

# **UNIVERSITY OF SWAZILAND**

## **RE SIT EXAMINATION PAPER 2016**

|                        |                                                                                                                        |
|------------------------|------------------------------------------------------------------------------------------------------------------------|
| <b>TITLE OF PAPER:</b> | <b>GENERAL LINEAR MODELS</b>                                                                                           |
| <b>COURSE CODE:</b>    | <b>STA 215</b>                                                                                                         |
| <b>TIME ALLOCATED:</b> | <b>2 (TWO) HOURS</b>                                                                                                   |
| <b>REQUIREMENTS:</b>   | <b>STATISTICAL TABLES AND CALCULATOR</b>                                                                               |
| <b>INSTRUCTION:</b>    | <b>ANSWER ANY 4 (FOUR) QUESTIONS OF YOUR CHOICE. ALL QUESTIONS CARRY THE MARKS AS INDICATED WITHIN THE PARENTHESIS</b> |

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.**

**QUESTION ONE****[2+3+6+7+7]**

A study was conducted on  $n = 10$  plots that had been fertilized in varies degrees to observe the changes in yield of corn. The following results were obtained from running the regression model,  $Y = \beta_0 + \beta_1 X_1 + \varepsilon_i$  using the study data.

**ANOVA**

| Model |            | Sum of Squares | Df | Mean Square | Fo    |
|-------|------------|----------------|----|-------------|-------|
| 1     | Regression | 964.587        | 1  | 964.587     | 1.478 |
|       | Residual   | 5221.413       | 8  | 652.677     |       |
|       | Total      | 8186.0000      | 9  |             |       |

**Coefficients**

| Model |          | Unstandardized Coefficients |            | Standardized Coefficients |
|-------|----------|-----------------------------|------------|---------------------------|
|       |          | B                           | Std. Error | Beta                      |
| 1     | Constant | 10.115                      | 3.108      |                           |
|       | x        | 2.784                       | 1.926      | .602                      |

- 1.1 Identify the dependent variable and the independent variable.
- 1.2 State the fitted regression line.
- 1.3 Perform the F test and clearly state the hypothesis and the conclusion.
- 1.4 Test  $\beta_0 = 10$  against  $\beta_0 \neq 10$  at  $\alpha = 0.05$
- 1.5 Test  $\beta_1 = 3$  against  $\beta_1 > 3$  at  $\alpha = 0.01$

**QUESTION TWO****[3+6+8+8]**

Some fairly used cars were displayed for sales in an automobile sales compound and the data below shows the prices and the year in which the car was first used.

| Price (000)       | 80   | 57   | 58   | 55   | 70   | 88   | 43   | 60   | 69   | 63   | 118  |
|-------------------|------|------|------|------|------|------|------|------|------|------|------|
| Year of first use | 2005 | 2007 | 2006 | 2006 | 2005 | 2004 | 2007 | 2006 | 2005 | 2005 | 2002 |

- 2.1 State which is the x and y variables respectively and why.
- 2.2 Using the observed data calculate the estimates of the slope and intercept parameter of the regression model.

- 2.3 Test for significance of regression. Predict the price of buying a car that was first used in 2010 and 2013.
- 2.4 Analyze the regression model adequacy using the Lack of fit method and comment on model adequacy.

**QUESTION THREE****[3+3+8+4+6+1]**

- 3.1 State 3 assumptions of any Multiple Regression Model.

A study was performed to investigate the shear strength of soil (Y) as it related to depth in feet ( $X_1$ ) and moisture content ( $X_2$ ). Ten observations were collected, and the following summary quantities obtained:

$$\begin{aligned}
 n &= 10 & \sum X_{i1} &= 223 & \sum X_{i2} &= 553 & \sum Y_i &= 1,916 \\
 \sum X_{i1}^2 &= 5,200.9 & \sum X_{i2}^2 &= 31,729 & \sum X_{i1} X_{i2} &= 12,352 & \sum X_{i1} Y_i &= 43,550.8 \\
 \sum X_{i2} Y_i &= 104,736.8 & \sum Y_i^2 &= 371,595.6
 \end{aligned}$$

- 3.2 Set up the least squares normal equations for the model
- $$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$
- 3.3 Estimate the parameters in the model in part (3.2) using the matrix method and give the final model for the shear strength of soil.
- 3.4 Estimate  $\sigma^2$
- 3.5 Construct the t-test on each regression coefficient using  $\alpha = 0.05$
- 3.6 What is the predicted strength when  $X_1 = 18$  feet and  $X_2 = 43\%$ ?

**QUESTION FOUR****[6+3+5+5+2+4]**

- 4.1 Compare the Cell Means Model and Factor Effects Model for single factor studies in terms of similarities and differences in the F test analysis of Variance.

In Design and Analysis of Experiments, 5th edition (John Wiley & Sons, 2001), D. C. Montgomery describes an experiment that determined the effect of four different types of tips in a hardness tester on the observed hardness of a metal alloy. Four specimens of the alloy were obtained, and each tip was tested once on each specimen, producing the following data:

| Type of Tip | Specimen |     |      |      |
|-------------|----------|-----|------|------|
|             | 1        | 2   | 3    | 4    |
| 1           | 9.3      | 9.4 | 9.6  | 10.0 |
| 2           | 9.4      | 9.3 | 9.8  | 9.9  |
| 3           | 9.2      | 9.4 | 9.5  | 9.7  |
| 4           | 9.7      | 9.6 | 10.0 | 10.2 |

- 4.2 Identify the dependent variable, factor studied and factor levels.
- 4.3 Complete the computation of the ANOVA table.
- 4.4 Use the 0.05 level in determining whether the four different Types of Tips could be equally effective. Clearly state all the steps in the test between the hypotheses and the conclusion.
- 4.5 Which Tip Type would you prefer? Explain.
- 4.6 Analyze the residuals from this experiment.

#### QUESTION FIVE

[8+6+11]

- 5.1 State the Cell Means Model for a single factor studies with all its important features.
- 5.2 Define the Factor effects Model for a two factor studies with all its important features.

Aircraft primer paints are applied to aluminium surfaces by two methods: dipping and spraying. The purpose of the primer is to improve paint adhesion, and some parts can be primed using either application method. The process engineering group responsible for this operation is interested in learning whether three different primers differ in their adhesion properties. A factorial experiment was performed to investigate the effect of paint primer type and application method on paint adhesion. For each combination of primer type and application method, three specimens were painted, then a finish paint was applied, and the adhesion force was measured. The data from the experiment is shown below.

| Primer Type | Dipping       | Spraying      |
|-------------|---------------|---------------|
| 1           | 4.0, 4.5, 4.3 | 5.4, 4.9, 5.6 |
| 2           | 5.6, 4.9, 5.4 | 5.8, 6.1, 6.3 |
| 3           | 3.8, 3.7, 4.0 | 5.5, 5.0, 5.0 |

- 5.3 Test the appropriate hypotheses and draw conclusions using the analysis of variance with  $\alpha = 0.05$ .

# STATISTICAL TABLES

Cumulative normal distribution

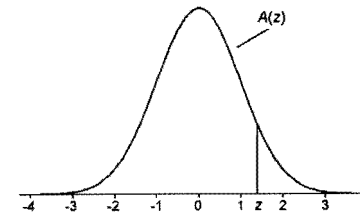
Critical values of the  $t$  distribution

Critical values of the  $F$  distribution

Critical values of the chi-squared distribution

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$  is the integral of the standardized normal distribution from  $-\infty$  to  $z$  (in other words, the area under the curve to the left of  $z$ ). It gives the probability of a normal random variable not being more than  $z$  standard deviations above its mean. Values of  $z$  of particular importance:

| $z$   | $A(z)$ |                                 |
|-------|--------|---------------------------------|
| 1.645 | 0.9500 | Lower limit of right 5% tail    |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail  |
| 2.326 | 0.9900 | Lower limit of right 1% tail    |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail  |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail  |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| $z$ | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 |        |        |        |        |        |        |        |

