UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2016

TITLE OF PAPER	:	NONPARAMETRIC ANALYSIS
COURSE CODE	:	ST409
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 10+10]

(a) In an attempt to assess the effectiveness of a political candidate's campaign oratory, a group of 60 subjects were asked the question prior to and after a prepared speech by the candidate, "If the election were held right now, would you vote for Candidate X against the incumbent?" The results are as follows:

	After	
Before	-	+
+	2	25
-	20	13

Are the responses before and after the speech associated? Use $\alpha = 0.05$.

(b) The following data are the annual incomes (in thousands of emalangeni) for a random sample of $n = 10$ adults from a certain area

12.8, 13.6, 14.7, 16.2, 17.9, 18.6, 19.2, 19.8, 20.3, 24.4

Use the *sign* test to test that the median income doesn't exceed SZL20000 in this particular area. Use the exact null distribution of your test statistic to determine the critical value. Report your conclusions.

Question 2 [20 marks, 10+10]

(a) In a survey conducted in 1992, senior high school students were asked if they had ever used marijuana. Of the females sampled, 445 said yes and 675 said no; of the males sampled, 515 said yes and 641 said no. Are male high school students more likely to use marijuana?

(b) It is desired to design a given automobile to allow enough headroom to accommodate comfortably all but the tallest 5% of the people who drive. Former studies indicate that the 95^{th} percentile was 70.3 inches. In order to see if the former studies are still valid, a random sample of size 100 is selected. It is found that the 12 tallest persons in the sample have the following heights.

72.6,70.0,71.3,70.5,70.8,76.0,70.1,72.5,71.1,70.6,71.9,72.8

Is it still reasonable to use 70.3 as the 95^{th} percentile.

Question 3 [20 marks, 8+12]

(a) Find the exact distribution of the Wilcoxon Signed Rank statistic for $n = 4$.

(b) The data in Table 1 are a subset of data obtained by Kaneto, Kosaka and Nakao (1967). The experiment investigated the effect of vagus nerve stimulation on insulin secretion. The subjects were mongrel dogs with varying bodyweights. Table 1 gives the ammount of immunoreactive insulin in pancreatic venous plasma just before stimulation of the left vagus nerve (X) and the amount measured 5 minutes after stimulation (Y) for seven dogs.

Table 1: Blood levels of immunoreactive insulin $\mu U/ml$.

Sample i	X_i	Y_i
1	350	480
2	200	130
3	240	250
4	290	310
5	90	280
6	370	1450
7	240	280

Test the hypothesis of no effect against the alternative that stimulation of the vagus nerve increases the blood level of immunoreactive insulin.

Question 4 [20 marks, 10+10]

(a) Carry out a Kolmogorov-Smirnov test of the hypothesis that the measurements

5.1, 6.2, 3.4, 2.2, 4.7, 3.3, 1.6, 4.6, 5.0, 4.3

come from the distribution.

$$f(x) = \frac{\lambda x^{\lambda-1}}{\theta^\lambda} \exp(-(x/\theta)^\lambda) \qquad x > 0$$

(b) A commuter in a large city can travel to work by car, bicycle or bus. She times four journeys by each method with the following results, in minutes.

Car	Bicycle	Bus
27	34	26
45	38	41
33	43	35
31	42	46

Test at the 5% significance level whether there are differences in the average journey times for the three modes of transport.

Question 5 [20 marks, 10+10]

(a) An experimentor was involved in a study to determine if there is an association between stress and mental health. The Hopkins Symptom Checklist was used to determine the level of symptoms experiences and a scale was developed to measure stress levels. For data obtained, refer to the following table. Use **Kendall's tau** to perform the appropriate test at the 1% level of significance.

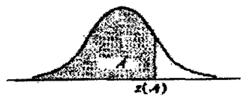
Participant	Stress (X)	Symptoms (Y)
1	33	100
2	30	92
3	11	83
4	18	69
5	5	98
6	16	105
7	3	65
8	21	84
9	21	70
10	35	120

(b) On 20 successive trips between Manzini and Mbabane a bus carried 24, 19, 32, 28, 21, 23, 26, 17, 20, 28, 30, 24, 13, 35, 26, 21, 19, 29, 27, and 23 passengers. Test whether it is reasonable to treat these data as if they constitute a random sample at $\alpha = 0.01$.

