

UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2017/18

COURSE TITLE: INFERENTIAL STATISTICS I

COURSE CODE: STA 232/ST 232

TIME ALLOWED: THREE (3) HOURS

INSTRUCTION: ANSWER ALL QUESTIONS IN SECTION A &
ANY THREE QUESTIONS IN SECTION B

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS AND STATISTICAL TABLES

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Question 1

The pass rate for a certain course is 65%. From a random sample of fifty students in this course, X is the number failing the course. Determine

- a) $P(X \geq 25);$
- b) $P(X < 25);$
- c) $P(20 < X < 30).$

(3+3+4 marks)

Question 2

In the course of three days 210 car accidents occurred within a certain municipal area. These accidents are divided into

| Fatal (deaths) | Serious injuries | Minor injuries | No injuries | Total |
|-------------------|---------------------|-------------------|----------------|-------|
| 3 | 12 | 23 | 172 | 210 |

- a) Estimate the proportion of accidents in the municipal area that (i) resulted in no injuries, and (ii) were fatal.
- b) What are the standard errors of the estimations in (a)?
- c) If the data above is representative of traffic accidents in the area for the whole month, find for the month, a 95% confidence interval for the proportion of accidents that did not result in injuries.

(2+2+2+4 marks)

Question 3

All newly appointed representatives at a firm are required to complete a two-week compulsory training course in sales techniques. The course has been presented for ten years at a current cost of £3,000 per person. A consultant approaches the firm and offers to train newly appointed staff with the aid of a modern course in sales techniques at a total estimated cost of £5,000 per person. This course also lasts for two weeks. The firm's board of directors has to decide whether the newly appointed representatives would take the traditional course in sales techniques or, would instead, change to a new course offered by the consultant. Suppose the first-year sales figures of ten representatives selected at random, who completed the new course are as follows:

| | | | | |
|----------|----------|----------|----------|----------|
| R287 900 | R419 400 | R338 300 | R287 500 | R310 850 |
| R292 600 | R390 050 | R369 850 | R430 400 | R338 450 |

It follows that $\bar{x} = R346\ 530$ and $s = R53\ 767.58$. Suppose the firm's decision is based on a choice between the following two hypotheses:

$$H_0: \mu = 300\ 000 \text{ against } H_1: \mu > 300\ 000$$

with 300 000 being the mean first-year sales of representative who completed the traditional course.
What would be your conclusions? (10 marks)

Question 4

An insurance company investigates various methods of cutting down on operating costs. One problem is the excessively high monthly telephone account. A preliminary analysis of the account shows that more than half of the outgoing calls are non-official. Since unmarried staff members are suspected of being largely responsible, a random sample is drawn from married and unmarried staff without the knowledge of either group. The following table is a summary of the sample information with regard to the total duration of private calls made in one month.

| Staff | Sample size | Mean | Variance |
|---------|-------------|------------------|----------------|
| Single | $n_1=65$ | $\bar{x}_1=5.25$ | $s_1^2 = 14.5$ |
| Married | $n_2=90$ | $\bar{x}_2=3.89$ | $s_2^2=13.7$ |

Test at a 5% level of significance to see if, on average, private calls by unmarried persons last longer than those by married persons. **(10 marks)**

Question 5

Assume a government agency desires to estimate the miles per gallon that a particular new (make and model) car is capable of attaining. To do this, the agency acquires one of these cars, fills the gas tank, and then a trained driver drives the car for 100 miles: the gas tank is then refilled and the same driver drives the car 100 miles, at which time it is again refilled and driven 100 miles, and so on. This operation is performed a total of $n = 10$ times; the number of gallons needed to fill the gas tank these 10 times are: 4.78, 4.42, 3.94, 4.15, 4.90, 3.92, 3.94, 4.68, 4.32, 4.23.

Assuming these values constitute a random sample from a normal population with mean μ and variance σ^2 ,

- (a) Compute estimates for μ and σ .
- (b) Compute the 90% confidence limits for μ , the number of gallons needed to drive the car a 100 miles. **(10 marks)**

Question 6

A bag contains 10 flashbulbs, 8 of which are good. If 5 flashbulbs are chosen from the bag at random, what is the probability function for the number of good flashbulbs? For the number of bad bulbs? **(10 marks)**

SECTION B**Question 7**

- (a) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having cancer is 0.06. **(8 marks)**

- (i) What is the probability that a person is diagnosed as having cancer?
- (ii) What is the probability that a person diagnosed as having cancer actually has the disease?

(b) An agriculturist claims that she has cultivated a new variety of orange that has a significantly higher content of ascorbic acid (vitamin C) than the variety that is the current top seller. The accompanying data shows the ascorbic acid content (mg/100g) for a random sample of fifteen oranges of the new variety and twelve of the current top seller.

Ascorbic content of two varieties of orange

| New variety ($n_1 = 15$) | | | Current top seller ($n_2 = 12$) | | |
|----------------------------|-------|-------|-----------------------------------|------|------|
| 95.1 | 104.3 | 100.9 | 92.5 | 92.1 | 90.4 |
| 99.2 | 99.8 | 99.4 | 96.4 | 93.1 | 90.1 |
| 99.3 | 100.3 | 103.5 | 100.8 | 95.1 | 96.4 |
| 97.5 | 102.8 | 99.5 | 94.5 | 91.7 | 93.6 |
| 100.3 | 95.7 | 100.6 | | | |

Lets μ_1 and μ_2 indicate the population mean ascorbic acid content of the new variety and of the current top seller. Test the claim made by the agriculturist at 1% significance level.

(12 marks)

Question 8

The data in the following table was collected using completely randomised design. Achievement scores for four different groups of students, each group taught by a different teaching technique. The objective of the experiment was to test the hypothesis of no difference in the population distributions of achievement test scores versus the alternative that they differ in location, that is, at least one of the distributions is shifted above the others. Conduct the test using the Kruskal-Wallis test using $\alpha = 0.05$.

| 1 | 2 | 3 | 4 |
|----|----|----|----|
| 65 | 75 | 59 | 94 |
| 87 | 69 | 78 | 89 |
| 73 | 83 | 67 | 80 |
| 79 | 81 | 62 | 88 |
| 81 | 72 | 83 | |
| 69 | 79 | 76 | |
| | 90 | | |

(20 marks)

Question 9

A survey was conducted among 400 “affluent” Americans with household incomes of \$750, 000 or more per year to examine the relationship between the use of a financial advisor and ownership of stocks. The results of the survey are reflected in the following table:

| | | Own Stocks | Do Not Own Stocks |
|-----------------------|-----|------------|-------------------|
| Use financial advisor | Yes | 165 | 135 |
| | No | 43 | 57 |

At 5% significance level, can you conclude that the use of a financial advisor is independent of stock ownership for all affluent Americans? **(20 marks)**

Question 10

The numbers of incoming phone calls at a company switchboard during 1-minute intervals are believed to have a Poisson distribution. Use $\alpha=0.10$ and the following data to test the assumption that the incoming phone calls follow a Poisson distribution. **(20 marks)**

Number of incoming phone calls

| <u>During 1-minute Interval</u> | <u>Observed Frequency</u> |
|---------------------------------|---------------------------|
| 0 | 15 |
| 1 | 31 |
| 2 | 20 |
| 3 | 15 |
| 4 | 13 |
| 5 | 4 |
| 6 | 2 |
| Total | 100 |

END OF EXAM!!!!