TABLE A.2  
t Distribution: Critical Values of  $t$

| Degrees of freedom | Two-tailed test:<br>One-tailed test: | Significance level |            |          |            |              |               |
|--------------------|--------------------------------------|--------------------|------------|----------|------------|--------------|---------------|
|                    |                                      | 10%<br>5%          | 5%<br>2.5% | 2%<br>1% | 1%<br>0.5% | 0.2%<br>0.1% | 0.1%<br>0.05% |
| 1                  |                                      | 6.314              | 12.706     | 31.821   | 63.657     | 318.309      | 636.619       |
| 2                  |                                      | 2.920              | 4.303      | 6.965    | 9.925      | 22.327       | 31.599        |
| 3                  |                                      | 2.353              | 3.182      | 4.541    | 5.841      | 10.215       | 12.924        |
| 4                  |                                      | 2.132              | 2.776      | 3.747    | 4.604      | 7.173        | 8.610         |
| 5                  |                                      | 2.015              | 2.571      | 3.365    | 4.032      | 5.893        | 6.869         |
| 6                  |                                      | 1.943              | 2.447      | 3.143    | 3.707      | 5.208        | 5.959         |
| 7                  |                                      | 1.894              | 2.365      | 2.998    | 3.499      | 4.785        | 5.408         |
| 8                  |                                      | 1.860              | 2.306      | 2.896    | 3.355      | 4.501        | 5.041         |
| 9                  |                                      | 1.833              | 2.262      | 2.821    | 3.250      | 4.297        | 4.781         |
| 10                 |                                      | 1.812              | 2.228      | 2.764    | 3.169      | 4.144        | 4.587         |
| 11                 |                                      | 1.796              | 2.201      | 2.718    | 3.106      | 4.025        | 4.437         |
| 12                 |                                      | 1.782              | 2.179      | 2.681    | 3.055      | 3.930        | 4.318         |
| 13                 |                                      | 1.771              | 2.160      | 2.650    | 3.012      | 3.852        | 4.221         |
| 14                 |                                      | 1.761              | 2.145      | 2.624    | 2.977      | 3.787        | 4.140         |
| 15                 |                                      | 1.753              | 2.131      | 2.602    | 2.947      | 3.733        | 4.073         |
| 16                 |                                      | 1.746              | 2.120      | 2.583    | 2.921      | 3.686        | 4.015         |
| 17                 |                                      | 1.740              | 2.110      | 2.567    | 2.898      | 3.646        | 3.965         |
| 18                 |                                      | 1.734              | 2.101      | 2.552    | 2.878      | 3.610        | 3.922         |
| 19                 |                                      | 1.729              | 2.093      | 2.539    | 2.861      | 3.579        | 3.883         |
| 20                 |                                      | 1.725              | 2.086      | 2.528    | 2.845      | 3.552        | 3.850         |
| 21                 |                                      | 1.721              | 2.080      | 2.518    | 2.831      | 3.527        | 3.819         |
| 22                 |                                      | 1.717              | 2.074      | 2.508    | 2.819      | 3.505        | 3.792         |
| 23                 |                                      | 1.714              | 2.069      | 2.500    | 2.807      | 3.485        | 3.768         |
| 24                 |                                      | 1.711              | 2.064      | 2.492    | 2.797      | 3.467        | 3.745         |
| 25                 |                                      | 1.708              | 2.060      | 2.485    | 2.787      | 3.450        | 3.725         |
| 26                 |                                      | 1.706              | 2.056      | 2.479    | 2.779      | 3.435        | 3.707         |
| 27                 |                                      | 1.703              | 2.052      | 2.473    | 2.771      | 3.421        | 3.690         |
| 28                 |                                      | 1.701              | 2.048      | 2.467    | 2.763      | 3.408        | 3.674         |
| 29                 |                                      | 1.699              | 2.045      | 2.462    | 2.756      | 3.396        | 3.659         |
| 30                 |                                      | 1.697              | 2.042      | 2.457    | 2.750      | 3.385        | 3.646         |
| 32                 |                                      | 1.694              | 2.037      | 2.449    | 2.738      | 3.365        | 3.622         |
| 34                 |                                      | 1.691              | 2.032      | 2.441    | 2.728      | 3.348        | 3.601         |
| 36                 |                                      | 1.688              | 2.028      | 2.434    | 2.719      | 3.333        | 3.582         |
| 38                 |                                      | 1.686              | 2.024      | 2.429    | 2.712      | 3.319        | 3.566         |
| 40                 |                                      | 1.684              | 2.021      | 2.423    | 2.704      | 3.307        | 3.551         |
| 42                 |                                      | 1.682              | 2.018      | 2.418    | 2.698      | 3.296        | 3.538         |
| 44                 |                                      | 1.680              | 2.015      | 2.414    | 2.692      | 3.286        | 3.526         |
| 46                 |                                      | 1.679              | 2.013      | 2.410    | 2.687      | 3.277        | 3.515         |
| 48                 |                                      | 1.677              | 2.011      | 2.407    | 2.682      | 3.269        | 3.505         |
| 50                 |                                      | 1.676              | 2.009      | 2.403    | 2.678      | 3.261        | 3.496         |
| 60                 |                                      | 1.671              | 2.000      | 2.390    | 2.660      | 3.232        | 3.460         |
| 70                 |                                      | 1.667              | 1.994      | 2.381    | 2.648      | 3.211        | 3.435         |
| 80                 |                                      | 1.664              | 1.990      | 2.374    | 2.639      | 3.195        | 3.416         |
| 90                 |                                      | 1.662              | 1.987      | 2.368    | 2.632      | 3.183        | 3.402         |
| 100                |                                      | 1.660              | 1.984      | 2.364    | 2.626      | 3.174        | 3.390         |
| 120                |                                      | 1.658              | 1.980      | 2.358    | 2.617      | 3.160        | 3.373         |
| 150                |                                      | 1.655              | 1.976      | 2.351    | 2.609      | 3.145        | 3.357         |
| 200                |                                      | 1.653              | 1.972      | 2.345    | 2.601      | 3.131        | 3.340         |
| 300                |                                      | 1.650              | 1.968      | 2.339    | 2.592      | 3.118        | 3.323         |
| 400                |                                      | 1.649              | 1.966      | 2.336    | 2.588      | 3.111        | 3.315         |
| 500                |                                      | 1.648              | 1.965      | 2.334    | 2.586      | 3.107        | 3.310         |
| 600                |                                      | 1.647              | 1.964      | 2.333    | 2.584      | 3.104        | 3.307         |
| ∞                  |                                      | 1.645              | 1.960      | 2.326    | 2.576      | 3.090        | 3.291         |

TABLE A.3

F Distribution: Critical Values of  $F$  (5% significance level)

