

UNIVERSITY OF SWAZILAND



MAIN EXAMINATION PAPER 2017/2018

TITLE OF PAPER : QUANTITATIVE METHODS IN DEMOGRAPHY

COURSE CODE : DEM 214

TIME ALLOWED : TWO (2) HOURS

INSTRUCTIONS : ANSWER QUESTION 1 AND ANY OTHER 2.

REQUIREMENTS : SCIENTIFIC CALCULATOR AND  
STATISTICAL TABLES.

### Question 1

- a. Using the information provided in the abstract below, define and identify the following
- i. Independent Variable [2 marks]
  - ii. Dependent Variable [2 marks]
  - iii. Data Source [2 marks]
  - iv. Sample size [2 marks]
  - v. Hypothesis [4 marks]

#### A B S T R A C T

In African countries including South Africa, the nexus between social cohesion and health remains under-researched. Using data from the 2012 South African social attitudes survey with a sample of 1988 adults in South Africa aged 18 years or older, we used the collective efficacy theory by Sampson and colleagues to examine the relationship between social cohesion and self-rated health in an African sample. We also examined how this relationship differed by race. Results from the multivariate analysis after adjusting for covariates suggested that adults in the highest tertile of social cohesion were more likely to report moderate or good health compared to those in the lowest tertile. Sub-group analysis provided no evidence that the relationship was moderated by race. These findings corroborate prior evidence that social cohesion is important for improving the health of adults.

- b. The abridged output below presents the results of a simple linear regression analysis of the relationship between current age of respondents (in single years) and total number of children ever born (CEB) among women in Malawi.

Number of observations= 8,444

Prob > F= 0.041

R-squared = 0.554

CEB	Coefficient	Standard error	P> t	[95% Confidence interval]
Age	0.201	0.003	0.49	0.197, 0.205
Constant	-3.054	0.062	0.00	-3.176, -2.931

- i. In a form of a mathematical equation, express the relationship between the two variables? [2 mark]
- ii. If the current age of the respondent is 45 years, predict what the woman's CEB [3 marks]
- iii. What is the strength of the relationship between the explanatory variable and the response variable? [3 marks]

[20 marks]

## Question 2

- a. A recent demographic survey in Swaziland revealed that about 22% of girls are pregnant before their 18th birthday. Using the binomial probability formula. Suppose that this survey included a random sample of 17 high school girls. Find the probability that:
- Six or more girls will be pregnant before her 18th birthday. [3 marks]
  - More than two but less than four girls will be pregnant before her 18th birthday. [3 marks]
- b. An immigration officer has been presented with a list of 24 persons coming into Swaziland through the Ngwenya border. Of the 24 immigrants, only 18 immigrants had the appropriate travel document. If the immigration officer decides to randomly select 4 persons for screening, what is the probability that:
- More than one, but less than 4 persons will have illegal travel documents? [3 marks]
  - More than 2 persons will have illegal travel documents [3 marks]
- c. The mean age at first marriage in Swaziland is assumed to be normally distributed with mean 21 years and variance 25. What is the probability that?
- Suppose that 23% of women in this sample are classified as having married late, what is the minimum age that woman has to be married to have married late? [3 marks]
  - Approximately what proportion of the women married years before the average. [3 marks]
- d. Among UNISWA students who hold part-time jobs during the 2018 academic year, the distribution of the time spent working per week is approximate normally distributed with a mean of 20.20 hours and standard deviation of 2.60 hours. Find the probability that the average time spent working per week for 18 randomly selected students who hold part-time jobs during the year is not within 1 hour of the population mean. [2 marks]

[20 marks]

### Question 3

- a. Briefly describe each of the following
- i. Confidence interval [1 marks]
  - ii. Type I error [1 marks]
  - iii. Type II errors [1 marks]
  - iv. Level of significance [1 marks]
  - v. Regression model [1 marks]
- b. Differentiate between the following
- i. a null and alternative hypothesis [3 marks]
  - ii. Directional and non-directional hypothesis [3 marks]
- c. In a 2007 Swaziland national survey, about 53% of young adults said reported that they have never had sex. In a recent random sample of 1,163 young adults, 48% of the young adults reported that they had never had sex. Find the p-value to test the hypothesis that the current percentage of young adults who have never had sex is different from 53%. What is your conclusion if the significance level is 5%? [3 marks]
- d. A recent national survey claims that the mean time to first marriage among young women is 26 years. A random sample of 26 young women selected from the country showed that the mean time to first marriage is 27 years with a standard deviation of 3 years. Assuming that the age at first marriage for these women have a normal distribution.
- i. Find the p-value for the test that the mean time to first marriage is different from 26 years [3 marks]
  - ii. Using a 1% level of significance, what would you conclude? [3 marks]

