UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2018

TITLE OF PAPER	: TOPICS IN STATISTICS (STATISTICAL MODELLING)
COURSE CODE	: ST 405
TIME ALLOWED	: THREE (3) HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	: ANSWER ANY FIVE QUESTIONS

Question 1

a) Consider the following data from a women's health study (MI is myocardial infarction, i.e. heart attack).

	MI	
Oral contraceptives	**Yes**	**No**
Used	**23**	**34**
Never Used	**35**	**132**

(i) Construct a 95% confidence interval for the population odds ratio.

(ii) Suppose that the answer to part (a) is (1.3, 4.9). Does it seem plausible that the variables are independent? Explain.

b) For adults who sailed on the Titanic on its fateful voyage, the odds ratio between gender (female, male) and survival (yes, no) was 11.4.

(i) What is wrong with the interpretation, "The probability of survival for females was 11.4 times that for males"?

(ii) When would the quoted interpretation be approximately correct? Why?

(iii) The odds of survival for females equalled 2.9. For each gender, find the proportion who survived.

(4+4+4+4+4 Marks)

Question 2

The following table and subsequent analysis are based on sample survey data on the usage of alcohol, cigarettes and marijuana among high school students;

Alcohol Use	Cigarette Use	Marijuana Use: Yes	Marijuana Use: No
Yes	Yes	911	538
	No	44	456
No	Yes	3	43
	No	2	279

R code

```
A<-c(1,1,1,1,0,0,0,0); ## 1--Alcohol use 0--otherwise
C<-c(1,1,0,0,1,1,0,0); ## 1---Cigarette use 0---otherwise
M<-c(1,0,1,0,1,0,1,0); ## 1-Marijuana use 0-otherwise
count<-c(911,538,44,456,3,43,2,279);
 AC<-A*C; AM<-A*M; CM<-C*M; ACM<-A*C*M;

##Model (AM,CM,AC) fit
drug.log<-glm(count~A+C+M+AM+CM+AC,family=poisson(link="log"))
summary(drug.log)

## output
 Call: glm(formula = count ~ A + C + M + AM + CM + AC, family =
poisson(link = "log"))  Coefficients:
              Estimate  Std. Error   z value          Pr(>|z|)
(Intercept)   5.63342     0.05970   94.361            < 2e-16***
A             0.48772     0.07577    6.437          1.22e-10 ***
C            -1.88667     0.16270  -11.596           < 2e-16 ***
M            -5.30904     0.47520  -11.172           < 2e-16***
AM            2.98601     0.46468    6.426         <1.31e-10***
CM            2.84789     0.16384   17.382          < 2e-16 ***
AC            2.05453     0.17406   11.803          < 2e-16 ***
 Null deviance: 2851.46098,   Residual deviance:    0.37399
 ##Estimated covariance matrix between AM and CM

         AM           CM
 AM  0.215925578 -0.004968391
 CM -0.004968391 0.026843349
```

Let X; Y and Z denote the variables Alcohol, Cigarette and Marijuana use respectively.

a) Write down the loglinear regression model and identify the associated estimates. (5 Marks)

b) Compute the estimated odds ratio between any two variables of Alcohol, Cigarette, and Marijuana use controlling for the third variable.

(5 Marks)

c) Construct the 95% confidence interval for the true odds ratio between Alcohol and Cigarette use controlling for Marijuana use.

(5 Marks)

d) Test if the true odds ratio between Alcohol and Marijuana use controlling for Cigarette use equals the true odds ratio between Cigarette and Marijuana use controlling for Alcohol use at $\alpha = 5\%$.

(8 Marks)

Question 3

a) For each of the following densities for a random variable Y, show that Y or some transformation of Y has an exponential family distribution. Derive the mean and variance of the exponential family distributed quantity in each case using the mean and variance formulas that hold in general within the exponential family distribution.

(i) $f(y;\mu,\lambda) = (2\pi y^3/\lambda)^{-1/2} exp\left\{-\lambda/_{2u^2}(y-\mu)^2/y\right\}, \qquad y,\lambda,\mu > 0.$

(ii) $f(y;\theta) = \theta a^\theta / y^{(\theta+1)}, \qquad y > a, \theta > 0, a > 0.$

(20 Marks)

Question 4

For a classical linear model $= \boldsymbol{X\beta} + \varepsilon$, where y, ε are n vectors, $\boldsymbol{\beta}$ has dimension p, X has n×p, and $\varepsilon_i's$ are i.i.d. N(0,σ^2), show that the information matrix of $\boldsymbol{\beta}$ is $\sigma^{-2}\boldsymbol{X}^T\boldsymbol{X}$.