| $v_1$ | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     | 12     | 14     | 16     | 18     | 20     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1     | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.36 | 246.46 | 247.32 | 248.01 |
| 2     | 18.51  | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  | 19.40  | 19.41  | 19.42  | 19.43  | 19.44  | 19.45  |
| 3     | 10.13  | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   | 8.79   | 8.74   | 8.71   | 8.69   | 8.67   | 8.66   |
| 4     | 7.71   | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   | 5.96   | 5.91   | 5.87   | 5.84   | 5.82   | 5.80   |
| 5     | 6.61   | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   | 4.74   | 4.68   | 4.64   | 4.60   | 4.58   | 4.56   |
| 6     | 5.99   | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   | 4.06   | 4.00   | 3.96   | 3.92   | 3.90   | 3.87   |
| 7     | 5.59   | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   | 3.64   | 3.57   | 3.53   | 3.49   | 3.47   | 3.44   |
| 8     | 5.32   | 4.46   | 4.07   | 3.84   | 3.69   | 3.58   | 3.50   | 3.44   | 3.39   | 3.35   | 3.28   | 3.24   | 3.20   | 3.17   | 3.15   |
| 9     | 5.12   | 4.26   | 3.86   | 3.63   | 3.48   | 3.37   | 3.29   | 3.23   | 3.18   | 3.14   | 3.07   | 3.03   | 2.99   | 2.96   | 2.94   |
| 10    | 4.96   | 4.10   | 3.71   | 3.48   | 3.33   | 3.22   | 3.14   | 3.07   | 3.02   | 2.98   | 2.91   | 2.86   | 2.83   | 2.80   | 2.77   |
| 11    | 4.84   | 3.98   | 3.59   | 3.36   | 3.20   | 3.09   | 3.01   | 2.95   | 2.90   | 2.85   | 2.79   | 2.74   | 2.70   | 2.67   | 2.65   |
| 12    | 4.75   | 3.89   | 3.49   | 3.26   | 3.11   | 3.00   | 2.91   | 2.85   | 2.80   | 2.75   | 2.69   | 2.64   | 2.60   | 2.57   | 2.54   |
| 13    | 4.67   | 3.81   | 3.41   | 3.18   | 3.03   | 2.92   | 2.83   | 2.77   | 2.71   | 2.67   | 2.60   | 2.55   | 2.51   | 2.48   | 2.46   |
| 14    | 4.60   | 3.74   | 3.34   | 3.11   | 2.96   | 2.85   | 2.76   | 2.70   | 2.65   | 2.60   | 2.53   | 2.48   | 2.44   | 2.41   | 2.39   |
| 15    | 4.54   | 3.68   | 3.29   | 3.06   | 2.90   | 2.79   | 2.71   | 2.64   | 2.59   | 2.54   | 2.48   | 2.42   | 2.38   | 2.35   | 2.33   |
| 16    | 4.49   | 3.63   | 3.24   | 3.01   | 2.85   | 2.74   | 2.66   | 2.59   | 2.54   | 2.49   | 2.42   | 2.37   | 2.33   | 2.30   | 2.28   |
| 17    | 4.45   | 3.59   | 3.20   | 2.96   | 2.81   | 2.70   | 2.61   | 2.55   | 2.49   | 2.45   | 2.38   | 2.33   | 2.29   | 2.26   | 2.23   |
| 18    | 4.41   | 3.55   | 3.16   | 2.93   | 2.77   | 2.66   | 2.58   | 2.51   | 2.46   | 2.41   | 2.34   | 2.29   | 2.25   | 2.22   | 2.19   |
| 19    | 4.38   | 3.52   | 3.13   | 2.90   | 2.74   | 2.63   | 2.54   | 2.48   | 2.42   | 2.38   | 2.31   | 2.26   | 2.21   | 2.18   | 2.16   |
| 20    | 4.35   | 3.49   | 3.10   | 2.87   | 2.71   | 2.60   | 2.51   | 2.45   | 2.39   | 2.35   | 2.28   | 2.22   | 2.18   | 2.15   | 2.12   |
| 21    | 4.32   | 3.47   | 3.07   | 2.84   | 2.68   | 2.57   | 2.49   | 2.42   | 2.37   | 2.32   | 2.25   | 2.20   | 2.16   | 2.12   | 2.10   |
| 22    | 4.30   | 3.44   | 3.05   | 2.82   | 2.66   | 2.55   | 2.46   | 2.40   | 2.34   | 2.30   | 2.23   | 2.17   | 2.13   | 2.10   | 2.07   |
| 23    | 4.28   | 3.42   | 3.03   | 2.80   | 2.64   | 2.53   | 2.44   | 2.37   | 2.32   | 2.27   | 2.20   | 2.15   | 2.11   | 2.08   | 2.05   |
| 24    | 4.26   | 3.40   | 3.01   | 2.78   | 2.62   | 2.51   | 2.42   | 2.36   | 2.30   | 2.25   | 2.18   | 2.13   | 2.09   | 2.05   | 2.03   |
| 25    | 4.24   | 3.39   | 2.99   | 2.76   | 2.60   | 2.49   | 2.40   | 2.34   | 2.28   | 2.24   | 2.16   | 2.11   | 2.07   | 2.04   | 2.01   |
| 26    | 4.22   | 3.37   | 2.98   | 2.74   | 2.59   | 2.47   | 2.39   | 2.32   | 2.27   | 2.22   | 2.15   | 2.09   | 2.05   | 2.02   | 1.99   |
| 27    | 4.21   | 3.35   | 2.96   | 2.73   | 2.57   | 2.46   | 2.37   | 2.31   | 2.25   | 2.20   | 2.13   | 2.08   | 2.04   | 2.00   | 1.97   |
| 28    | 4.20   | 3.34   | 2.95   | 2.71   | 2.56   | 2.45   | 2.36   | 2.29   | 2.24   | 2.19   | 2.12   | 2.06   | 2.02   | 1.99   | 1.96   |
| 29    | 4.18   | 3.33   | 2.93   | 2.70   | 2.55   | 2.43   | 2.35   | 2.28   | 2.22   | 2.18   | 2.10   | 2.05   | 2.01   | 1.97   | 1.94   |
| 30    | 4.17   | 3.32   | 2.92   | 2.69   | 2.53   | 2.42   | 2.33   | 2.27   | 2.21   | 2.16   | 2.09   | 2.04   | 1.99   | 1.96   | 1.93   |
| 35    | 4.12   | 3.27   | 2.87   | 2.64   | 2.49   | 2.37   | 2.29   | 2.22   | 2.16   | 2.11   | 2.04   | 1.99   | 1.94   | 1.91   | 1.88   |
| 40    | 4.08   | 3.23   | 2.84   | 2.61   | 2.45   | 2.34   | 2.25   | 2.18   | 2.12   | 2.08   | 2.00   | 1.95   | 1.90   | 1.87   | 1.84   |
| 50    | 4.03   | 3.18   | 2.79   | 2.56   | 2.40   | 2.29   | 2.20   | 2.13   | 2.07   | 2.03   | 1.95   | 1.89   | 1.85   | 1.81   | 1.78   |
| 60    | 4.00   | 3.15   | 2.76   | 2.53   | 2.37   | 2.25   | 2.17   | 2.10   | 2.04   | 1.99   | 1.92   | 1.86   | 1.82   | 1.78   | 1.75   |
| 70    | 3.98   | 3.13   | 2.74   | 2.50   | 2.35   | 2.23   | 2.14   | 2.07   | 2.02   | 1.97   | 1.89   | 1.84   | 1.79   | 1.75   | 1.72   |
| 80    | 3.96   | 3.11   | 2.72   | 2.49   | 2.33   | 2.21   | 2.13   | 2.06   | 2.00   | 1.95   | 1.88   | 1.82   | 1.77   | 1.73   | 1.70   |
| 90    | 3.95   | 3.10   | 2.71   | 2.47   | 2.32   | 2.20   | 2.11   | 2.04   | 1.99   | 1.94   | 1.86   | 1.80   | 1.76   | 1.72   | 1.69   |
| 100   | 3.94   | 3.09   | 2.70   | 2.46   | 2.31   | 2.19   | 2.10   | 2.03   | 1.97   | 1.93   | 1.85   | 1.79   | 1.75   | 1.71   | 1.68   |
| 120   | 3.92   | 3.07   | 2.68   | 2.45   | 2.29   | 2.18   | 2.09   | 2.02   | 1.96   | 1.91   | 1.83   | 1.78   | 1.73   | 1.69   | 1.66   |
| 150   | 3.90   | 3.06   | 2.66   | 2.43   | 2.27   | 2.16   | 2.07   | 2.00   | 1.94   | 1.89   | 1.82   | 1.76   | 1.71   | 1.67   | 1.64   |
| 200   | 3.89   | 3.04   | 2.65   | 2.42   | 2.26   | 2.14   | 2.06   | 1.98   | 1.93   | 1.88   | 1.80   | 1.74   | 1.69   | 1.66   | 1.62   |
| 250   | 3.88   | 3.03   | 2.64   | 2.41   | 2.25   | 2.13   | 2.05   | 1.98   | 1.92   | 1.87   | 1.79   | 1.73   | 1.68   | 1.65   | 1.61   |
| 300   | 3.87   | 3.03   | 2.63   | 2.40   | 2.24   | 2.13   | 2.04   | 1.97   | 1.91   | 1.86   | 1.78   | 1.72   | 1.68   | 1.64   | 1.61   |
| 400   | 3.86   | 3.02   | 2.63   | 2.39   | 2.24   | 2.12   | 2.03   | 1.96   | 1.90   | 1.85   | 1.78   | 1.72   | 1.67   | 1.63   | 1.60   |
| 500   | 3.86   | 3.01   | 2.62   | 2.39   | 2.23   | 2.12   | 2.03   | 1.96   | 1.90   | 1.85   | 1.77   | 1.71   | 1.66   | 1.62   | 1.59   |
| 600   | 3.86   | 3.01   | 2.62   | 2.39   | 2.23   | 2.11   | 2.02   | 1.95   | 1.90   | 1.85   | 1.77   | 1.71   | 1.66   | 1.62   | 1.59   |
| 750   | 3.85   | 3.01   | 2.62   | 2.38   | 2.23   | 2.11   | 2.02   | 1.95   | 1.89   | 1.84   | 1.77   | 1.70   | 1.66   | 1.62   | 1.58   |
| 1000  | 3.85   | 3.00   | 2.61   | 2.38   | 2.22   | 2.11   | 2.02   | 1.95   | 1.89   | 1.84   | 1.76   | 1.70   | 1.65   | 1.61   | 1.58   |

TABLE A.3 (continued)

F Distribution: Critical Values of F (5% significance level)