STATISTICAL TABLES

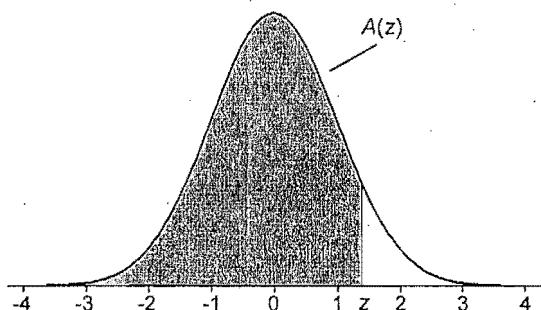
Cumulative normal distribution

Critical values of the t distribution

Critical values of the F distribution

Critical values of the chi-squared distribution

TABLE A.1
Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

| z | $A(z)$ | |
|-------|--------|---------------------------------|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

t Table

| cum. prob | $t_{.50}$ | $t_{.75}$ | $t_{.80}$ | $t_{.85}$ | $t_{.90}$ | $t_{.95}$ | $t_{.975}$ | $t_{.99}$ | $t_{.995}$ | $t_{.999}$ | $t_{.9995}$ |
|-----------|------------------|-----------|-----------|-----------|-----------|-----------|------------|-----------|------------|------------|-------------|
| one-tail | 0.50 | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| two-tails | 1.00 | 0.50 | 0.40 | 0.30 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 | 0.002 | 0.001 |
| df | | | | | | | | | | | |
| 1 | 0.000 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 318.31 | 636.62 |
| 2 | 0.000 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | 0.000 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 0.000 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 0.000 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 0.000 | 0.718 | 0.906 | 1.134 | 1.440 | 1.946 | 2.477 | 3.143 | 3.797 | 5.208 | 5.963 |
| 7 | 0.000 | 0.711 | 0.895 | 1.119 | 1.415 | 1.895 | 2.433 | 2.993 | 3.495 | 4.785 | 5.416 |
| 8 | 0.000 | 0.706 | 0.886 | 1.105 | 1.397 | 1.850 | 2.386 | 2.896 | 3.355 | 4.501 | 5.104 |
| 9 | 0.000 | 0.703 | 0.883 | 1.100 | 1.393 | 1.843 | 2.352 | 2.821 | 3.250 | 4.297 | 4.733 |
| 10 | 0.000 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.325 | 2.764 | 3.159 | 4.122 | 4.587 |
| 11 | 0.000 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 0.000 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | 0.000 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | 0.000 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | 0.000 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | 0.000 | 0.690 | 0.863 | 1.071 | 1.337 | 1.746 | 2.120 | 2.589 | 2.921 | 3.636 | 4.015 |
| 17 | 0.000 | 0.689 | 0.860 | 1.069 | 1.333 | 1.731 | 2.104 | 2.559 | 2.893 | 3.646 | 4.065 |
| 18 | 0.000 | 0.688 | 0.859 | 1.067 | 1.330 | 1.724 | 2.091 | 2.552 | 2.878 | 3.610 | 4.022 |
| 19 | 0.000 | 0.688 | 0.856 | 1.066 | 1.328 | 1.719 | 2.083 | 2.553 | 2.861 | 3.579 | 4.086 |
| 20 | 0.000 | 0.687 | 0.860 | 1.064 | 1.325 | 1.712 | 2.086 | 2.528 | 2.845 | 3.552 | 4.050 |
| 21 | 0.000 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | 0.000 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | 0.000 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | 0.000 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | 0.000 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | 0.000 | 0.684 | 0.856 | 1.058 | 1.315 | 1.705 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | 0.000 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | 0.000 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.046 | 2.467 | 2.768 | 3.408 | 3.674 |
| 29 | 0.000 | 0.683 | 0.854 | 1.057 | 1.312 | 1.699 | 2.043 | 2.465 | 2.765 | 3.395 | 3.659 |
| 30 | 0.000 | 0.682 | 0.854 | 1.055 | 1.310 | 1.697 | 2.040 | 2.460 | 2.760 | 3.383 | 3.640 |
| 40 | 0.000 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 60 | 0.000 | 0.679 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 80 | 0.000 | 0.678 | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 100 | 0.000 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 1000 | 0.000 | 0.675 | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.330 | 2.581 | 3.098 | 3.300 |
| 23 | 0.000 | 0.674 | 0.842 | 1.036 | 1.281 | 1.643 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |
| | 0% | 50% | 60% | 70% | 80% | 90% | 95% | 98% | 99% | 99.8% | 99.9% |
| | Confidence Level | | | | | | | | | | |

TABLE A.3

F Distribution: Critical Values of F (5% significance level)