[20 marks]

#### Question 4

There is an outbreak of cholera in your community and this has raised concerns among policy makers and the international community. About 373 persons out of the 582 persons residing in the community are already infected (I). About 252 infected cases are among women (W). The ministry of health is aware that you are taking a course on quantitative methods in demography. Applying your knowledge to solve the problem in your community, find and interpret:

- a)  $p(I)$  (2 marks)
- b)  $p(W)$  (2 marks)
- c)  $p(W \cap I)$  (2 marks)
- d)  $p(W \cap I)^c$  (2 marks)
- e)  $p(W \cup I)$  (2 marks)
- f)  $p(W | I^c)$  (2 marks)
- g)  $p(W^c | I)$  (2 marks)
- h)  $p(I^c)$  (2 marks)
- i) Is  $p(W \text{ or } I) = p(W) + p(I)$ ? (2 marks)
- j) Is  $p(W \text{ and } I) = p(W) \cdot p(I)$ ? (2 marks)

**TOTAL [20 marks]**

## Standard Normal Probabilities

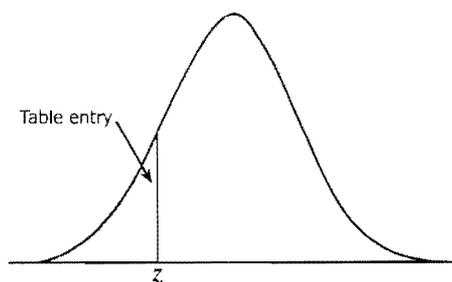


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.

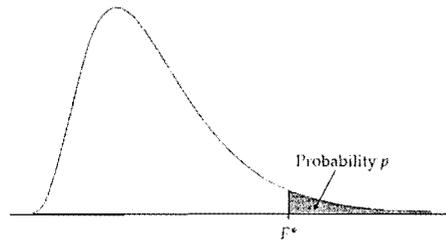


Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.

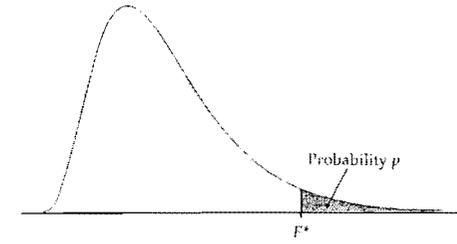


TABLE E										
F critical values										
Degrees of freedom in the numerator										
$p$	1	2	3	4	5	6	7	8	9	
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.06	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33

TABLE E												
F critical values (continued)												
Degrees of freedom in the numerator												
	10	12	15	20	25	30	40	50	60	1000		
1	.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
	.025	968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
	.010	6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
	.001	605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
2	.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
	.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
	.010	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
	.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
3	.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
	.025	14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
	.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
4	.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
	.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
	.025	8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
	.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
5	.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
	.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
	.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	.001	26.42	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
6	.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
	.025	5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
	.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
7	.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
	.025	4.76	4.67	4.57	4.47	4.40	4.36	4.28	4.25	4.23	4.20	4.15
	.010	6.62	6.47	6.31	6.16	6.06	5.99	5.86	5.82	5.82	5.74	5.66
	.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72

(Continued)

**TABLE E**  
F critical values (continued)

		Degrees of freedom in the numerator								
		1	2	3	4	5	6	7	8	9
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.010	11.26	8.65	7.39	7.01	6.63	6.37	6.18	6.03	5.91
	.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75

**TABLE E**  
F critical values (continued)

		Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
8	.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
	.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
	.025	4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
	.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
	.001	11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
9	.100	2.47	2.43	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
	.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
	.025	3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
	.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
	.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
10	.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
	.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
	.025	3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
	.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.09	4.00	3.92
	.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
11	.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
	.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
	.025	3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
	.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
	.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
12	.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
	.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
	.025	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73
	.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
	.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44
13	.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
	.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
	.025	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60
	.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
	.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
14	.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
	.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
	.025	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50
	.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
	.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
15	.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
	.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
	.025	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40
	.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
	.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
16	.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
	.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
	.025	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32
	.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
	.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.44	4.39	4.23	4.08
17	.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
	.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
	.025	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26
	.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
	.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87