| $\nu_1$ | 25     | 30     | 35     | 40     | 50     | 60     | 75     | 100    | 150    | 200    |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1       | 249.26 | 250.10 | 250.69 | 251.14 | 251.77 | 252.20 | 252.62 | 253.04 | 253.46 | 253.68 |
| 2       | 19.46  | 19.46  | 19.47  | 19.47  | 19.48  | 19.48  | 19.48  | 19.49  | 19.49  | 19.49  |
| 3       | 8.63   | 8.62   | 8.60   | 8.59   | 8.58   | 8.57   | 8.56   | 8.55   | 8.54   | 8.54   |
| 4       | 5.77   | 5.75   | 5.73   | 5.72   | 5.70   | 5.69   | 5.68   | 5.66   | 5.65   | 5.65   |
| 5       | 4.52   | 4.50   | 4.48   | 4.46   | 4.44   | 4.43   | 4.42   | 4.41   | 4.39   | 4.39   |
| 6       | 3.83   | 3.81   | 3.79   | 3.77   | 3.75   | 3.74   | 3.73   | 3.71   | 3.70   | 3.69   |
| 7       | 3.40   | 3.38   | 3.36   | 3.34   | 3.32   | 3.30   | 3.29   | 3.27   | 3.26   | 3.25   |
| 8       | 3.11   | 3.08   | 3.06   | 3.04   | 3.02   | 3.01   | 2.99   | 2.97   | 2.96   | 2.95   |
| 9       | 2.89   | 2.86   | 2.84   | 2.83   | 2.80   | 2.79   | 2.77   | 2.76   | 2.74   | 2.73   |
| 10      | 2.73   | 2.70   | 2.68   | 2.66   | 2.64   | 2.62   | 2.60   | 2.59   | 2.57   | 2.56   |
| 11      | 2.60   | 2.57   | 2.55   | 2.53   | 2.51   | 2.49   | 2.47   | 2.46   | 2.44   | 2.43   |
| 12      | 2.50   | 2.47   | 2.44   | 2.43   | 2.40   | 2.38   | 2.37   | 2.35   | 2.33   | 2.32   |
| 13      | 2.41   | 2.38   | 2.36   | 2.34   | 2.31   | 2.30   | 2.28   | 2.26   | 2.24   | 2.23   |
| 14      | 2.34   | 2.31   | 2.28   | 2.27   | 2.24   | 2.22   | 2.21   | 2.19   | 2.17   | 2.16   |
| 15      | 2.28   | 2.25   | 2.22   | 2.20   | 2.18   | 2.16   | 2.14   | 2.12   | 2.10   | 2.10   |
| 16      | 2.23   | 2.19   | 2.17   | 2.15   | 2.12   | 2.11   | 2.09   | 2.07   | 2.05   | 2.04   |
| 17      | 2.18   | 2.15   | 2.12   | 2.10   | 2.08   | 2.06   | 2.04   | 2.02   | 2.00   | 1.99   |
| 18      | 2.14   | 2.11   | 2.08   | 2.06   | 2.04   | 2.02   | 2.00   | 1.98   | 1.96   | 1.95   |
| 19      | 2.11   | 2.07   | 2.05   | 2.03   | 2.00   | 1.98   | 1.96   | 1.94   | 1.92   | 1.91   |
| 20      | 2.07   | 2.04   | 2.01   | 1.99   | 1.97   | 1.95   | 1.93   | 1.91   | 1.89   | 1.88   |
| 21      | 2.05   | 2.01   | 1.98   | 1.96   | 1.94   | 1.92   | 1.90   | 1.88   | 1.86   | 1.84   |
| 22      | 2.02   | 1.98   | 1.96   | 1.94   | 1.91   | 1.89   | 1.87   | 1.85   | 1.83   | 1.82   |
| 23      | 2.00   | 1.96   | 1.93   | 1.91   | 1.88   | 1.86   | 1.84   | 1.82   | 1.80   | 1.79   |
| 24      | 1.97   | 1.94   | 1.91   | 1.89   | 1.86   | 1.84   | 1.82   | 1.80   | 1.78   | 1.77   |
| 25      | 1.96   | 1.92   | 1.89   | 1.87   | 1.84   | 1.82   | 1.80   | 1.78   | 1.76   | 1.75   |
| 26      | 1.94   | 1.90   | 1.87   | 1.85   | 1.82   | 1.80   | 1.78   | 1.76   | 1.74   | 1.73   |
| 27      | 1.92   | 1.88   | 1.86   | 1.84   | 1.81   | 1.79   | 1.76   | 1.74   | 1.72   | 1.71   |
| 28      | 1.91   | 1.87   | 1.84   | 1.82   | 1.79   | 1.77   | 1.75   | 1.73   | 1.70   | 1.69   |
| 29      | 1.89   | 1.85   | 1.83   | 1.81   | 1.77   | 1.75   | 1.73   | 1.71   | 1.69   | 1.67   |
| 30      | 1.88   | 1.84   | 1.81   | 1.79   | 1.76   | 1.74   | 1.72   | 1.70   | 1.67   | 1.66   |
| 35      | 1.82   | 1.79   | 1.76   | 1.74   | 1.70   | 1.68   | 1.66   | 1.63   | 1.61   | 1.60   |
| 40      | 1.78   | 1.74   | 1.72   | 1.69   | 1.66   | 1.64   | 1.61   | 1.59   | 1.56   | 1.55   |
| 50      | 1.73   | 1.69   | 1.66   | 1.63   | 1.60   | 1.58   | 1.55   | 1.52   | 1.50   | 1.48   |
| 60      | 1.69   | 1.65   | 1.62   | 1.59   | 1.56   | 1.53   | 1.51   | 1.48   | 1.45   | 1.44   |
| 70      | 1.66   | 1.62   | 1.59   | 1.57   | 1.53   | 1.50   | 1.48   | 1.45   | 1.42   | 1.40   |
| 80      | 1.64   | 1.60   | 1.57   | 1.54   | 1.51   | 1.48   | 1.45   | 1.43   | 1.39   | 1.38   |
| 90      | 1.63   | 1.59   | 1.55   | 1.53   | 1.49   | 1.46   | 1.44   | 1.41   | 1.38   | 1.36   |
| 100     | 1.62   | 1.57   | 1.54   | 1.52   | 1.48   | 1.45   | 1.42   | 1.39   | 1.36   | 1.34   |
| 120     | 1.60   | 1.55   | 1.52   | 1.50   | 1.46   | 1.43   | 1.40   | 1.37   | 1.33   | 1.32   |
| 150     | 1.58   | 1.54   | 1.50   | 1.48   | 1.44   | 1.41   | 1.38   | 1.34   | 1.31   | 1.29   |
| 200     | 1.56   | 1.52   | 1.48   | 1.46   | 1.41   | 1.39   | 1.35   | 1.32   | 1.28   | 1.26   |
| 250     | 1.55   | 1.50   | 1.47   | 1.44   | 1.40   | 1.37   | 1.34   | 1.31   | 1.27   | 1.25   |
| 300     | 1.54   | 1.50   | 1.46   | 1.43   | 1.39   | 1.36   | 1.33   | 1.30   | 1.26   | 1.23   |
| 400     | 1.53   | 1.49   | 1.45   | 1.42   | 1.38   | 1.35   | 1.32   | 1.28   | 1.24   | 1.22   |
| 500     | 1.53   | 1.48   | 1.45   | 1.42   | 1.38   | 1.35   | 1.31   | 1.28   | 1.23   | 1.21   |
| 600     | 1.52   | 1.48   | 1.44   | 1.41   | 1.37   | 1.34   | 1.31   | 1.27   | 1.23   | 1.20   |
| 750     | 1.52   | 1.47   | 1.44   | 1.41   | 1.37   | 1.34   | 1.30   | 1.26   | 1.22   | 1.20   |
| 1000    | 1.52   | 1.47   | 1.43   | 1.41   | 1.36   | 1.33   | 1.30   | 1.26   | 1.22   | 1.19   |