Normal Distribution

Table C-1. Cumulative Probabilities of the Standard Normal Distribution.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



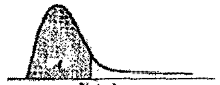
$z(A)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Chi-Square Distribution

Table C-2. Percentiles of the χ^2 Distribution

Entry is $\chi^2(A;\nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A;\nu)\} = A$



$\chi^2(A;\nu)$

ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	$0.0^{4}393$	$0.0^{3}157$	$0.0^{3}982$	$0.0^{2}393$	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Table C-3. Binomial Distribution

$$B(x;n,p) = \sum_{0 \le y \le x} b(y;n,p)$$

The values of $B(x;n,p)$ for $0.5 < p < 1.0$ are obtained by using the formula

$$B(x;n,1-p) = 1 - B(n-1-x;n,p)$$

		p									
n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
5	0	0.774	0.590	0.444	0.328	0.237	0.168	0.116	0.078	0.050	0.031
	1	0.977	0.919	0.835	0.737	0.633	0.528	0.428	0.337	0.256	0.188
	2	0.999	0.991	0.973	0.942	0.896	0.837	0.765	0.683	0.593	0.500
	3	1.000	1.000	0.998	0.993	0.984	0.969	0.946	0.913	0.869	0.813
	4	1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.990	0.982	0.969
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	0	0.599	0.349	0.197	0.107	0.056	0.028	0.013	0.006	0.003	0.001
	1	0.914	0.736	0.544	0.376	0.244	0.149	0.086	0.046	0.023	0.011
	2	0.988	0.930	0.820	0.678	0.526	0.383	0.262	0.167	0.100	0.055
	3	0.999	0.987	0.950	0.879	0.776	0.650	0.514	0.382	0.266	0.172
	4	1.000	0.998	0.990	0.967	0.922	0.850	0.751	0.633	0.504	0.377
	5	1.000	1.000	0.999	0.994	0.980	0.953	0.905	0.834	0.738	0.623
	6	1.000	1.000	1.000	0.999	0.996	0.989	0.974	0.945	0.898	0.828
	7	1.000	1.000	1.000	1.000	1.000	0.998	0.995	0.988	0.973	0.945
	8	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.989
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0	0.463	0.206	0.087	0.035	0.013	0.005	0.002	0.000	0.000	0.000
	1	0.829	0.549	0.319	0.167	0.080	0.035	0.014	0.005	0.002	0.000
	2	0.964	0.816	0.604	0.398	0.236	0.127	0.062	0.027	0.011	0.004
	3	0.995	0.944	0.823	0.648	0.461	0.297	0.173	0.091	0.042	0.018
	4	0.999	0.987	0.938	0.836	0.686	0.515	0.352	0.217	0.120	0.059
	5	1.000	0.998	0.983	0.939	0.852	0.722	0.564	0.403	0.261	0.151
	6	1.000	1.000	0.996	0.982	0.943	0.869	0.755	0.610	0.452	0.304
	7	1.000	1.000	0.999	0.996	0.983	0.950	0.887	0.787	0.654	0.500
	8	1.000	1.000	1.000	0.999	0.996	0.985	0.958	0.905	0.818	0.696
	9	1.000	1.000	1.000	1.000	0.999	0.996	0.988	0.966	0.923	0.849
	10	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.991	0.975	0.941
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994	0.982
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table C-3 (Continued) Binomial Distribution