| v_1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| v_2 | | | | | | | | | | | | | | | |
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.36 | 246.46 | 247.32 | 248.01 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.42 | 19.43 | 19.44 | 19.45 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.71 | 8.69 | 8.67 | 8.66 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.87 | 5.84 | 5.82 | 5.80 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.64 | 4.60 | 4.58 | 4.56 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.96 | 3.92 | 3.90 | 3.87 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.53 | 3.49 | 3.47 | 3.44 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.24 | 3.20 | 3.17 | 3.15 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.03 | 2.99 | 2.96 | 2.94 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.86 | 2.83 | 2.80 | 2.77 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.74 | 2.70 | 2.67 | 2.65 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.64 | 2.60 | 2.57 | 2.54 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.55 | 2.51 | 2.48 | 2.46 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.48 | 2.44 | 2.41 | 2.39 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.42 | 2.38 | 2.35 | 2.33 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.37 | 2.33 | 2.30 | 2.28 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.33 | 2.29 | 2.26 | 2.23 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.29 | 2.25 | 2.22 | 2.19 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.26 | 2.21 | 2.18 | 2.16 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.22 | 2.18 | 2.15 | 2.12 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.20 | 2.16 | 2.12 | 2.10 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.17 | 2.13 | 2.10 | 2.07 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.15 | 2.11 | 2.08 | 2.05 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.13 | 2.09 | 2.05 | 2.03 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.11 | 2.07 | 2.04 | 2.01 |
| 26 | 4.22 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.09 | 2.05 | 2.02 | 1.99 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.06 | 2.02 | 1.99 | 1.96 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.05 | 2.01 | 1.97 | 1.94 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.04 | 1.99 | 1.96 | 1.93 |
| 35 | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 2.04 | 1.99 | 1.94 | 1.91 | 1.88 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.95 | 1.90 | 1.87 | 1.84 |
| 50 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.95 | 1.89 | 1.85 | 1.81 | 1.78 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.86 | 1.82 | 1.78 | 1.75 |
| 70 | 3.98 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 | 1.89 | 1.84 | 1.79 | 1.75 | 1.72 |
| 80 | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 | 1.88 | 1.82 | 1.77 | 1.73 | 1.70 |
| 90 | 3.95 | 3.10 | 2.71 | 2.47 | 2.32 | 2.20 | 2.11 | 2.04 | 1.99 | 1.94 | 1.86 | 1.80 | 1.76 | 1.72 | 1.69 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.85 | 1.79 | 1.75 | 1.71 | 1.68 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 |
| 150 | 3.90 | 3.06 | 2.66 | 2.43 | 2.27 | 2.16 | 2.07 | 2.00 | 1.94 | 1.89 | 1.82 | 1.76 | 1.71 | 1.67 | 1.64 |
| 200 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.80 | 1.74 | 1.69 | 1.66 | 1.62 |
| 250 | 3.88 | 3.03 | 2.64 | 2.41 | 2.25 | 2.13 | 2.05 | 1.98 | 1.92 | 1.87 | 1.79 | 1.73 | 1.68 | 1.65 | 1.61 |
| 300 | 3.87 | 3.03 | 2.63 | 2.40 | 2.24 | 2.13 | 2.04 | 1.97 | 1.91 | 1.86 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 |
| 400 | 3.86 | 3.02 | 2.63 | 2.39 | 2.24 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.78 | 1.72 | 1.67 | 1.63 | 1.60 |
| 500 | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |
| 600 | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.11 | 2.02 | 1.95 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |
| 750 | 3.85 | 3.01 | 2.62 | 2.38 | 2.23 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.77 | 1.70 | 1.66 | 1.62 | 1.58 |
| 1000 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.76 | 1.70 | 1.65 | 1.61 | 1.58 |

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (5% significance level)