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

| $\nu_1$ | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      | 12      | 14      | 16      | 18      | 20      |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\nu_1$ | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6106.32 | 6142.67 | 6170.10 | 6191.53 | 6208.73 |
| 2       | 98.50   | 99.00   | 99.17   | 99.25   | 99.30   | 99.33   | 99.36   | 99.37   | 99.39   | 99.40   | 99.42   | 99.43   | 99.44   | 99.44   | 99.45   |
| 3       | 34.12   | 30.82   | 29.46   | 28.71   | 28.24   | 27.91   | 27.67   | 27.49   | 27.35   | 27.23   | 27.05   | 26.92   | 26.83   | 26.75   | 26.69   |
| 4       | 21.20   | 18.00   | 16.69   | 15.98   | 15.52   | 15.21   | 14.98   | 14.80   | 14.66   | 14.55   | 14.37   | 14.25   | 14.15   | 14.08   | 14.02   |
| 5       | 16.26   | 13.27   | 12.06   | 11.39   | 10.97   | 10.67   | 10.46   | 10.29   | 10.16   | 10.05   | 9.89    | 9.77    | 9.68    | 9.61    | 9.55    |
| 6       | 13.75   | 10.92   | 9.78    | 9.15    | 8.75    | 8.47    | 8.26    | 8.10    | 7.98    | 7.87    | 7.72    | 7.60    | 7.52    | 7.45    | 7.40    |
| 7       | 12.25   | 9.55    | 8.45    | 7.85    | 7.46    | 7.19    | 6.99    | 6.84    | 6.72    | 6.62    | 6.47    | 6.36    | 6.28    | 6.21    | 6.16    |
| 8       | 11.26   | 8.65    | 7.59    | 7.01    | 6.63    | 6.37    | 6.18    | 6.03    | 5.91    | 5.81    | 5.67    | 5.56    | 5.48    | 5.41    | 5.36    |
| 9       | 10.56   | 8.02    | 6.99    | 6.42    | 6.06    | 5.80    | 5.61    | 5.47    | 5.35    | 5.26    | 5.11    | 5.01    | 4.92    | 4.86    | 4.81    |
| 10      | 10.04   | 7.56    | 6.55    | 5.99    | 5.64    | 5.39    | 5.20    | 5.06    | 4.94    | 4.85    | 4.71    | 4.60    | 4.52    | 4.46    | 4.41    |
| 11      | 9.65    | 7.21    | 6.22    | 5.67    | 5.32    | 5.07    | 4.89    | 4.74    | 4.63    | 4.54    | 4.40    | 4.29    | 4.21    | 4.15    | 4.10    |
| 12      | 9.33    | 6.93    | 5.95    | 5.41    | 5.06    | 4.82    | 4.64    | 4.50    | 4.39    | 4.30    | 4.16    | 4.05    | 3.97    | 3.91    | 3.86    |
| 13      | 9.07    | 6.70    | 5.74    | 5.21    | 4.86    | 4.62    | 4.44    | 4.30    | 4.19    | 4.10    | 3.96    | 3.86    | 3.78    | 3.72    | 3.66    |
| 14      | 8.86    | 6.51    | 5.56    | 5.04    | 4.69    | 4.46    | 4.28    | 4.14    | 4.03    | 3.94    | 3.80    | 3.70    | 3.62    | 3.56    | 3.51    |
| 15      | 8.68    | 6.36    | 5.42    | 4.89    | 4.56    | 4.32    | 4.14    | 4.00    | 3.89    | 3.80    | 3.67    | 3.56    | 3.49    | 3.42    | 3.37    |
| 16      | 8.53    | 6.23    | 5.29    | 4.77    | 4.44    | 4.20    | 4.03    | 3.89    | 3.78    | 3.69    | 3.55    | 3.45    | 3.37    | 3.31    | 3.26    |
| 17      | 8.40    | 6.11    | 5.18    | 4.67    | 4.34    | 4.10    | 3.93    | 3.79    | 3.68    | 3.59    | 3.46    | 3.35    | 3.27    | 3.21    | 3.16    |
| 18      | 8.29    | 6.01    | 5.09    | 4.58    | 4.25    | 4.01    | 3.84    | 3.71    | 3.60    | 3.51    | 3.37    | 3.27    | 3.19    | 3.13    | 3.08    |
| 19      | 8.18    | 5.93    | 5.01    | 4.50    | 4.17    | 3.94    | 3.77    | 3.63    | 3.52    | 3.43    | 3.30    | 3.19    | 3.12    | 3.05    | 3.00    |
| 20      | 8.10    | 5.85    | 4.94    | 4.43    | 4.10    | 3.87    | 3.70    | 3.56    | 3.46    | 3.37    | 3.23    | 3.13    | 3.05    | 2.99    | 2.94    |
| 21      | 8.02    | 5.78    | 4.87    | 4.37    | 4.04    | 3.81    | 3.64    | 3.51    | 3.40    | 3.31    | 3.17    | 3.07    | 2.99    | 2.93    | 2.88    |
| 22      | 7.95    | 5.72    | 4.82    | 4.31    | 3.99    | 3.76    | 3.59    | 3.45    | 3.35    | 3.26    | 3.12    | 3.02    | 2.94    | 2.88    | 2.83    |
| 23      | 7.88    | 5.66    | 4.76    | 4.26    | 3.94    | 3.71    | 3.54    | 3.41    | 3.30    | 3.21    | 3.07    | 2.97    | 2.89    | 2.83    | 2.78    |
| 24      | 7.82    | 5.61    | 4.72    | 4.22    | 3.90    | 3.67    | 3.50    | 3.36    | 3.26    | 3.17    | 3.03    | 2.93    | 2.85    | 2.79    | 2.74    |
| 25      | 7.77    | 5.57    | 4.68    | 4.18    | 3.85    | 3.63    | 3.46    | 3.32    | 3.22    | 3.13    | 2.99    | 2.89    | 2.81    | 2.75    | 2.70    |
| 26      | 7.72    | 5.53    | 4.64    | 4.14    | 3.82    | 3.59    | 3.42    | 3.29    | 3.18    | 3.09    | 2.96    | 2.86    | 2.78    | 2.72    | 2.66    |
| 27      | 7.68    | 5.49    | 4.60    | 4.11    | 3.78    | 3.56    | 3.39    | 3.26    | 3.15    | 3.06    | 2.93    | 2.82    | 2.75    | 2.68    | 2.63    |
| 28      | 7.64    | 5.45    | 4.57    | 4.07    | 3.75    | 3.53    | 3.36    | 3.23    | 3.12    | 3.03    | 2.90    | 2.79    | 2.72    | 2.65    | 2.60    |
| 29      | 7.60    | 5.42    | 4.54    | 4.04    | 3.73    | 3.50    | 3.33    | 3.20    | 3.09    | 3.00    | 2.87    | 2.77    | 2.69    | 2.63    | 2.57    |
| 30      | 7.56    | 5.39    | 4.51    | 4.02    | 3.70    | 3.47    | 3.30    | 3.17    | 3.07    | 2.98    | 2.84    | 2.74    | 2.66    | 2.60    | 2.55    |
| 35      | 7.42    | 5.27    | 4.40    | 3.91    | 3.59    | 3.37    | 3.20    | 3.07    | 2.96    | 2.88    | 2.74    | 2.64    | 2.56    | 2.50    | 2.44    |
| 40      | 7.31    | 5.18    | 4.31    | 3.83    | 3.51    | 3.29    | 3.12    | 2.99    | 2.89    | 2.80    | 2.66    | 2.56    | 2.48    | 2.42    | 2.37    |
| 50      | 7.17    | 5.06    | 4.20    | 3.72    | 3.41    | 3.19    | 3.02    | 2.89    | 2.78    | 2.70    | 2.56    | 2.46    | 2.38    | 2.32    | 2.27    |
| 60      | 7.08    | 4.98    | 4.13    | 3.65    | 3.34    | 3.12    | 2.95    | 2.82    | 2.72    | 2.63    | 2.50    | 2.39    | 2.31    | 2.25    | 2.20    |
| 70      | 7.01    | 4.92    | 4.07    | 3.60    | 3.29    | 3.07    | 2.91    | 2.78    | 2.67    | 2.59    | 2.45    | 2.35    | 2.27    | 2.20    | 2.15    |
| 80      | 6.96    | 4.88    | 4.04    | 3.56    | 3.26    | 3.04    | 2.87    | 2.74    | 2.64    | 2.55    | 2.42    | 2.31    | 2.23    | 2.17    | 2.12    |
| 90      | 6.93    | 4.85    | 4.01    | 3.53    | 3.23    | 3.01    | 2.84    | 2.72    | 2.61    | 2.52    | 2.39    | 2.29    | 2.21    | 2.14    | 2.09    |
| 100     | 6.90    | 4.82    | 3.98    | 3.51    | 3.21    | 2.99    | 2.82    | 2.69    | 2.59    | 2.50    | 2.37    | 2.27    | 2.19    | 2.12    | 2.07    |
| 120     | 6.85    | 4.79    | 3.95    | 3.48    | 3.17    | 2.96    | 2.79    | 2.66    | 2.56    | 2.47    | 2.34    | 2.23    | 2.15    | 2.09    | 2.03    |
| 150     | 6.81    | 4.75    | 3.91    | 3.45    | 3.14    | 2.92    | 2.76    | 2.63    | 2.53    | 2.44    | 2.31    | 2.20    | 2.12    | 2.06    | 2.00    |
| 200     | 6.76    | 4.71    | 3.88    | 3.41    | 3.11    | 2.89    | 2.73    | 2.60    | 2.50    | 2.41    | 2.27    | 2.17    | 2.09    | 2.03    | 1.97    |
| 250     | 6.74    | 4.69    | 3.86    | 3.40    | 3.09    | 2.87    | 2.71    | 2.58    | 2.48    | 2.39    | 2.26    | 2.15    | 2.07    | 2.01    | 1.95    |
| 300     | 6.72    | 4.68    | 3.85    | 3.38    | 3.08    | 2.86    | 2.70    | 2.57    | 2.47    | 2.38    | 2.24    | 2.14    | 2.06    | 1.99    | 1.94    |
| 400     | 6.70    | 4.66    | 3.83    | 3.37    | 3.06    | 2.85    | 2.68    | 2.56    | 2.45    | 2.37    | 2.23    | 2.13    | 2.05    | 1.98    | 1.92    |
| 500     | 6.69    | 4.65    | 3.82    | 3.36    | 3.05    | 2.84    | 2.68    | 2.55    | 2.44    | 2.36    | 2.22    | 2.12    | 2.04    | 1.97    | 1.92    |
| 600     | 6.68    | 4.64    | 3.81    | 3.35    | 3.05    | 2.83    | 2.67    | 2.54    | 2.44    | 2.35    | 2.21    | 2.11    | 2.03    | 1.96    | 1.91    |
| 750     | 6.67    | 4.63    | 3.81    | 3.34    | 3.04    | 2.83    | 2.66    | 2.53    | 2.43    | 2.34    | 2.21    | 2.11    | 2.02    | 1.96    | 1.90    |
| 1000    | 6.66    | 4.63    | 3.80    | 3.34    | 3.04    | 2.82    | 2.66    | 2.53    | 2.43    | 2.34    | 2.20    | 2.10    | 2.02    | 1.95    | 1.90    |

