS
INSTRUCTION:	ANSWER QUESTION ONE AND ANY THREE QUESTIONS EACH QUESTION IS WORTH 25 MARKS
SPECIAL REQUIREMENTS:	SCIENTIFIC CALCULATORS AND GRAPH PAPER

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Question 1

Solve the following problem for the optimal values of the decision variables and the objective function using the graphical method. **(12 marks)**

Maximize $Z = 5x_1 + 8x_2$
subject to
$3x_1 + 12x_2 \geq 36$
$11x_1 + 3x_2 \geq 33$
$2x_1 + 2x_2 \leq 16$
$x_1, x_1 \geq 0$

(b) Do any of the constraints have surplus or slack? If so, which one(s) and how much surplus/slack? **(3 marks)**

(c) Formulate the dual of this problem:

Maximize $Z = 50x_1 + 80x_2$
Subject to
$3x_1 + 5x_2 \leq 45$
$4x_1 + 2x_2 \geq 16$
$6x_1 + 6x_2 = 30$
$x_1 x_2 \geq 0$

(10 marks)

Question 2

Solve the following LP model using the Dual Simplex method:

Minimise $Z = x_1 + 2x_2$
subject to
$x_1 \leq 10$
$x_2 \leq 10$
$x_1 + x_2 \geq 15$
$x_1, x_2 \geq 0$

(25 marks)

Question 3

Consider the following transportation table where the values in the upper-right-hand corner of each cell represent unit shipping cost:

To: / From:	Warehouse #1	Warehouse #2	Warehouse #3	Supply
Plant A	50	32	40	700
Plant B	16	30	20	200
Plant C	35	28	42	200
Demand	300	400	400	

a. Use the VAM to find the initial feasible solution.
b. Use the MODI method to test for optimality.
c. Develop the Improved Solution using the Stepping Stone Method.
d. If route 3-A is unacceptable for some reason, what distribution plan would be optimal? What would be its total cost? **(25 marks)**

Question 4

a. The approximate travel times for officiating crews for college soccer for four games scheduled for a weekend are shown in the following table:

	Game Site			
Crew	**Siphofaneni**	**BigBend**	**Gilgal**	**Sithobelweni**
A	1.2	1.4	0.2	1.5
B	1.0	2.0	0.5	1.0
C	1.2	3.4	2.4	0.5
D	2.1	3.1	1.1	0.8

*Time in Hours

i. What set of assignments will minimise travel time?
ii. What is the total travel time required for the optimal assignment? **(10 marks)**

b. Suppose that a soccer game in Duze has been cancelled, freeing up another crew (Crew E) to select from. Crew E's travel times to Siphofaneni, Big Bend, Gilgal and Sithobelweni are 1.8, 0.5, 0.6 and 2.0 hours, respectively. Assume also, that the D-Sithobelweni assignment is undesirable.

i. Determine the set of assignments that will minimise total travel time.
ii. Which crew will not be assigned?
iii. Is there an alternative optimal solution? Explain. **(15 marks)**

Question 5

Solve the following LP problem using the Simplex method:

Max $Z = 5x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 50$

$3x_1 + x_2 \geq 90$

$x_2 = 10$

$x_1, x_2 \geq 0$

(25 marks)

Question 6

A LP formulation for a Maximization problem is as follows:

Max $Z = 6x_1 + 2x_2 + 8x_3$

Subject to

$x_1 + 2x_2 + x_3 \leq 500$

$3x_1 + 2x_2 \leq 460$

$2x_1 + 8x_2 \leq 840$

$x_1, x_2, x_3 \geq 0$

Using the optimal tableau given below, answer the following questions:

	Cj	6	2	8	0	0	0	
BASIS		X1	X2	X3	S1	S2	S3	Bj
S1	0	-1	0	0	1	-0.5	-0.25	60
X3	8	1.5	0	1	0	0.5	0	230
X2	2	0.25	1	0	0	0	0.125	105
	Zj	12.5	2	8	0	4	0.25	2050
	Cj-Zj	-6.5	0	0	0	-4	-0.25	

(a) What is the optimal production schedule for this firm? Are there alternative optimal production schedules?

(b) What are the optimal values for the dual decision variable?

(c) What are the optimal values for the dual surplus variables?

(d) What are the shadow prices for the three resources, how are these interpreted?

(e) How is the Cj-Zj value for decision variable 1 interpreted? **(25 marks)**

END OF EXAM!!