| <i>v₁</i> | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| <i>v₂</i> | | | | | | | | | | |
| 1 | 249.26 | 250.10 | 250.69 | 251.14 | 251.77 | 252.20 | 252.62 | 253.04 | 253.46 | 253.68 |
| 2 | 19.46 | 19.46 | 19.47 | 19.47 | 19.48 | 19.48 | 19.48 | 19.49 | 19.49 | 19.49 |
| 3 | 8.63 | 8.62 | 8.60 | 8.59 | 8.58 | 8.57 | 8.56 | 8.55 | 8.54 | 8.54 |
| 4 | 5.77 | 5.75 | 5.73 | 5.72 | 5.70 | 5.69 | 5.68 | 5.66 | 5.65 | 5.65 |
| 5 | 4.52 | 4.50 | 4.48 | 4.46 | 4.44 | 4.43 | 4.42 | 4.41 | 4.39 | 4.39 |
| 6 | 3.83 | 3.81 | 3.79 | 3.77 | 3.75 | 3.74 | 3.73 | 3.71 | 3.70 | 3.69 |
| 7 | 3.40 | 3.38 | 3.36 | 3.34 | 3.32 | 3.30 | 3.29 | 3.27 | 3.26 | 3.25 |
| 8 | 3.11 | 3.08 | 3.06 | 3.04 | 3.02 | 3.01 | 2.99 | 2.97 | 2.96 | 2.95 |
| 9 | 2.89 | 2.86 | 2.84 | 2.83 | 2.80 | 2.79 | 2.77 | 2.76 | 2.74 | 2.73 |
| 10 | 2.73 | 2.70 | 2.68 | 2.66 | 2.64 | 2.62 | 2.60 | 2.59 | 2.57 | 2.56 |
| 11 | 2.60 | 2.57 | 2.55 | 2.53 | 2.51 | 2.49 | 2.47 | 2.46 | 2.44 | 2.43 |
| 12 | 2.50 | 2.47 | 2.44 | 2.43 | 2.40 | 2.38 | 2.37 | 2.35 | 2.33 | 2.32 |
| 13 | 2.41 | 2.38 | 2.36 | 2.34 | 2.31 | 2.30 | 2.28 | 2.26 | 2.24 | 2.23 |
| 14 | 2.34 | 2.31 | 2.28 | 2.27 | 2.24 | 2.22 | 2.21 | 2.19 | 2.17 | 2.16 |
| 15 | 2.28 | 2.25 | 2.22 | 2.20 | 2.18 | 2.16 | 2.14 | 2.12 | 2.10 | 2.10 |
| 16 | 2.23 | 2.19 | 2.17 | 2.15 | 2.12 | 2.11 | 2.09 | 2.07 | 2.05 | 2.04 |
| 17 | 2.18 | 2.15 | 2.12 | 2.10 | 2.08 | 2.06 | 2.04 | 2.02 | 2.00 | 1.99 |
| 18 | 2.14 | 2.11 | 2.08 | 2.06 | 2.04 | 2.02 | 2.00 | 1.98 | 1.96 | 1.95 |
| 19 | 2.11 | 2.07 | 2.05 | 2.03 | 2.00 | 1.98 | 1.96 | 1.94 | 1.92 | 1.91 |
| 20 | 2.07 | 2.04 | 2.01 | 1.99 | 1.97 | 1.95 | 1.93 | 1.91 | 1.89 | 1.88 |
| 21 | 2.05 | 2.01 | 1.98 | 1.96 | 1.94 | 1.92 | 1.90 | 1.88 | 1.86 | 1.84 |
| 22 | 2.02 | 1.98 | 1.96 | 1.94 | 1.91 | 1.89 | 1.87 | 1.85 | 1.83 | 1.82 |
| 23 | 2.00 | 1.96 | 1.93 | 1.91 | 1.88 | 1.86 | 1.84 | 1.82 | 1.80 | 1.79 |
| 24 | 1.97 | 1.94 | 1.91 | 1.89 | 1.86 | 1.84 | 1.82 | 1.80 | 1.78 | 1.77 |
| 25 | 1.96 | 1.92 | 1.89 | 1.87 | 1.84 | 1.82 | 1.80 | 1.78 | 1.76 | 1.75 |
| 26 | 1.94 | 1.90 | 1.87 | 1.85 | 1.82 | 1.80 | 1.78 | 1.76 | 1.74 | 1.73 |
| 27 | 1.92 | 1.88 | 1.86 | 1.84 | 1.81 | 1.79 | 1.76 | 1.74 | 1.72 | 1.71 |
| 28 | 1.91 | 1.87 | 1.84 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 1.70 | 1.69 |
| 29 | 1.89 | 1.85 | 1.83 | 1.81 | 1.77 | 1.75 | 1.73 | 1.71 | 1.69 | 1.67 |
| 30 | 1.88 | 1.84 | 1.81 | 1.79 | 1.76 | 1.74 | 1.72 | 1.70 | 1.67 | 1.66 |
| 35 | 1.82 | 1.79 | 1.76 | 1.74 | 1.70 | 1.68 | 1.66 | 1.63 | 1.61 | 1.60 |
| 40 | 1.78 | 1.74 | 1.72 | 1.69 | 1.66 | 1.64 | 1.61 | 1.59 | 1.56 | 1.55 |
| 50 | 1.73 | 1.69 | 1.66 | 1.63 | 1.60 | 1.58 | 1.55 | 1.52 | 1.50 | 1.48 |
| 60 | 1.69 | 1.65 | 1.62 | 1.59 | 1.56 | 1.53 | 1.51 | 1.48 | 1.45 | 1.44 |
| 70 | 1.66 | 1.62 | 1.59 | 1.57 | 1.53 | 1.50 | 1.48 | 1.45 | 1.42 | 1.40 |
| 80 | 1.64 | 1.60 | 1.57 | 1.54 | 1.51 | 1.48 | 1.45 | 1.43 | 1.39 | 1.38 |
| 90 | 1.63 | 1.59 | 1.55 | 1.53 | 1.49 | 1.46 | 1.44 | 1.41 | 1.38 | 1.36 |
| 100 | 1.62 | 1.57 | 1.54 | 1.52 | 1.48 | 1.45 | 1.42 | 1.39 | 1.36 | 1.34 |
| 120 | 1.60 | 1.55 | 1.52 | 1.50 | 1.46 | 1.43 | 1.40 | 1.37 | 1.33 | 1.32 |
| 150 | 1.58 | 1.54 | 1.50 | 1.48 | 1.44 | 1.41 | 1.38 | 1.34 | 1.31 | 1.29 |
| 200 | 1.56 | 1.52 | 1.48 | 1.46 | 1.41 | 1.39 | 1.35 | 1.32 | 1.28 | 1.26 |
| 250 | 1.55 | 1.50 | 1.47 | 1.44 | 1.40 | 1.37 | 1.34 | 1.31 | 1.27 | 1.25 |
| 300 | 1.54 | 1.50 | 1.46 | 1.43 | 1.39 | 1.36 | 1.33 | 1.30 | 1.26 | 1.23 |
| 400 | 1.53 | 1.49 | 1.45 | 1.42 | 1.38 | 1.35 | 1.32 | 1.28 | 1.24 | 1.22 |
| 500 | 1.53 | 1.48 | 1.45 | 1.42 | 1.38 | 1.35 | 1.31 | 1.28 | 1.23 | 1.21 |
| 600 | 1.52 | 1.48 | 1.44 | 1.41 | 1.37 | 1.34 | 1.31 | 1.27 | 1.23 | 1.20 |
| 750 | 1.52 | 1.47 | 1.44 | 1.41 | 1.37 | 1.34 | 1.30 | 1.26 | 1.22 | 1.20 |
| 1000 | 1.52 | 1.47 | 1.43 | 1.41 | 1.36 | 1.33 | 1.30 | 1.26 | 1.22 | 1.19 |

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

| v_1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| v_2 | | | | | | | | | | | | | | | |