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

| $v_1$ | 25      | 30      | 35      | 40      | 50      | 60      | 75      | 100     | 150     | 200     |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1     | 6239.83 | 6260.65 | 6275.57 | 6286.78 | 6302.52 | 6313.03 | 6323.56 | 6334.11 | 6344.68 | 6349.97 |
| 2     | 99.46   | 99.47   | 99.47   | 99.47   | 99.48   | 99.48   | 99.49   | 99.49   | 99.49   | 99.49   |
| 3     | 26.58   | 26.50   | 26.45   | 26.41   | 26.35   | 26.32   | 26.28   | 26.24   | 26.20   | 26.18   |
| 4     | 13.91   | 13.84   | 13.79   | 13.75   | 13.69   | 13.65   | 13.61   | 13.58   | 13.54   | 13.52   |
| 5     | 9.45    | 9.38    | 9.33    | 9.29    | 9.24    | 9.20    | 9.17    | 9.13    | 9.09    | 9.08    |
| 6     | 7.30    | 7.23    | 7.18    | 7.14    | 7.09    | 7.06    | 7.02    | 6.99    | 6.95    | 6.93    |
| 7     | 6.06    | 5.99    | 5.94    | 5.91    | 5.86    | 5.82    | 5.79    | 5.75    | 5.72    | 5.70    |
| 8     | 5.26    | 5.20    | 5.15    | 5.12    | 5.07    | 5.03    | 5.00    | 4.96    | 4.93    | 4.91    |
| 9     | 4.71    | 4.65    | 4.60    | 4.57    | 4.52    | 4.48    | 4.45    | 4.41    | 4.38    | 4.36    |
| 10    | 4.31    | 4.25    | 4.20    | 4.17    | 4.12    | 4.08    | 4.05    | 4.01    | 3.98    | 3.96    |
| 11    | 4.01    | 3.94    | 3.89    | 3.86    | 3.81    | 3.78    | 3.74    | 3.71    | 3.67    | 3.66    |
| 12    | 3.76    | 3.70    | 3.65    | 3.62    | 3.57    | 3.54    | 3.50    | 3.47    | 3.43    | 3.41    |
| 13    | 3.57    | 3.51    | 3.46    | 3.43    | 3.38    | 3.34    | 3.31    | 3.27    | 3.24    | 3.22    |
| 14    | 3.41    | 3.35    | 3.30    | 3.27    | 3.22    | 3.18    | 3.15    | 3.11    | 3.08    | 3.06    |
| 15    | 3.28    | 3.21    | 3.17    | 3.13    | 3.08    | 3.05    | 3.01    | 2.98    | 2.94    | 2.92    |
| 16    | 3.16    | 3.10    | 3.05    | 3.02    | 2.97    | 2.93    | 2.90    | 2.86    | 2.83    | 2.81    |
| 17    | 3.07    | 3.00    | 2.96    | 2.92    | 2.87    | 2.83    | 2.80    | 2.76    | 2.73    | 2.71    |
| 18    | 2.98    | 2.92    | 2.87    | 2.84    | 2.78    | 2.75    | 2.71    | 2.68    | 2.64    | 2.62    |
| 19    | 2.91    | 2.84    | 2.80    | 2.76    | 2.71    | 2.67    | 2.64    | 2.60    | 2.57    | 2.55    |
| 20    | 2.84    | 2.78    | 2.73    | 2.69    | 2.64    | 2.61    | 2.57    | 2.54    | 2.50    | 2.48    |
| 21    | 2.79    | 2.72    | 2.67    | 2.64    | 2.58    | 2.55    | 2.51    | 2.48    | 2.44    | 2.42    |
| 22    | 2.73    | 2.67    | 2.62    | 2.58    | 2.53    | 2.50    | 2.46    | 2.42    | 2.38    | 2.36    |
| 23    | 2.69    | 2.62    | 2.57    | 2.54    | 2.48    | 2.45    | 2.41    | 2.37    | 2.34    | 2.32    |
| 24    | 2.64    | 2.58    | 2.53    | 2.49    | 2.44    | 2.40    | 2.37    | 2.33    | 2.29    | 2.27    |
| 25    | 2.60    | 2.54    | 2.49    | 2.45    | 2.40    | 2.36    | 2.33    | 2.29    | 2.25    | 2.23    |
| 26    | 2.57    | 2.50    | 2.45    | 2.42    | 2.36    | 2.33    | 2.29    | 2.25    | 2.21    | 2.19    |
| 27    | 2.54    | 2.47    | 2.42    | 2.38    | 2.33    | 2.29    | 2.26    | 2.22    | 2.18    | 2.16    |
| 28    | 2.51    | 2.44    | 2.39    | 2.35    | 2.30    | 2.26    | 2.23    | 2.19    | 2.15    | 2.13    |
| 29    | 2.48    | 2.41    | 2.36    | 2.33    | 2.27    | 2.23    | 2.20    | 2.16    | 2.12    | 2.10    |
| 30    | 2.45    | 2.39    | 2.34    | 2.30    | 2.25    | 2.21    | 2.17    | 2.13    | 2.09    | 2.07    |
| 35    | 2.35    | 2.28    | 2.23    | 2.19    | 2.14    | 2.10    | 2.06    | 2.02    | 1.98    | 1.96    |
| 40    | 2.27    | 2.20    | 2.15    | 2.11    | 2.06    | 2.02    | 1.98    | 1.94    | 1.90    | 1.87    |
| 50    | 2.17    | 2.10    | 2.05    | 2.01    | 1.95    | 1.91    | 1.87    | 1.82    | 1.78    | 1.76    |
| 60    | 2.10    | 2.03    | 1.98    | 1.94    | 1.88    | 1.84    | 1.79    | 1.75    | 1.70    | 1.68    |
| 70    | 2.05    | 1.98    | 1.93    | 1.89    | 1.83    | 1.78    | 1.74    | 1.70    | 1.65    | 1.62    |
| 80    | 2.01    | 1.94    | 1.89    | 1.85    | 1.79    | 1.75    | 1.70    | 1.65    | 1.61    | 1.58    |
| 90    | 1.99    | 1.92    | 1.86    | 1.82    | 1.76    | 1.72    | 1.67    | 1.62    | 1.57    | 1.55    |
| 100   | 1.97    | 1.89    | 1.84    | 1.80    | 1.74    | 1.69    | 1.65    | 1.60    | 1.55    | 1.52    |
| 120   | 1.93    | 1.86    | 1.81    | 1.76    | 1.70    | 1.66    | 1.61    | 1.56    | 1.51    | 1.48    |
| 150   | 1.90    | 1.83    | 1.77    | 1.73    | 1.66    | 1.62    | 1.57    | 1.52    | 1.46    | 1.43    |
| 200   | 1.87    | 1.79    | 1.74    | 1.69    | 1.63    | 1.58    | 1.53    | 1.48    | 1.42    | 1.39    |
| 250   | 1.85    | 1.77    | 1.72    | 1.67    | 1.61    | 1.56    | 1.51    | 1.46    | 1.40    | 1.36    |
| 300   | 1.84    | 1.76    | 1.70    | 1.66    | 1.59    | 1.55    | 1.50    | 1.44    | 1.38    | 1.35    |
| 400   | 1.82    | 1.75    | 1.69    | 1.64    | 1.58    | 1.53    | 1.48    | 1.42    | 1.36    | 1.32    |
| 500   | 1.81    | 1.74    | 1.68    | 1.63    | 1.57    | 1.52    | 1.47    | 1.41    | 1.34    | 1.31    |
| 600   | 1.80    | 1.73    | 1.67    | 1.63    | 1.56    | 1.51    | 1.46    | 1.40    | 1.34    | 1.30    |
| 750   | 1.80    | 1.72    | 1.66    | 1.62    | 1.55    | 1.50    | 1.45    | 1.39    | 1.33    | 1.29    |
| 1000  | 1.79    | 1.72    | 1.66    | 1.61    | 1.54    | 1.50    | 1.44    | 1.38    | 1.32    | 1.28    |