(Continued)

<b>TABLE E</b>		<b>F critical values (continued)</b>									
		Degrees of freedom in the numerator									
<i>p</i>		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
		.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
		.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
		.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
		.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
	19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
		.050	4.36	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
		.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
		.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
		.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
	20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
		.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
		.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
		.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
		.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
	21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
		.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
		.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
		.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
		.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	
25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	
	.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.76	2.68	
	.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	
	.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	
26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	
	.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	
	.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	
	.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	
27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	
	.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	
	.010	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	
	.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	

<b>TABLE E</b>		<b>F critical values (continued)</b>											
		Degrees of freedom in the numerator											
<i>p</i>		10	12	15	20	25	30	40	50	60	120	1000	
Degrees of freedom in the denominator	18	.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
		.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
		.025	2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20
		.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
		.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
	19	.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
		.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
		.025	2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.30	2.27	2.20	2.14
		.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
		.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
	20	.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
		.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
		.025	2.77	2.68	2.57	2.46	2.40	2.35	2.29	2.25	2.22	2.16	2.09
		.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
		.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
	21	.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
		.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
		.025	2.73	2.64	2.53	2.42	2.36	2.31	2.25	2.21	2.18	2.11	2.05
		.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
		.001	4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
22	.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57	
	.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79	
	.025	2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01	
	.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32	
	.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17	
23	.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55	
	.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76	
	.025	2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98	
	.010	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27	
	.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08	
24	.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54	
	.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74	
	.025	2.64	2.54	2.44	2.33	2.26	2.21	2.15	2.11	2.08	2.01	1.94	
	.010	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22	
	.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99	
25	.100	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52	
	.050	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72	
	.025	2.61	2.51	2.41	2.30	2.23	2.18	2.12	2.08	2.05	1.98	1.91	
	.010	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18	
	.001	4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91	
26	.100	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51	
	.050	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70	
	.025	2.59	2.49	2.39	2.28	2.21	2.16	2.09	2.05	2.03	1.95	1.89	
	.010	3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14	
	.001	4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84	
27	.100	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50	
	.050	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68	
	.025	2.57	2.47	2.36	2.25	2.18	2.13	2.07	2.03	2.00	1.93	1.86	
	.010	3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11	
	.001	4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78	

(Continued)

**TABLE E**

*F* critical values (continued)

		Degrees of freedom in the numerator									
		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
		.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
		.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
		.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
		.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
		.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
		.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
		.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
		.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
		.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
	30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
		.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
		.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
		.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
		.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
	40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
		.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
		.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
		.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
		.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
	50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
		.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
		.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
		.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
		.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
	60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
		.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
		.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
		.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
		.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
	100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
		.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
		.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
		.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
		.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
	200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
		.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
		.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
		.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
		.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
	1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
		.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
		.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
		.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
		.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

**TABLE E**

*F* critical values (continued)

		Degrees of freedom in the numerator										
		10	12	15	20	25	30	40	50	60	120	1000
Degrees of freedom in the denominator	28	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
		2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
		2.55	2.45	2.34	2.23	2.16	2.11	2.05	2.01	1.98	1.91	1.84
		3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
		4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
		1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
		2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
		2.53	2.43	2.32	2.21	2.14	2.09	2.03	1.99	1.96	1.89	1.82
		3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
		4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
	30	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
		2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
		2.51	2.41	2.31	2.20	2.12	2.07	2.01	1.97	1.94	1.87	1.80
		2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
		4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
	40	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
		2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
		2.39	2.29	2.18	2.07	1.99	1.94	1.88	1.83	1.80	1.72	1.65
		2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
		3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
	50	1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
		2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
		2.32	2.22	2.11	1.99	1.92	1.87	1.80	1.75	1.72	1.64	1.56
		2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
		3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05
	60	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
		1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
		2.27	2.17	2.06	1.94	1.87	1.82	1.74	1.70	1.67	1.58	1.49
		2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
		3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
	100	1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
		1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
		2.18	2.08	1.97	1.85	1.77	1.71	1.64	1.59	1.56	1.46	1.36
		2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
		3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.84	1.64
	200	1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
		1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
		2.11	2.01	1.90	1.78	1.70	1.64	1.56	1.51	1.47	1.37	1.25
		2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
		3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
	1000	1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
		1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
		2.06	1.96	1.85	1.72	1.64	1.58	1.50	1.45	1.41	1.29	1.13
		2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
		2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22