| 1 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6106.32 | 6142.67 | 6170.10 | 6191.53 | 6208.73 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.44 | 99.44 | 99.45 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.92 | 26.83 | 26.75 | 26.69 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.25 | 14.15 | 14.08 | 14.02 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89 | 9.77 | 9.68 | 9.61 | 9.55 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.60 | 7.52 | 7.45 | 7.40 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.36 | 6.28 | 6.21 | 6.16 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.56 | 5.48 | 5.41 | 5.36 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 5.01 | 4.92 | 4.86 | 4.81 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.60 | 4.52 | 4.46 | 4.41 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.29 | 4.21 | 4.15 | 4.10 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.05 | 3.97 | 3.91 | 3.86 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.86 | 3.78 | 3.72 | 3.66 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.70 | 3.62 | 3.56 | 3.51 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.56 | 3.49 | 3.42 | 3.37 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.45 | 3.37 | 3.31 | 3.26 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.35 | 3.27 | 3.21 | 3.16 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.27 | 3.19 | 3.13 | 3.08 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.19 | 3.12 | 3.05 | 3.00 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.13 | 3.05 | 2.99 | 2.94 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.07 | 2.99 | 2.93 | 2.88 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 3.02 | 2.94 | 2.88 | 2.83 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.97 | 2.89 | 2.83 | 2.78 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.93 | 2.85 | 2.79 | 2.74 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.89 | 2.81 | 2.75 | 2.70 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.86 | 2.78 | 2.72 | 2.66 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.82 | 2.75 | 2.68 | 2.63 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.79 | 2.72 | 2.65 | 2.60 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.77 | 2.69 | 2.63 | 2.57 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.74 | 2.66 | 2.60 | 2.55 |
| 35 | 7.42 | 5.27 | 4.40 | 3.91 | 3.59 | 3.37 | 3.20 | 3.07 | 2.96 | 2.88 | 2.74 | 2.64 | 2.56 | 2.50 | 2.44 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.56 | 2.48 | 2.42 | 2.37 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 | 2.56 | 2.46 | 2.38 | 2.32 | 2.27 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.39 | 2.31 | 2.25 | 2.20 |
| 70 | 7.01 | 4.92 | 4.07 | 3.60 | 3.29 | 3.07 | 2.91 | 2.78 | 2.67 | 2.59 | 2.45 | 2.35 | 2.27 | 2.20 | 2.15 |
| 80 | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.64 | 2.55 | 2.42 | 2.31 | 2.23 | 2.17 | 2.12 |
| 90 | 6.93 | 4.85 | 4.01 | 3.53 | 3.23 | 3.01 | 2.84 | 2.72 | 2.61 | 2.52 | 2.39 | 2.29 | 2.21 | 2.14 | 2.09 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.37 | 2.27 | 2.19 | 2.12 | 2.07 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.23 | 2.15 | 2.09 | 2.03 |
| 150 | 6.81 | 4.75 | 3.91 | 3.45 | 3.14 | 2.92 | 2.76 | 2.63 | 2.53 | 2.44 | 2.31 | 2.20 | 2.12 | 2.06 | 2.00 |
| 200 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.27 | 2.17 | 2.09 | 2.03 | 1.97 |
| 250 | 6.74 | 4.69 | 3.86 | 3.40 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 | 2.26 | 2.15 | 2.07 | 2.01 | 1.95 |
| 300 | 6.72 | 4.68 | 3.85 | 3.38 | 3.08 | 2.86 | 2.70 | 2.57 | 2.47 | 2.38 | 2.24 | 2.14 | 2.06 | 1.99 | 1.94 |
| 400 | 6.70 | 4.66 | 3.83 | 3.37 | 3.06 | 2.85 | 2.68 | 2.56 | 2.45 | 2.37 | 2.23 | 2.13 | 2.05 | 1.98 | 1.92 |
| 500 | 6.69 | 4.65 | 3.82 | 3.36 | 3.05 | 2.84 | 2.68 | 2.55 | 2.44 | 2.36 | 2.22 | 2.12 | 2.04 | 1.97 | 1.92 |
| 600 | 6.68 | 4.64 | 3.81 | 3.35 | 3.05 | 2.83 | 2.67 | 2.54 | 2.44 | 2.35 | 2.21 | 2.11 | 2.03 | 1.96 | 1.91 |
| 750 | 6.67 | 4.63 | 3.81 | 3.34 | 3.04 | 2.83 | 2.66 | 2.53 | 2.43 | 2.34 | 2.21 | 2.11 | 2.02 | 1.96 | 1.90 |
| 1000 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.20 | 2.10 | 2.02 | 1.95 | 1.90 |