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

| $v_1$ | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      | 12      | 14      | 16      | 18      | 20      |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $p_1$ |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| 1     | 4.05e05 | 5.00e05 | 5.40e05 | 5.62e05 | 5.76e05 | 5.86e05 | 5.93e05 | 5.98e05 | 6.02e05 | 6.06e05 | 6.11e05 | 6.14e05 | 6.17e05 | 6.19e05 | 6.21e05 |
| 2     | 998.50  | 999.00  | 999.17  | 999.25  | 999.30  | 999.33  | 999.36  | 999.37  | 999.39  | 999.40  | 999.42  | 999.43  | 999.44  | 999.44  | 999.45  |
| 3     | 167.03  | 148.50  | 141.11  | 137.10  | 134.58  | 132.85  | 131.58  | 130.62  | 129.86  | 129.25  | 128.32  | 127.64  | 127.14  | 126.74  | 126.42  |
| 4     | 74.14   | 61.25   | 56.18   | 53.44   | 51.71   | 50.53   | 49.66   | 49.00   | 48.47   | 48.05   | 47.41   | 46.95   | 46.60   | 46.32   | 46.10   |
| 5     | 47.18   | 37.12   | 33.20   | 31.09   | 29.75   | 28.83   | 28.16   | 27.65   | 27.24   | 26.92   | 26.42   | 26.06   | 25.78   | 25.57   | 25.39   |
| 6     | 35.51   | 27.00   | 23.70   | 21.92   | 20.80   | 20.03   | 19.46   | 19.03   | 18.69   | 18.41   | 17.99   | 17.68   | 17.45   | 17.27   | 17.12   |
| 7     | 29.25   | 21.69   | 18.77   | 17.20   | 16.21   | 15.52   | 15.02   | 14.63   | 14.33   | 14.08   | 13.71   | 13.43   | 13.23   | 13.06   | 12.93   |
| 8     | 25.41   | 18.49   | 15.83   | 14.39   | 13.48   | 12.86   | 12.40   | 12.05   | 11.77   | 11.54   | 11.19   | 10.94   | 10.75   | 10.60   | 10.48   |
| 9     | 22.86   | 16.39   | 13.90   | 12.56   | 11.71   | 11.13   | 10.70   | 10.37   | 10.11   | 9.89    | 9.57    | 9.33    | 9.15    | 9.01    | 8.90    |
| 10    | 21.04   | 14.91   | 12.55   | 11.28   | 10.48   | 9.93    | 9.52    | 9.20    | 8.96    | 8.75    | 8.45    | 8.22    | 8.05    | 7.91    | 7.80    |
| 11    | 19.69   | 13.81   | 11.56   | 10.35   | 9.58    | 9.05    | 8.66    | 8.35    | 8.12    | 7.92    | 7.63    | 7.41    | 7.24    | 7.11    | 7.01    |
| 12    | 18.64   | 12.97   | 10.80   | 9.63    | 8.89    | 8.38    | 8.00    | 7.71    | 7.48    | 7.29    | 7.00    | 6.79    | 6.63    | 6.51    | 6.40    |
| 13    | 17.82   | 12.31   | 10.21   | 9.07    | 8.35    | 7.86    | 7.49    | 7.21    | 6.98    | 6.80    | 6.52    | 6.31    | 6.16    | 6.03    | 5.93    |
| 14    | 17.14   | 11.78   | 9.73    | 8.62    | 7.92    | 7.44    | 7.08    | 6.80    | 6.58    | 6.40    | 6.13    | 5.93    | 5.78    | 5.66    | 5.56    |
| 15    | 16.59   | 11.34   | 9.34    | 8.25    | 7.57    | 7.09    | 6.74    | 6.47    | 6.26    | 6.08    | 5.81    | 5.62    | 5.46    | 5.35    | 5.25    |
| 16    | 16.12   | 10.97   | 9.01    | 7.94    | 7.27    | 6.80    | 6.46    | 6.19    | 5.98    | 5.81    | 5.55    | 5.35    | 5.20    | 5.09    | 4.99    |
| 17    | 15.72   | 10.66   | 8.73    | 7.68    | 7.02    | 6.56    | 6.22    | 5.96    | 5.75    | 5.58    | 5.32    | 5.13    | 4.99    | 4.87    | 4.78    |
| 18    | 15.38   | 10.39   | 8.49    | 7.46    | 6.81    | 6.35    | 6.02    | 5.76    | 5.56    | 5.39    | 5.13    | 4.94    | 4.80    | 4.68    | 4.59    |
| 19    | 15.08   | 10.16   | 8.28    | 7.27    | 6.62    | 6.18    | 5.85    | 5.59    | 5.39    | 5.22    | 4.97    | 4.78    | 4.64    | 4.52    | 4.43    |
| 20    | 14.82   | 9.95    | 8.10    | 7.10    | 6.46    | 6.02    | 5.69    | 5.44    | 5.24    | 5.08    | 4.82    | 4.64    | 4.49    | 4.38    | 4.29    |
| 21    | 14.59   | 9.77    | 7.94    | 6.95    | 6.32    | 5.88    | 5.56    | 5.31    | 5.11    | 4.95    | 4.70    | 4.51    | 4.37    | 4.26    | 4.17    |
| 22    | 14.38   | 9.61    | 7.80    | 6.81    | 6.19    | 5.76    | 5.44    | 5.19    | 4.99    | 4.83    | 4.58    | 4.40    | 4.26    | 4.15    | 4.06    |
| 23    | 14.20   | 9.47    | 7.67    | 6.70    | 6.08    | 5.65    | 5.33    | 5.09    | 4.89    | 4.73    | 4.48    | 4.30    | 4.16    | 4.05    | 3.96    |
| 24    | 14.03   | 9.34    | 7.55    | 6.59    | 5.98    | 5.55    | 5.23    | 4.99    | 4.80    | 4.64    | 4.39    | 4.21    | 4.07    | 3.96    | 3.87    |
| 25    | 13.88   | 9.22    | 7.45    | 6.49    | 5.89    | 5.46    | 5.15    | 4.91    | 4.71    | 4.56    | 4.31    | 4.13    | 3.99    | 3.88    | 3.79    |
| 26    | 13.74   | 9.12    | 7.36    | 6.41    | 5.80    | 5.38    | 5.07    | 4.83    | 4.64    | 4.48    | 4.24    | 4.06    | 3.92    | 3.81    | 3.72    |
| 27    | 13.61   | 9.02    | 7.27    | 6.33    | 5.73    | 5.31    | 5.00    | 4.76    | 4.57    | 4.41    | 4.17    | 3.99    | 3.86    | 3.75    | 3.66    |
| 28    | 13.50   | 8.93    | 7.19    | 6.25    | 5.66    | 5.24    | 4.93    | 4.69    | 4.50    | 4.35    | 4.11    | 3.93    | 3.80    | 3.69    | 3.60    |
| 29    | 13.39   | 8.85    | 7.12    | 6.19    | 5.59    | 5.18    | 4.87    | 4.64    | 4.45    | 4.29    | 4.05    | 3.88    | 3.74    | 3.63    | 3.54    |
| 30    | 13.29   | 8.77    | 7.05    | 6.12    | 5.53    | 5.12    | 4.82    | 4.58    | 4.39    | 4.24    | 4.00    | 3.82    | 3.69    | 3.58    | 3.49    |
| 35    | 12.90   | 8.47    | 6.79    | 5.88    | 5.30    | 4.89    | 4.59    | 4.36    | 4.18    | 4.03    | 3.79    | 3.62    | 3.48    | 3.38    | 3.29    |
| 40    | 12.61   | 8.25    | 6.59    | 5.70    | 5.13    | 4.73    | 4.44    | 4.21    | 4.02    | 3.87    | 3.64    | 3.47    | 3.34    | 3.23    | 3.14    |
| 50    | 12.22   | 7.96    | 6.34    | 5.46    | 4.90    | 4.51    | 4.22    | 4.00    | 3.82    | 3.67    | 3.44    | 3.27    | 3.14    | 3.04    | 2.95    |
| 60    | 11.97   | 7.77    | 6.17    | 5.31    | 4.76    | 4.37    | 4.09    | 3.86    | 3.69    | 3.54    | 3.32    | 3.15    | 3.02    | 2.91    | 2.83    |
| 70    | 11.80   | 7.64    | 6.06    | 5.20    | 4.66    | 4.28    | 3.99    | 3.77    | 3.60    | 3.45    | 3.23    | 3.06    | 2.93    | 2.83    | 2.74    |
| 80    | 11.67   | 7.54    | 5.97    | 5.12    | 4.58    | 4.20    | 3.92    | 3.70    | 3.53    | 3.39    | 3.16    | 3.00    | 2.87    | 2.76    | 2.68    |
| 90    | 11.57   | 7.47    | 5.91    | 5.06    | 4.53    | 4.15    | 3.87    | 3.65    | 3.48    | 3.34    | 3.11    | 2.95    | 2.82    | 2.71    | 2.63    |
| 100   | 11.50   | 7.41    | 5.86    | 5.02    | 4.48    | 4.11    | 3.83    | 3.61    | 3.44    | 3.30    | 3.07    | 2.91    | 2.78    | 2.68    | 2.59    |
| 120   | 11.38   | 7.32    | 5.78    | 4.95    | 4.42    | 4.04    | 3.77    | 3.55    | 3.38    | 3.24    | 3.02    | 2.85    | 2.72    | 2.62    | 2.53    |
| 150   | 11.27   | 7.24    | 5.71    | 4.88    | 4.35    | 3.98    | 3.71    | 3.49    | 3.32    | 3.18    | 2.96    | 2.80    | 2.67    | 2.56    | 2.48    |
| 200   | 11.15   | 7.15    | 5.63    | 4.81    | 4.29    | 3.92    | 3.65    | 3.43    | 3.26    | 3.12    | 2.90    | 2.74    | 2.61    | 2.51    | 2.42    |
| 250   | 11.09   | 7.10    | 5.59    | 4.77    | 4.25    | 3.88    | 3.61    | 3.40    | 3.23    | 3.09    | 2.87    | 2.71    | 2.58    | 2.48    | 2.39    |
| 300   | 11.04   | 7.07    | 5.56    | 4.75    | 4.22    | 3.86    | 3.59    | 3.38    | 3.21    | 3.07    | 2.85    | 2.69    | 2.56    | 2.46    | 2.37    |
| 400   | 10.99   | 7.03    | 5.53    | 4.71    | 4.19    | 3.83    | 3.56    | 3.35    | 3.18    | 3.04    | 2.82    | 2.66    | 2.53    | 2.43    | 2.34    |
| 500   | 10.96   | 7.00    | 5.51    | 4.69    | 4.18    | 3.81    | 3.54    | 3.33    | 3.16    | 3.02    | 2.81    | 2.64    | 2.52    | 2.41    | 2.33    |
| 600   | 10.94   | 6.99    | 5.49    | 4.68    | 4.16    | 3.80    | 3.53    | 3.32    | 3.15    | 3.01    | 2.80    | 2.63    | 2.51    | 2.40    | 2.32    |
| 750   | 10.91   | 6.97    | 5.48    | 4.67    | 4.15    | 3.79    | 3.52    | 3.31    | 3.14    | 3.00    | 2.78    | 2.62    | 2.49    | 2.39    | 2.31    |
| 1000  | 10.89   | 6.96    | 5.46    | 4.65    | 4.14    | 3.78    | 3.51    | 3.30    | 3.13    | 2.99    | 2.77    | 2.61    | 2.48    | 2.38    | 2.30    |

TABLE A.3 (continued)

F Distribution: Critical Values of  $F$  (0.1% significance level)

| $\nu_1$ | 25      | 30      | 35      | 40      | 50      | 60      | 75      | 100     | 150     | 200     |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1       | 6.24e05 | 6.26e05 | 6.28e05 | 6.29e05 | 6.30e05 | 6.31e05 | 6.32e05 | 6.33e05 | 6.35e05 | 6.35e05 |
| 2       | 999.46  | 999.47  | 999.47  | 999.47  | 999.48  | 999.48  | 999.49  | 999.49  | 999.49  | 999.49  |
| 3       | 125.84  | 125.45  | 125.17  | 124.96  | 124.66  | 124.47  | 124.27  | 124.07  | 123.87  | 123.77  |
| 4       | 45.70   | 45.43   | 45.23   | 45.09   | 44.88   | 44.75   | 44.61   | 44.47   | 44.33   | 44.26   |
| 5       | 25.08   | 24.87   | 24.72   | 24.60   | 24.44   | 24.33   | 24.22   | 24.12   | 24.01   | 23.95   |
| 6       | 16.85   | 16.67   | 16.54   | 16.44   | 16.31   | 16.21   | 16.12   | 16.03   | 15.93   | 15.89   |
| 7       | 12.69   | 12.53   | 12.41   | 12.33   | 12.20   | 12.12   | 12.04   | 11.95   | 11.87   | 11.82   |
| 8       | 10.26   | 10.11   | 10.00   | 9.92    | 9.80    | 9.73    | 9.65    | 9.57    | 9.49    | 9.45    |
| 9       | 8.69    | 8.55    | 8.46    | 8.37    | 8.26    | 8.19    | 8.11    | 8.04    | 7.96    | 7.93    |
| 10      | 7.60    | 7.47    | 7.37    | 7.30    | 7.19    | 7.12    | 7.05    | 6.98    | 6.91    | 6.87    |
| 11      | 6.81    | 6.68    | 6.59    | 6.52    | 6.42    | 6.35    | 6.28    | 6.21    | 6.14    | 6.10    |
| 12      | 6.22    | 6.09    | 6.00    | 5.93    | 5.83    | 5.76    | 5.70    | 5.63    | 5.56    | 5.52    |
| 13      | 5.75    | 5.63    | 5.54    | 5.47    | 5.37    | 5.30    | 5.24    | 5.17    | 5.10    | 5.07    |
| 14      | 5.38    | 5.25    | 5.17    | 5.10    | 5.00    | 4.94    | 4.87    | 4.81    | 4.74    | 4.71    |
| 15      | 5.07    | 4.95    | 4.86    | 4.80    | 4.70    | 4.64    | 4.57    | 4.51    | 4.44    | 4.41    |
| 16      | 4.82    | 4.70    | 4.61    | 4.54    | 4.45    | 4.39    | 4.32    | 4.26    | 4.19    | 4.16    |
| 17      | 4.60    | 4.48    | 4.40    | 4.33    | 4.24    | 4.18    | 4.11    | 4.05    | 3.98    | 3.95    |
| 18      | 4.42    | 4.30    | 4.22    | 4.15    | 4.06    | 4.00    | 3.93    | 3.87    | 3.80    | 3.77    |
| 19      | 4.26    | 4.14    | 4.06    | 3.99    | 3.90    | 3.84    | 3.78    | 3.71    | 3.65    | 3.61    |
| 20      | 4.12    | 4.00    | 3.92    | 3.86    | 3.77    | 3.70    | 3.64    | 3.58    | 3.51    | 3.48    |
| 21      | 4.00    | 3.88    | 3.80    | 3.74    | 3.64    | 3.58    | 3.52    | 3.46    | 3.39    | 3.36    |
| 22      | 3.89    | 3.78    | 3.70    | 3.63    | 3.54    | 3.48    | 3.41    | 3.35    | 3.28    | 3.25    |
| 23      | 3.79    | 3.68    | 3.60    | 3.53    | 3.44    | 3.38    | 3.32    | 3.25    | 3.19    | 3.16    |
| 24      | 3.71    | 3.59    | 3.51    | 3.45    | 3.36    | 3.29    | 3.23    | 3.17    | 3.10    | 3.07    |
| 25      | 3.63    | 3.52    | 3.43    | 3.37    | 3.28    | 3.22    | 3.15    | 3.09    | 3.03    | 2.99    |
| 26      | 3.56    | 3.44    | 3.36    | 3.30    | 3.21    | 3.15    | 3.08    | 3.02    | 2.95    | 2.92    |
| 27      | 3.49    | 3.38    | 3.30    | 3.23    | 3.14    | 3.08    | 3.02    | 2.96    | 2.89    | 2.86    |
| 28      | 3.43    | 3.32    | 3.24    | 3.18    | 3.09    | 3.02    | 2.96    | 2.90    | 2.83    | 2.80    |
| 29      | 3.38    | 3.27    | 3.18    | 3.12    | 3.03    | 2.97    | 2.91    | 2.84    | 2.78    | 2.74    |
| 30      | 3.33    | 3.22    | 3.13    | 3.07    | 2.98    | 2.92    | 2.86    | 2.79    | 2.73    | 2.69    |
| 35      | 3.13    | 3.02    | 2.93    | 2.87    | 2.78    | 2.72    | 2.66    | 2.59    | 2.52    | 2.49    |
| 40      | 2.98    | 2.87    | 2.79    | 2.73    | 2.64    | 2.57    | 2.51    | 2.44    | 2.38    | 2.34    |
| 50      | 2.79    | 2.68    | 2.60    | 2.53    | 2.44    | 2.38    | 2.31    | 2.25    | 2.18    | 2.14    |
| 60      | 2.67    | 2.55    | 2.47    | 2.41    | 2.32    | 2.25    | 2.19    | 2.12    | 2.05    | 2.01    |
| 70      | 2.58    | 2.47    | 2.39    | 2.32    | 2.23    | 2.16    | 2.10    | 2.03    | 1.95    | 1.92    |
| 80      | 2.52    | 2.41    | 2.32    | 2.26    | 2.16    | 2.10    | 2.03    | 1.96    | 1.89    | 1.85    |
| 90      | 2.47    | 2.36    | 2.27    | 2.21    | 2.11    | 2.05    | 1.98    | 1.91    | 1.83    | 1.79    |
| 100     | 2.43    | 2.32    | 2.24    | 2.17    | 2.08    | 2.01    | 1.94    | 1.87    | 1.79    | 1.75    |
| 120     | 2.37    | 2.26    | 2.18    | 2.11    | 2.02    | 1.95    | 1.88    | 1.81    | 1.73    | 1.68    |
| 150     | 2.32    | 2.21    | 2.12    | 2.06    | 1.96    | 1.89    | 1.82    | 1.74    | 1.66    | 1.62    |
| 200     | 2.26    | 2.15    | 2.07    | 2.00    | 1.90    | 1.83    | 1.76    | 1.68    | 1.60    | 1.55    |
| 250     | 2.23    | 2.12    | 2.03    | 1.97    | 1.87    | 1.80    | 1.72    | 1.65    | 1.56    | 1.51    |
| 300     | 2.21    | 2.10    | 2.01    | 1.94    | 1.85    | 1.78    | 1.70    | 1.62    | 1.53    | 1.48    |
| 400     | 2.18    | 2.07    | 1.98    | 1.92    | 1.82    | 1.75    | 1.67    | 1.59    | 1.50    | 1.45    |
| 500     | 2.17    | 2.05    | 1.97    | 1.90    | 1.80    | 1.73    | 1.65    | 1.57    | 1.48    | 1.43    |
| 600     | 2.16    | 2.04    | 1.96    | 1.89    | 1.79    | 1.72    | 1.64    | 1.56    | 1.46    | 1.41    |
| 750     | 2.15    | 2.03    | 1.95    | 1.88    | 1.78    | 1.71    | 1.63    | 1.55    | 1.45    | 1.40    |
| 1000    | 2.14    | 2.02    | 1.94    | 1.87    | 1.77    | 1.69    | 1.62    | 1.53    | 1.44    | 1.38    |

TABLE A.4

 $\chi^2$  (Chi-Squared) Distribution: Critical Values of  $\chi^2$ 

| Degrees of freedom | Significance level |        |        |
|--------------------|--------------------|--------|--------|
|                    | 5%                 | 1%     | 0.1%   |
| 1                  | 3.841              | 6.635  | 10.828 |
| 2                  | 5.991              | 9.210  | 13.816 |
| 3                  | 7.815              | 11.345 | 16.266 |
| 4                  | 9.488              | 13.277 | 18.467 |
| 5                  | 11.070             | 15.086 | 20.515 |
| 6                  | 12.592             | 16.812 | 22.458 |
| 7                  | 14.067             | 18.475 | 24.322 |
| 8                  | 15.507             | 20.090 | 26.124 |
| 9                  | 16.919             | 21.666 | 27.877 |
| 10                 | 18.307             | 23.209 | 29.588 |