		p									
n	x	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
20	0	0.358	0.122	0.039	0.012	0.003	0.001	0.000	0.000	0.000	0.000
	1	0.736	0.392	0.176	0.069	0.024	0.008	0.002	0.001	0.000	0.000
	2	0.925	0.677	0.405	0.206	0.091	0.035	0.012	0.004	0.001	0.000
	3	0.984	0.867	0.648	0.411	0.225	0.107	0.044	0.016	0.005	0.001
	4	0.997	0.957	0.830	0.630	0.415	0.238	0.118	0.051	0.019	0.006
	5	1.000	0.989	0.933	0.804	0.617	0.416	0.245	0.126	0.055	0.021
	6	1.000	0.998	0.978	0.913	0.786	0.608	0.417	0.250	0.130	0.058
	7	1.000	1.000	0.994	0.968	0.898	0.772	0.601	0.416	0.252	0.132
	8	1.000	1.000	0.999	0.990	0.959	0.887	0.762	0.596	0.414	0.252
	9	1.000	1.000	1.000	0.997	0.986	0.952	0.878	0.755	0.591	0.412
	10	1.000	1.000	1.000	0.999	0.996	0.983	0.947	0.872	0.751	0.588
	11	1.000	1.000	1.000	1.000	0.999	0.995	0.980	0.943	0.869	0.748
	12	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.979	0.942	0.868
	13	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994	0.979	0.942
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994	0.979
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	0	0.277	0.072	0.017	0.004	0.001	0.000	0.000	0.000	0.000	0.000
	1	0.642	0.271	0.093	0.027	0.007	0.002	0.000	0.000	0.000	0.000
	2	0.873	0.537	0.254	0.098	0.032	0.009	0.002	0.000	0.000	0.000
	3	0.966	0.764	0.471	0.234	0.096	0.033	0.010	0.002	0.000	0.000
	4	0.993	0.902	0.682	0.421	0.214	0.090	0.032	0.009	0.002	0.000
	5	0.999	0.967	0.838	0.617	0.378	0.193	0.083	0.029	0.009	0.002
	6	1.000	0.991	0.930	0.780	0.561	0.341	0.173	0.074	0.026	0.007
	7	1.000	0.998	0.975	0.891	0.727	0.512	0.306	0.154	0.064	0.022
	8	1.000	1.000	0.992	0.953	0.851	0.677	0.467	0.274	0.134	0.054
	9	1.000	1.000	0.998	0.983	0.929	0.811	0.630	0.425	0.242	0.115
	10	1.000	1.000	1.000	0.994	0.970	0.902	0.771	0.586	0.384	0.212
	11	1.000	1.000	1.000	0.998	0.989	0.956	0.875	0.732	0.543	0.345
	12	1.000	1.000	1.000	1.000	0.997	0.983	0.940	0.846	0.694	0.500
	13	1.000	1.000	1.000	1.000	0.999	0.994	0.975	0.922	0.817	0.655
	14	1.000	1.000	1.000	1.000	1.000	0.998	0.991	0.966	0.904	0.788
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.987	0.956	0.885
	16	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.946
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.978
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.993
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Quantiles of the Mann-Whitney Test Statistic

Table C-6 Quantiles of the Mann-Whitney Test Statistic

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	.001	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	.005	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4
	.01	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
	.025	3	3	3	3	3	3	4	4	4	5	5	5	5	5	5	6	6	6	6
	.05	3	3	3	4	4	4	5	5	5	5	6	6	7	7	7	7	8	8	8
	.10	3	4	4	5	5	5	6	6	7	7	8	8	8	9	9	10	10	11	11
3	.001	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7	7	7
	.005	6	6	6	6	6	6	6	7	7	7	8	8	8	9	9	9	9	10	10
	.01	6	6	6	6	6	7	7	8	8	8	9	9	9	10	10	11	11	11	12
	.025	6	6	6	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15
	.05	6	7	7	8	9	9	10	11	11	12	12	13	14	14	15	16	16	17	18
	.10	7	8	8	9	10	11	12	12	13	14	15	16	17	17	18	19	20	21	22
4	.001	10	10	10	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	14
	.005	10	10	10	10	11	11	12	12	13	13	14	14	15	16	16	17	17	18	19
	.01	10	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	20	21
	.025	10	10	11	12	13	14	15	15	16	17	18	19	20	21	22	22	23	24	25
	.05	10	11	12	13	14	15	16	17	18	19	20	21	22	23	25	26	27	28	29
	.10	11	12	14	15	16	17	18	20	21	22	23	24	26	27	28	29	31	32	33
5	.001	15	15	15	15	15	15	16	17	17	18	18	19	19	20	21	21	22	23	23
	.005	15	15	15	16	17	17	18	19	20	21	22	23	23	24	25	26	27	28	29
	.01	15	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
	.025	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35	36
	.05	16	17	18	20	21	22	24	25	27	28	29	31	32	34	35	36	38	39	41
	.10	17	18	20	21	23	24	26	28	29	31	33	34	36	38	39	41	43	44	46
6	.001	21	21	21	21	21	21	23	24	25	26	26	27	28	29	30	31	32	33	34
	.005	21	21	22	23	24	25	26	27	28	29	31	32	33	34	35	37	38	39	40
	.01	21	21	23	24	25	26	28	29	30	31	33	34	35	37	38	40	41	42	44
	.025	21	23	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49
	.05	22	24	25	27	29	30	32	34	36	38	39	41	43	45	47	48	50	52	54
	.10	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	56	58	60
7	.001	28	28	28	28	29	30	31	32	34	35	36	37	38	39	40	42	43	44	45
	.005	28	28	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51	53
	.01	28	29	30	32	33	35	36	38	40	41	43	45	46	48	50	52	53	55	57
	.025	28	30	32	34	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
	.05	29	31	33	35	37	40	42	44	46	48	50	53	55	57	59	62	64	66	68
	.10	30	33	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75
8	.001	36	36	36	37	38	39	41	42	43	45	46	48	49	51	52	54	55	57	58
	.005	36	36	38	39	41	43	44	46	48	50	52	54	55	57	59	61	63	65	67
	.01	36	37	39	41	43	44	46	48	50	52	54	56	59	61	63	65	67	69	71
	.025	37	39	41	43	45	47	50	52	54	56	59	61	63	66	68	71	73	75	78
	.05	38	40	42	45	47	50	52	55	57	60	63	65	68	70	73	76	78	81	84
	.10	39	42	44	47	50	53	56	59	61	64	67	70	73	76	79	82	85	88	91
9	.001	45	45	45	47	48	49	51	53	54	56	58	60	61	63	65	67	69	71	72
	.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	73	75	77	79	82
	.01	45	47	49	51	53	55	57	60	62	64	67	69	72	74	77	79	82	84	86
	.025	46	48	50	53	56	58	61	63	66	69	72	74	77	80	83	85	88	91	94
	.05	47	50	52	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	100
	.10	48	51	55	58	61	64	68	71	74	77	81	84	87	91	94	98	101	104	108
10	.001	55	55	56	57	59	61	62	64	66	68	70	73	75	77	79	81	83	85	88
	.005	55	56	58	60	62	65	67	69	72	74	77	80	82	85	87	90	93	95	98
	.01	55	57	59	62	64	67	69	72	75	78	80	83	86	89	92	94	97	100	103
	.025	56	59	61	64	67	70	73	76	79	82	85	89	92	95	98	101	104	108	111
	.05	57	60	63	67	70	73	76	80	83	87	90	93	97	100	104	107	111	114	118
	.10	59	62	66	69	73	77	80	84	88	92	95	99	103	107	110	114	118	122	126