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (1% significance level)

| <i>v</i> ₁ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|-----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| <i>v</i> ₂ | | | | | | | | | | |
| 1 | 6239.83 | 6260.65 | 6275.57 | 6286.78 | 6302.52 | 6313.03 | 6323.56 | 6334.11 | 6344.68 | 6349.97 |
| 2 | 99.46 | 99.47 | 99.47 | 99.47 | 99.48 | 99.48 | 99.49 | 99.49 | 99.49 | 99.49 |
| 3 | 26.58 | 26.50 | 26.45 | 26.41 | 26.35 | 26.32 | 26.28 | 26.24 | 26.20 | 26.18 |
| 4 | 13.91 | 13.84 | 13.79 | 13.75 | 13.69 | 13.65 | 13.61 | 13.58 | 13.54 | 13.52 |
| 5 | 9.45 | 9.38 | 9.33 | 9.29 | 9.24 | 9.20 | 9.17 | 9.13 | 9.09 | 9.08 |
| 6 | 7.30 | 7.23 | 7.18 | 7.14 | 7.09 | 7.06 | 7.02 | 6.99 | 6.95 | 6.93 |
| 7 | 6.06 | 5.99 | 5.94 | 5.91 | 5.86 | 5.82 | 5.79 | 5.75 | 5.72 | 5.70 |
| 8 | 5.26 | 5.20 | 5.15 | 5.12 | 5.07 | 5.03 | 5.00 | 4.96 | 4.93 | 4.91 |
| 9 | 4.71 | 4.65 | 4.60 | 4.57 | 4.52 | 4.48 | 4.45 | 4.41 | 4.38 | 4.36 |
| 10 | 4.31 | 4.25 | 4.20 | 4.17 | 4.12 | 4.08 | 4.05 | 4.01 | 3.98 | 3.96 |
| 11 | 4.01 | 3.94 | 3.89 | 3.86 | 3.81 | 3.78 | 3.74 | 3.71 | 3.67 | 3.66 |
| 12 | 3.76 | 3.70 | 3.65 | 3.62 | 3.57 | 3.54 | 3.50 | 3.47 | 3.43 | 3.41 |
| 13 | 3.57 | 3.51 | 3.46 | 3.43 | 3.38 | 3.34 | 3.31 | 3.27 | 3.24 | 3.22 |
| 14 | 3.41 | 3.35 | 3.30 | 3.27 | 3.22 | 3.18 | 3.15 | 3.11 | 3.08 | 3.06 |
| 15 | 3.28 | 3.21 | 3.17 | 3.13 | 3.08 | 3.05 | 3.01 | 2.98 | 2.94 | 2.92 |
| 16 | 3.16 | 3.10 | 3.05 | 3.02 | 2.97 | 2.93 | 2.90 | 2.86 | 2.83 | 2.81 |
| 17 | 3.07 | 3.00 | 2.96 | 2.92 | 2.87 | 2.83 | 2.80 | 2.76 | 2.73 | 2.71 |
| 18 | 2.98 | 2.92 | 2.87 | 2.84 | 2.78 | 2.75 | 2.71 | 2.68 | 2.64 | 2.62 |
| 19 | 2.91 | 2.84 | 2.80 | 2.76 | 2.71 | 2.67 | 2.64 | 2.60 | 2.57 | 2.55 |
| 20 | 2.84 | 2.78 | 2.73 | 2.69 | 2.64 | 2.61 | 2.57 | 2.54 | 2.50 | 2.48 |
| 21 | 2.79 | 2.72 | 2.67 | 2.64 | 2.58 | 2.55 | 2.51 | 2.48 | 2.44 | 2.42 |
| 22 | 2.73 | 2.67 | 2.62 | 2.58 | 2.53 | 2.50 | 2.46 | 2.42 | 2.38 | 2.36 |
| 23 | 2.69 | 2.62 | 2.57 | 2.54 | 2.48 | 2.45 | 2.41 | 2.37 | 2.34 | 2.32 |
| 24 | 2.64 | 2.58 | 2.53 | 2.49 | 2.44 | 2.40 | 2.37 | 2.33 | 2.29 | 2.27 |
| 25 | 2.60 | 2.54 | 2.49 | 2.45 | 2.40 | 2.36 | 2.33 | 2.29 | 2.25 | 2.23 |
| 26 | 2.57 | 2.50 | 2.45 | 2.42 | 2.36 | 2.33 | 2.29 | 2.25 | 2.21 | 2.19 |
| 27 | 2.54 | 2.47 | 2.42 | 2.38 | 2.33 | 2.29 | 2.26 | 2.22 | 2.18 | 2.16 |
| 28 | 2.51 | 2.44 | 2.39 | 2.35 | 2.30 | 2.26 | 2.23 | 2.19 | 2.15 | 2.13 |
| 29 | 2.48 | 2.41 | 2.36 | 2.33 | 2.27 | 2.23 | 2.20 | 2.16 | 2.12 | 2.10 |
| 30 | 2.45 | 2.39 | 2.34 | 2.30 | 2.25 | 2.21 | 2.17 | 2.13 | 2.09 | 2.07 |
| 35 | 2.35 | 2.28 | 2.23 | 2.19 | 2.14 | 2.10 | 2.06 | 2.02 | 1.98 | 1.96 |
| 40 | 2.27 | 2.20 | 2.15 | 2.11 | 2.06 | 2.02 | 1.98 | 1.94 | 1.90 | 1.87 |
| 50 | 2.17 | 2.10 | 2.05 | 2.01 | 1.95 | 1.91 | 1.87 | 1.82 | 1.78 | 1.76 |
| 60 | 2.10 | 2.03 | 1.98 | 1.94 | 1.88 | 1.84 | 1.79 | 1.75 | 1.70 | 1.68 |
| 70 | 2.05 | 1.98 | 1.93 | 1.89 | 1.83 | 1.78 | 1.74 | 1.70 | 1.65 | 1.62 |
| 80 | 2.01 | 1.94 | 1.89 | 1.85 | 1.79 | 1.75 | 1.70 | 1.65 | 1.61 | 1.58 |
| 90 | 1.99 | 1.92 | 1.86 | 1.82 | 1.76 | 1.72 | 1.67 | 1.62 | 1.57 | 1.55 |
| 100 | 1.97 | 1.89 | 1.84 | 1.80 | 1.74 | 1.69 | 1.65 | 1.60 | 1.55 | 1.52 |
| 120 | 1.93 | 1.86 | 1.81 | 1.76 | 1.70 | 1.66 | 1.61 | 1.56 | 1.51 | 1.48 |
| 150 | 1.90 | 1.83 | 1.77 | 1.73 | 1.66 | 1.62 | 1.57 | 1.52 | 1.46 | 1.43 |
| 200 | 1.87 | 1.79 | 1.74 | 1.69 | 1.63 | 1.58 | 1.53 | 1.48 | 1.42 | 1.39 |
| 250 | 1.85 | 1.77 | 1.72 | 1.67 | 1.61 | 1.56 | 1.51 | 1.46 | 1.40 | 1.36 |
| 300 | 1.84 | 1.76 | 1.70 | 1.66 | 1.59 | 1.55 | 1.50 | 1.44 | 1.38 | 1.35 |
| 400 | 1.82 | 1.75 | 1.69 | 1.64 | 1.58 | 1.53 | 1.48 | 1.42 | 1.36 | 1.32 |
| 500 | 1.81 | 1.74 | 1.68 | 1.63 | 1.57 | 1.52 | 1.47 | 1.41 | 1.34 | 1.31 |
| 600 | 1.80 | 1.73 | 1.67 | 1.63 | 1.56 | 1.51 | 1.46 | 1.40 | 1.34 | 1.30 |
| 750 | 1.80 | 1.72 | 1.66 | 1.62 | 1.55 | 1.50 | 1.45 | 1.39 | 1.33 | 1.29 |
| 1000 | 1.79 | 1.72 | 1.66 | 1.61 | 1.54 | 1.50 | 1.44 | 1.38 | 1.32 | 1.28 |