(20 Marks)

Question 5

If the goal for this software expert is to build an email spam filter: based on observed characteristics of an email message she wants to build a classification rule for assigning the message either as spam(marked with a "1") or not spam ("0"). To build the filter she has data of 4601 emails, and for each message she has a human-assigned to label 1 for spam, 0 for not spam, and the following characteristics:

- **caps_avg** = the average of the lengths of strings of capital letters used in the email (e.g. "The" = 1, "HELLO" = 5)
- **c_paren**, **c_exclaim**, **c_dollar** = the percentage of characters in the message which are parentheses ("(", "[", ")", "]"), exclamation point ("!"), and dollar sign ("$") respectively. (Percentages are between 0 and 100.)

a) In the current context, what are the two types of errors that a classifier can make? In the present context, is one type of mistake "worse" than the other? Explain your reasoning.

Use the following output to answer parts (b) - (e).

```
Call:
glm(formula = spam ~ caps_avg + c_paren + c_exclaim + c_dollar,
    family = "binomial", data = spam)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)     -1.75       0.07     -25   <2e-16 ***
caps_avg         0.21       0.02      12   <2e-16 ***
c_paren         -1.66       0.23      -7    2e-13 ***
c_exclaim        1.38       0.11      12   <2e-16 ***
c_dollar        11.86       0.62      19   <2e-16 ***

    Null deviance: 6170.2  on 4600  degrees of freedom
Residual deviance: 4160.7  on 4596  degrees of freedom
AIC: 4171

Number of Fisher Scoring iterations: 15
```

b) Provide a precise, numerical interpretation of the coefficient estimate for **c_dollar**. Do you and this result credible? Why or why not?

c) Provide a precise, numerical interpretation of the coefficient estimate for **caps_avg**. Do you find this result credible? Why or why not?

d) For a message that is 2% parentheses, 2% exclamation points, has zero dollar signs, and never strings together more than one capital letter, what estimated probability this message is spam?

e) If she uses the regression in above to build a classification rule based on the predicted probabilities. For some number K, we will flag a message as spam if the estimated P[spam = 1|X] > K. Referring to your answer in part (a), would you prefer to choose K = 1/4, K = 1/2, or K = 3/4? Why?

(4+3+3+5+5 Marks)

Question 6

a) If the hazard function is $h(t) = a\sqrt{t}$, where $a > 0$, what are the survival and density functions?

(10 Marks)

b) If survival times in the absence of censoring are distributed according to a Weibull distribution with parameters κ and λ, the hazard and survival functions can be written as

$$h(t) = \lambda\kappa t^{k-1}$$

$$S(t) = \exp(-\lambda t^{k})$$

respectively. If we observed data of the form (t_i, δ_i), where $\delta_i = 1$ if individual i fails at time t_i and $\delta_i = 0$ if i is right-censored at t_i, for i = 1,...,m. What is the log-likelihood function? Explan briefly how you might find the maximum-likelihood estimates of k and λ.

(10 Marks)

Question 7

150 female rats were bought in 50 litters of 3 and randomly given a placebo (2 rats per litter) or a new drug (1 rat per litter). The rats were followed for 4 months, and the time at which they developed tumours was recorded. Some rats died without developing tumours and were recorded as right-censored at the time of death. The following R commands and output have been used to test whether rats given the drug have the same survival function as those given the placebo.

(20 Marks)

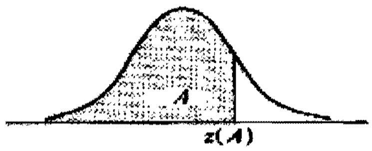
```
> survdiff(Surv(t,delta)~treat)
Call:
survdiff(formula = Surv(t, delta) ~ treat)

          N Observed Expected (O-E)^2/E (O-E)^2/V
treat=0 100       19     27.5      2.65       8.6
treat=1  50       21     12.5      5.86       8.6

 Chisq= 8.6  on 1 degrees of freedom, p= 0.00337


> coxph(Surv(t,delta)~treat)
Call:
coxph(formula = Surv(t, delta) ~ treat)


       coef exp(coef) se(coef)    z      p
treat 0.905      2.47    0.318 2.85 0.0044

Likelihood ratio test=7.97  on 1 df, p=0.00474  n= 150
```

What do you conclude?