Table C-6 (Continued) Quantiles of the Mann-Whitney Test Statistic

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	.001	66	66	67	69	71	73	75	77	79	82	84	87	89	91	94	96	99	101	104
	.005	66	67	69	72	74	77	80	83	85	88	91	94	97	100	103	106	109	112	115
	.01	66	68	71	74	76	79	82	85	89	92	95	98	101	104	108	111	114	117	120
	.025	67	70	73	76	80	83	86	90	93	97	100	104	107	111	114	118	122	125	129
	.05	68	72	75	79	83	86	90	94	98	101	105	109	113	117	121	124	128	132	136
	.10	70	74	78	82	86	90	94	98	103	107	111	115	119	124	128	132	136	140	145
12	.001	78	78	79	81	83	86	88	91	93	96	98	102	104	106	110	113	116	118	121
	.005	78	80	82	85	88	91	94	97	100	103	106	110	113	116	120	123	126	130	133
	.01	78	81	84	87	90	93	96	100	103	107	110	114	117	121	125	128	132	135	139
	.025	80	83	86	90	93	97	101	105	108	112	116	120	124	128	132	136	140	144	148
	.05	81	84	88	92	96	100	105	109	113	117	121	126	130	134	139	143	147	151	156
	.10	83	87	91	96	100	105	109	114	118	123	128	132	137	142	146	151	156	160	165
13	.001	91	91	93	95	97	100	103	106	109	112	115	118	121	124	127	130	134	137	140
	.005	91	93	95	99	102	105	109	112	116	119	123	126	130	134	137	141	145	149	152
	.01	92	94	97	101	104	108	112	115	119	123	127	131	135	139	143	147	151	155	159
	.025	93	96	100	104	108	112	116	120	125	129	133	137	142	146	151	155	159	164	168
	.05	94	98	102	107	111	116	120	125	129	134	139	143	148	153	157	162	167	172	176
	.10	96	101	105	110	115	120	125	130	135	140	145	150	155	160	166	171	176	181	186
14	.001	105	105	107	109	112	115	118	121	125	128	131	135	138	142	145	149	152	156	160
	.005	105	107	110	113	117	121	124	128	132	136	140	144	148	152	156	160	164	169	173
	.01	106	108	112	116	119	123	128	132	136	140	144	149	153	157	162	166	171	175	179
	.025	107	111	115	119	123	128	132	137	142	146	151	156	161	165	170	175	180	184	189
	.05	109	113	117	122	127	132	137	142	147	152	157	162	167	172	177	183	188	193	198
	.10	110	116	121	126	131	137	142	147	153	158	164	169	175	180	186	191	197	203	208
15	.001	120	120	122	125	128	133	135	138	142	145	149	153	157	161	164	168	172	176	180
	.005	120	123	126	129	133	137	141	145	150	154	158	163	167	172	176	181	185	190	194
	.01	121	124	128	132	136	140	145	149	154	158	163	168	172	177	182	187	191	196	201
	.025	122	126	131	135	140	145	150	155	160	165	170	175	180	185	191	196	201	206	211
	.05	124	128	133	139	144	149	154	160	165	171	176	182	187	193	198	204	209	215	221
	.10	126	131	137	143	148	154	160	166	172	178	184	189	195	201	207	213	219	225	231
16	.001	136	136	139	142	145	148	152	156	160	164	168	172	176	180	185	189	193	197	202
	.005	136	139	142	146	150	155	159	164	168	173	178	182	187	192	197	202	207	211	216
	.01	137	140	144	149	153	158	163	168	173	178	183	188	193	198	203	208	213	219	224
	.025	138	143	148	152	158	163	168	174	179	184	190	196	201	207	212	218	223	229	235
	.05	140	145	151	156	162	167	173	179	185	191	197	202	208	214	220	226	232	238	244
	.10	142	148	154	160	166	173	179	185	191	198	204	211	217	223	230	236	243	249	256
17	.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220	224
	.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235	240
	.01	154	158	162	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242	247
	.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253	259
	.05	157	163	169	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263	269
	.10	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274	281
18	.001	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243	248
	.005	171	174	178	183	188	193	198	203	209	214	219	225	230	236	242	247	253	259	264
	.01	172	176	181	186	191	196	202	208	213	219	225	231	237	242	248	254	260	266	272
	.025	174	179	184	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278	284
	.05	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288	295
	.10	178	185	192	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300	307
19	.001	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268	273
	.005	191	194	198	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284	290
	.01	192	195	200	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292	298
	.025	193	198	204	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304	310
	.05	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314	321
	.10	198	205	212	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326	334