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

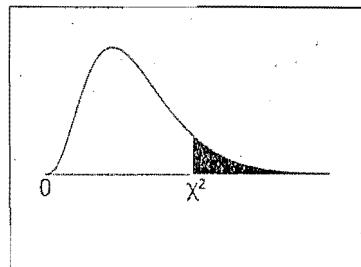
| <i>v₁</i> | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| <i>v₂</i> | | | | | | | | | | | | | | | |
| 1 | 4.05e05 | 5.00e05 | 5.40e05 | 5.62e05 | 5.76e05 | 5.86e05 | 5.93e05 | 5.98e05 | 6.02e05 | 6.06e05 | 6.11e05 | 6.14e05 | 6.17e05 | 6.19e05 | 6.21e05 |
| 2 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 | 999.40 | 999.42 | 999.43 | 999.44 | 999.44 | 999.45 |
| 3 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 | 129.25 | 128.32 | 127.64 | 127.14 | 126.74 | 126.42 |
| 4 | 74.14 | 61.25 | 56.18 | 53.44 | 51.71 | 50.53 | 49.66 | 49.00 | 48.47 | 48.05 | 47.41 | 46.95 | 46.60 | 46.32 | 46.10 |
| 5 | 47.18 | 37.12 | 33.20 | 31.09 | 29.75 | 28.83 | 28.16 | 27.65 | 27.24 | 26.92 | 26.42 | 26.06 | 25.78 | 25.57 | 25.39 |
| 6 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 | 18.69 | 18.41 | 17.99 | 17.68 | 17.45 | 17.27 | 17.12 |
| 7 | 29.25 | 21.69 | 18.77 | 17.20 | 16.21 | 15.52 | 15.02 | 14.63 | 14.33 | 14.08 | 13.71 | 13.43 | 13.23 | 13.06 | 12.93 |
| 8 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 | 11.54 | 11.19 | 10.94 | 10.75 | 10.60 | 10.48 |
| 9 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 | 9.89 | 9.57 | 9.33 | 9.15 | 9.01 | 8.90 |
| 10 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 | 8.45 | 8.22 | 8.05 | 7.91 | 7.80 |
| 11 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 | 7.92 | 7.63 | 7.41 | 7.24 | 7.11 | 7.01 |
| 12 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 | 7.29 | 7.00 | 6.79 | 6.63 | 6.51 | 6.40 |
| 13 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 | 6.80 | 6.52 | 6.31 | 6.16 | 6.03 | 5.93 |
| 14 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 | 6.40 | 6.13 | 5.93 | 5.78 | 5.66 | 5.56 |
| 15 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 | 5.81 | 5.62 | 5.46 | 5.35 | 5.25 |
| 16 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 | 5.81 | 5.55 | 5.35 | 5.20 | 5.09 | 4.99 |
| 17 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 | 5.58 | 5.32 | 5.13 | 4.99 | 4.87 | 4.78 |
| 18 | 15.38 | 10.39 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 | 5.39 | 5.13 | 4.94 | 4.80 | 4.68 | 4.59 |
| 19 | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 | 5.22 | 4.97 | 4.78 | 4.64 | 4.52 | 4.43 |
| 20 | 14.82 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 | 4.82 | 4.64 | 4.49 | 4.38 | 4.29 |
| 21 | 14.59 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 | 4.95 | 4.70 | 4.51 | 4.37 | 4.26 | 4.17 |
| 22 | 14.38 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 | 4.83 | 4.58 | 4.40 | 4.26 | 4.15 | 4.06 |
| 23 | 14.20 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 | 4.73 | 4.48 | 4.30 | 4.16 | 4.05 | 3.96 |
| 24 | 14.03 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 | 4.64 | 4.39 | 4.21 | 4.07 | 3.96 | 3.87 |
| 25 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 | 4.31 | 4.13 | 3.99 | 3.88 | 3.79 |
| 26 | 13.74 | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 | 4.48 | 4.24 | 4.06 | 3.92 | 3.81 | 3.72 |
| 27 | 13.61 | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 | 4.41 | 4.17 | 3.99 | 3.86 | 3.75 | 3.66 |
| 28 | 13.50 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 | 4.35 | 4.11 | 3.93 | 3.80 | 3.69 | 3.60 |
| 29 | 13.39 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 | 4.29 | 4.05 | 3.88 | 3.74 | 3.63 | 3.54 |
| 30 | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 | 4.00 | 3.82 | 3.69 | 3.58 | 3.49 |
| 35 | 12.90 | 8.47 | 6.79 | 5.88 | 5.30 | 4.89 | 4.59 | 4.36 | 4.18 | 4.03 | 3.79 | 3.62 | 3.48 | 3.38 | 3.29 |
| 40 | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 | 3.64 | 3.47 | 3.34 | 3.23 | 3.14 |
| 50 | 12.22 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 | 3.67 | 3.44 | 3.27 | 3.11 | 3.04 | 2.95 |
| 60 | 11.97 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 | 3.32 | 3.15 | 3.02 | 2.91 | 2.83 |
| 70 | 11.80 | 7.64 | 6.06 | 5.20 | 4.66 | 4.28 | 3.99 | 3.77 | 3.60 | 3.45 | 3.23 | 3.06 | 2.93 | 2.83 | 2.74 |
| 80 | 11.67 | 7.54 | 5.97 | 5.12 | 4.58 | 4.20 | 3.92 | 3.70 | 3.53 | 3.39 | 3.16 | 3.00 | 2.87 | 2.76 | 2.68 |
| 90 | 11.57 | 7.47 | 5.91 | 5.06 | 4.53 | 4.15 | 3.87 | 3.65 | 3.48 | 3.34 | 3.11 | 2.95 | 2.82 | 2.71 | 2.63 |
| 100 | 11.50 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 | 3.30 | 3.07 | 2.91 | 2.78 | 2.68 | 2.59 |
| 120 | 11.38 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 | 3.24 | 3.02 | 2.85 | 2.72 | 2.62 | 2.53 |
| 150 | 11.27 | 7.24 | 5.71 | 4.88 | 4.35 | 3.98 | 3.71 | 3.49 | 3.32 | 3.18 | 2.96 | 2.80 | 2.67 | 2.56 | 2.48 |
| 200 | 11.15 | 7.15 | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 | 3.12 | 2.90 | 2.74 | 2.61 | 2.51 | 2.42 |
| 250 | 11.09 | 7.10 | 5.59 | 4.77 | 4.25 | 3.88 | 3.61 | 3.40 | 3.23 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 |
| 300 | 11.04 | 7.07 | 5.56 | 4.75 | 4.22 | 3.86 | 3.59 | 3.38 | 3.21 | 3.07 | 2.85 | 2.69 | 2.56 | 2.46 | 2.37 |
| 400 | 10.99 | 7.03 | 5.53 | 4.71 | 4.19 | 3.83 | 3.56 | 3.35 | 3.18 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 |
| 500 | 10.96 | 7.00 | 5.51 | 4.69 | 4.18 | 3.81 | 3.54 | 3.33 | 3.16 | 3.02 | 2.81 | 2.64 | 2.52 | 2.41 | 2.33 |
| 600 | 10.94 | 6.99 | 5.49 | 4.68 | 4.16 | 3.80 | 3.53 | 3.32 | 3.15 | 3.01 | 2.80 | 2.63 | 2.51 | 2.40 | 2.32 |
| 750 | 10.91 | 6.97 | 5.48 | 4.67 | 4.15 | 3.79 | 3.52 | 3.31 | 3.14 | 3.00 | 2.78 | 2.62 | 2.49 | 2.39 | 2.31 |
| 1000 | 10.89 | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 | 2.99 | 2.77 | 2.61 | 2.48 | 2.38 | 2.30 |