Normal Distribution

Table C-1. Cumulative Probabilities of the Standard Normal Distribution.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$

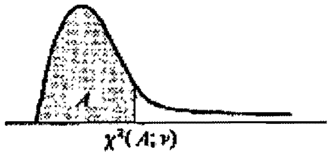


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Chi-Square Distribution

Table C-2. Percentiles of the χ^2 Distribution

Entry is $\chi^2(A;\ \nu)$ where $P\{\chi^2(\nu) \le \chi^2(A;\ \nu)\} = A$



	A									
ν	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Table C-3. Binomial Distribution

$$B(x;n,p) = \sum_{0 \le y \le x} b(y;n,p)$$

The values of $B(x;n,p)$ for $0.5 < p < 1.0$ are obtained by using the formula

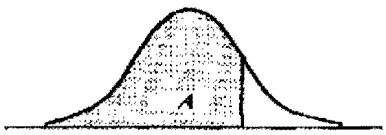
$$B(x;n,1-p) = 1 - B(n-1-x;n,p)$$

		p									
n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
5	0	0.774	0.590	0.444	0.328	0.237	0.168	0.116	0.078	0.050	0.031
	1	0.977	0.919	0.835	0.737	0.633	0.528	0.428	0.337	0.256	0.188
	2	0.999	0.991	0.973	0.942	0.896	0.837	0.765	0.683	0.593	0.500
	3	1.000	1.000	0.998	0.993	0.984	0.969	0.946	0.913	0.869	0.813
	4	1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.990	0.982	0.969
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	0	0.599	0.349	0.197	0.107	0.056	0.028	0.013	0.006	0.003	0.001
	1	0.914	0.736	0.544	0.376	0.244	0.149	0.086	0.046	0.023	0.011
	2	0.988	0.930	0.820	0.678	0.526	0.383	0.262	0.167	0.100	0.055
	3	0.999	0.987	0.950	0.879	0.776	0.650	0.514	0.382	0.266	0.172
	4	1.000	0.998	0.990	0.967	0.922	0.850	0.751	0.633	0.504	0.377
	5	1.000	1.000	0.999	0.994	0.980	0.953	0.905	0.834	0.738	0.623
	6	1.000	1.000	1.000	0.999	0.996	0.989	0.974	0.945	0.898	0.828
	7	1.000	1.000	1.000	1.000	1.000	0.998	0.995	0.988	0.973	0.945
	8	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.989
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0	0.463	0.206	0.087	0.035	0.013	0.005	0.002	0.000	0.000	0.000
	1	0.829	0.549	0.319	0.167	0.080	0.035	0.014	0.005	0.002	0.000
	2	0.964	0.816	0.604	0.398	0.236	0.127	0.062	0.027	0.011	0.004
	3	0.995	0.944	0.823	0.648	0.461	0.297	0.173	0.091	0.042	0.018
	4	0.999	0.987	0.938	0.836	0.686	0.515	0.352	0.217	0.120	0.059
	5	1.000	0.998	0.983	0.939	0.852	0.722	0.564	0.403	0.261	0.151
	6	1.000	1.000	0.996	0.982	0.943	0.869	0.755	0.610	0.452	0.304
	7	1.000	1.000	0.999	0.996	0.983	0.950	0.887	0.787	0.654	0.500
	8	1.000	1.000	1.000	0.999	0.996	0.985	0.958	0.905	0.818	0.696
	9	1.000	1.000	1.000	1.000	0.999	0.996	0.988	0.966	0.923	0.849
	10	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.991	0.975	0.941
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994	0.982
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Student's Distribution (t Distribution)

Table C-4 Percentiles of the t Distribution

Entry is $t(A;\ \nu)$ where $P\{t(\nu) \leq t(A;\ \nu)\} = A$



$t(A;\ \nu)$

ν	A = .60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table C-4 (Continued) Percentiles of the *t* Distribution

	A						
ν	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291