Table C-6 (Continued) Quantiles of the Mann-Whitney Test Statistic

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	.001	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293	299
	.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310	316
	.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318	325
20	.025	213	219	225	231	238	245	251	259	206	273	280	287	294	301	309	316	323	330	338
	.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341	349
	.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354	362

Quantiles of the Kruskal-Wallis Test Statistics for Small Sample Sizes

Table C-7 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes

Sample Sizes	$w_{0.90}$	$w_{0.95}$	$w_{0.99}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.773	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

Quantiles of Kendall's Test Statistic

Table C-9 Quantiles of the Kendall's Test Statistic

n	*p* = .900	.950	.975	.990	.995
4	4	4	6	6	6
5	6	6	8	8	10
6	7	9	11	11	13
7	9	11	13	15	17
8	10	14	16	18	20
9	12	16	18	22	24
10	15	19	21	25	27
11	17	21	25	29	31
12	18	24	28	34	36
13	22	26	32	38	42
14	23	31	35	41	45
15	27	33	39	47	51
16	28	36	44	50	56
17	32	40	48	56	62
18	35	43	51	61	67
19	37	47	55	65	73
20	40	50	60	70	78
21	42	54	64	76	84
22	45	59	69	81	89
23	49	63	73	87	97
24	52	66	78	92	102
25	56	70	84	98	108
26	59	75	89	105	115
27	61	79	93	111	123
28	66	84	98	116	128
29	68	88	104	124	136
30	73	93	109	129	143
31	75	97	115	135	149
32	80	102	120	142	158
33	84	106	126	150	164
34	87	111	131	155	173
35	91	115	137	163	179
36	94	120	144	170	188
37	98	126	150	176	196
38	103	131	155	183	203
39	107	137	161	191	211
40	110	142	168	198	220

Table C-9 (Continued) Quantiles of the Kendall's Test Statistic

n	*p* = .900	.950	.975	.990	.995
41	114	146	174	206	228
42	119	151	181	213	235
43	123	157	187	221	245
44	128	162	194	228	252
45	132	168	200	236	262
46	135	173	207	245	271
47	141	179	213	253	279
48	144	186	220	260	288
49	150	190	228	268	296
50	153	197	233	277	305
51	159	203	241	285	315
52	162	208	248	294	324
53	168	214	256	302	334
54	173	221	263	311	343
55	177	227	269	319	353
56	182	232	276	328	362
57	186	240	284	336	372
58	191	245	291	345	381
59	197	251	299	355	391
60	202	258	306	364	402