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

| <i>v₁</i> | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|----------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| <i>v₂</i> | | | | | | | | | | |
| 1 | 6.24e05 | 6.26e05 | 6.28e05 | 6.29e05 | 6.30e05 | 6.31e05 | 6.32e05 | 6.33e05 | 6.35e05 | 6.35e05 |
| 2 | 999.46 | 999.47 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 | 999.49 | 999.49 | 999.49 |
| 3 | 125.84 | 125.45 | 125.17 | 124.96 | 124.66 | 124.47 | 124.27 | 124.07 | 123.87 | 123.77 |
| 4 | 45.70 | 45.43 | 45.23 | 45.09 | 44.88 | 44.75 | 44.61 | 44.47 | 44.33 | 44.26 |
| 5 | 25.08 | 24.87 | 24.72 | 24.60 | 24.44 | 24.33 | 24.22 | 24.12 | 24.01 | 23.95 |
| 6 | 16.85 | 16.67 | 16.54 | 16.44 | 16.31 | 16.21 | 16.12 | 16.03 | 15.93 | 15.89 |
| 7 | 12.69 | 12.53 | 12.41 | 12.33 | 12.20 | 12.12 | 12.04 | 11.95 | 11.87 | 11.82 |
| 8 | 10.26 | 10.11 | 10.00 | 9.92 | 9.80 | 9.73 | 9.65 | 9.57 | 9.49 | 9.45 |
| 9 | 8.69 | 8.55 | 8.46 | 8.37 | 8.26 | 8.19 | 8.11 | 8.04 | 7.96 | 7.93 |
| 10 | 7.60 | 7.47 | 7.37 | 7.30 | 7.19 | 7.12 | 7.05 | 6.98 | 6.91 | 6.87 |
| 11 | 6.81 | 6.68 | 6.59 | 6.52 | 6.42 | 6.35 | 6.28 | 6.21 | 6.14 | 6.10 |
| 12 | 6.22 | 6.09 | 6.00 | 5.93 | 5.83 | 5.76 | 5.70 | 5.63 | 5.56 | 5.52 |
| 13 | 5.75 | 5.63 | 5.54 | 5.47 | 5.37 | 5.30 | 5.24 | 5.17 | 5.10 | 5.07 |
| 14 | 5.38 | 5.25 | 5.17 | 5.10 | 5.00 | 4.94 | 4.87 | 4.81 | 4.74 | 4.71 |
| 15 | 5.07 | 4.95 | 4.86 | 4.80 | 4.70 | 4.64 | 4.57 | 4.51 | 4.44 | 4.41 |
| 16 | 4.82 | 4.70 | 4.61 | 4.54 | 4.45 | 4.39 | 4.32 | 4.26 | 4.19 | 4.16 |
| 17 | 4.60 | 4.48 | 4.40 | 4.33 | 4.24 | 4.18 | 4.11 | 4.05 | 3.98 | 3.95 |
| 18 | 4.42 | 4.30 | 4.22 | 4.15 | 4.06 | 4.00 | 3.93 | 3.87 | 3.80 | 3.77 |
| 19 | 4.26 | 4.14 | 4.06 | 3.99 | 3.90 | 3.84 | 3.78 | 3.71 | 3.65 | 3.61 |
| 20 | 4.12 | 4.00 | 3.92 | 3.86 | 3.77 | 3.70 | 3.64 | 3.58 | 3.51 | 3.48 |
| 21 | 4.00 | 3.88 | 3.80 | 3.74 | 3.64 | 3.58 | 3.52 | 3.46 | 3.39 | 3.36 |
| 22 | 3.89 | 3.78 | 3.70 | 3.63 | 3.54 | 3.48 | 3.41 | 3.35 | 3.28 | 3.25 |
| 23 | 3.79 | 3.68 | 3.60 | 3.53 | 3.44 | 3.38 | 3.32 | 3.25 | 3.19 | 3.16 |
| 24 | 3.71 | 3.59 | 3.51 | 3.45 | 3.36 | 3.29 | 3.23 | 3.17 | 3.10 | 3.07 |
| 25 | 3.63 | 3.52 | 3.43 | 3.37 | 3.28 | 3.22 | 3.15 | 3.09 | 3.03 | 2.99 |
| 26 | 3.56 | 3.44 | 3.36 | 3.30 | 3.21 | 3.15 | 3.08 | 3.02 | 2.95 | 2.92 |
| 27 | 3.49 | 3.38 | 3.30 | 3.23 | 3.14 | 3.08 | 3.02 | 2.96 | 2.89 | 2.86 |
| 28 | 3.43 | 3.32 | 3.24 | 3.18 | 3.09 | 3.02 | 2.96 | 2.90 | 2.83 | 2.80 |
| 29 | 3.38 | 3.27 | 3.18 | 3.12 | 3.03 | 2.97 | 2.91 | 2.84 | 2.78 | 2.74 |
| 30 | 3.33 | 3.22 | 3.13 | 3.07 | 2.98 | 2.92 | 2.86 | 2.79 | 2.73 | 2.69 |
| 35 | 3.13 | 3.02 | 2.93 | 2.87 | 2.78 | 2.72 | 2.66 | 2.59 | 2.52 | 2.49 |
| 40 | 2.98 | 2.87 | 2.79 | 2.73 | 2.64 | 2.57 | 2.51 | 2.44 | 2.38 | 2.34 |
| 50 | 2.79 | 2.68 | 2.60 | 2.53 | 2.44 | 2.38 | 2.31 | 2.25 | 2.18 | 2.14 |
| 60 | 2.67 | 2.55 | 2.47 | 2.41 | 2.32 | 2.25 | 2.19 | 2.12 | 2.05 | 2.01 |
| 70 | 2.58 | 2.47 | 2.39 | 2.32 | 2.23 | 2.16 | 2.10 | 2.03 | 1.95 | 1.92 |
| 80 | 2.52 | 2.41 | 2.32 | 2.26 | 2.16 | 2.10 | 2.03 | 1.96 | 1.89 | 1.85 |
| 90 | 2.47 | 2.36 | 2.27 | 2.21 | 2.11 | 2.05 | 1.98 | 1.91 | 1.83 | 1.79 |
| 100 | 2.43 | 2.32 | 2.24 | 2.17 | 2.08 | 2.01 | 1.94 | 1.87 | 1.79 | 1.75 |
| 120 | 2.37 | 2.26 | 2.18 | 2.11 | 2.02 | 1.95 | 1.88 | 1.81 | 1.73 | 1.68 |
| 150 | 2.32 | 2.21 | 2.12 | 2.06 | 1.96 | 1.89 | 1.82 | 1.74 | 1.66 | 1.62 |
| 200 | 2.26 | 2.15 | 2.07 | 2.00 | 1.90 | 1.83 | 1.76 | 1.68 | 1.60 | 1.55 |
| 250 | 2.23 | 2.12 | 2.03 | 1.97 | 1.87 | 1.80 | 1.72 | 1.65 | 1.56 | 1.51 |
| 300 | 2.21 | 2.10 | 2.01 | 1.94 | 1.85 | 1.78 | 1.70 | 1.62 | 1.53 | 1.48 |
| 400 | 2.18 | 2.07 | 1.98 | 1.92 | 1.82 | 1.75 | 1.67 | 1.59 | 1.50 | 1.45 |
| 500 | 2.17 | 2.05 | 1.97 | 1.90 | 1.80 | 1.73 | 1.65 | 1.57 | 1.48 | 1.43 |
| 600 | 2.16 | 2.04 | 1.96 | 1.89 | 1.79 | 1.72 | 1.64 | 1.56 | 1.46 | 1.41 |
| 750 | 2.15 | 2.03 | 1.95 | 1.88 | 1.78 | 1.71 | 1.63 | 1.55 | 1.45 | 1.40 |
| 1000 | 2.14 | 2.02 | 1.94 | 1.87 | 1.77 | 1.69 | 1.62 | 1.53 | 1.44 | 1.38 |

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi_{\alpha}^2$.

| df | $\chi^2_{.995}$ | $\chi^2_{.990}$ | $\chi^2_{.975}$ | $\chi^2_{.950}$ | $\chi^2_{.900}$ | $\chi^2_{.100}$ | $\chi^2_{.050}$ | $\chi^2_{.025}$ | $\chi^2_{.010}$ | $\chi^2_{.005}$ |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |