

UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2015

TITLE OF PAPER : NON-PARAMETRIC ANALYSIS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

**REQUIREMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER QUESTION ONE AND ANY
THREE (3) QUESTIONS. ALL QUESTIONS
CARRY MARKS AS INDICATED WITHIN
THE PARENTHESIS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

ANSWER QUESTION ONE & ANY THREE QUESTIONS:

For all questions, clearly state the name of the test, type of the test (upper, lower, or two-sided), the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, the decision & the conclusions.

QUESTION ONE.

[20 + 20 marks]

Fifteen volunteers for an experiment are divided randomly into three groups to see if telescopic sight improves the ability to hit the target under twilight conditions. Group A is given rifles with telescopic sights (auto focus), group B is given same kind of rifles with telescopic sights but with manual focus, while group C has the same kind of rifle with open sights. After a reasonable learning period they are given a shooting test at twilight. These are their scores (100 is perfect).

<i>Group A:</i>	96	93	88	85	89
<i>Group B:</i>	92	97	87	91	96
<i>Group C:</i>	89	93	80	77	84

- Use an appropriate test to test whether any difference exists between the scores of auto and manual focus. Use 5% level of significance.
- Use another appropriate test to determine whether evidence exists to conclude that the assistance in sights tend to differ in hitting targets. Use $\alpha = 0.10$.

QUESTION TWO.

[10 + 10 marks]

A. According to the director of a tourist bureau, there is a median of 10 hours of sunshine per day during the summer months. For a random sample of 20 days during the past three summers, the number of hours of sunshine has been recorded as shown below:

8	9	8	10	9	7	7	9	7	7
9	8	11	9	10	7	8	11	8	12

Test the director's claim at 5% level of significance using an appropriate tests based on the binomial distribution. Also calculate the P-value.

B. The following table shows the information on cotton crop insurance for the years 1976 to 2000.

Year	#of Crops insured	Year	# of Crops insured
1976	19,479	1989	15,375
1977	26,667	1990	21,312
1978	63,969	1991	26,526
1979	57,715	1992	24,865
1980	38,086	1993	21,152
1981	38,434	1994	23,458
1982	24,196	1995	25,774
1983	19,319	1996	32,646
1984	29,975	1997	31,786
1985	25,451	1998	24,821
1986	20,410	1999	19,593
1987	19,940	2000	14,960
1988	15,628		

Do these data indicate a downward trend in the number of crops insured? Use the Cox-Stuart test for trend with $\alpha = 0.05$.

QUESTION THREE.

[20 marks]

Two judges rated the participants in a dance contest as follows:

<u>Judge</u>	<u>Contestant</u>							
	A	B	C	D	E	F	G	H
1	5	2	6	3	4	1	8	7
2	3	1	7	4	5	2	6	8

Are the two rankings independent? Use either Spearman's ρ test or Kendall's τ test with $\alpha = 0.05$.

QUESTION FOUR.

[20 marks]

Two computer software packages are being considered for use in the inventory control department of a small manufacturing firm. The firm has selected 12 different computing tasks that are typical of the kinds of jobs such a package would have to perform, and then recorded the number of seconds each package required to complete each task, given in the following table. Do the data present sufficient evidence to indicate that both software's performed the jobs at the same time? Analyse the data by using the Wilcoxon Signed Rank Test with $\alpha = 0.10$.

<u>Computing Task</u>	<u>Time Required for Software Packages X and Y</u>	
	<u>X_i</u>	<u>Y_i</u>
A	24.0	23.1
B	16.7	20.4
C	21.6	17.7
D	23.7	20.7
E	37.5	42.1
F	31.4	36.1
G	14.9	21.8
H	37.3	40.3
I	17.9	26.0
J	15.5	15.5
K	29.0	35.4
L	19.9	25.5

QUESTION FIVE.

[20 marks]

A random sample of five sixth-grade boys in one section of town were given a literacy test with the following results; 82, 74, 87, 86, 75. Another random sample of eight sixth-grade boys from a different section of town were given the same literacy test with these scores resulting: 88, 77, 91, 88, 94, 93, 83, 94. Using Smirnov test, find whether there is a difference in literacy in the two populations of sixth-grade boys.

TABLE A3 (Continued)

<i>n</i>	<i>y</i>	<i>p</i> = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000
	11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	-0.0086	0.0002
	14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020
	15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* Y has the binomial distribution with parameters n and p . The entries are the values of $P(Y \leq y) = \sum_{k=0}^y \binom{n}{k} p^k (1-p)^{n-k}$, for p ranging from 0.05 to 0.95.

For n larger than 20, the r th quantile y_r of a binomial random variable may be approximated using $y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the r th quantile of a standard normal random variable, obtained from Table A1.

TABLE A10 Quantiles of Spearman's ρ^a

n	$p = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p \approx \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

Source: Adapted from Glasser and Winter (1961), with corrections, with permission from the Biometrika Trustees.

*The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$p = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A12 (Continued)

	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.50}$	$w_{0.10}$	$w_{0.20}$	$w_{0.50}$	$w_{0.90}$	$w_{0.95}$	$\frac{n(n+1)}{2}$
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n + 1)/4] + z_p \sqrt{n(n + 1)(2n + 1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE. Adapted from Hartar and Owen (1970), with permission from the American Mathematical Society.

The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n + 1)/2 - w_{1-p}$$

where $n(n + 1)/2$ is given in the right hand column in the table. Note that $P(T^* < w_p) \leq p$ and $P(T^* > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^* less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic^a

One-Sided Test						Two-Sided Test					
	$p = 0.90$	0.95	0.975	0.99	0.995		$p = 0.90$	0.95	0.975	0.99	0.995
	$p = 0.80$	0.90	0.95	0.98	0.99		$p = 0.80$	0.90	0.95	0.98	0.99
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252
Approximation ^b for $n > 40$							1.07	1.22	1.36	1.52	1.63
							$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{n}}$

SOURCE. Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

*The entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.

TABLE A19 Quantiles of the Smirnov Test Statistic for Two Samples of Equal Size n^a

One-Sided Test: $p = 0.90$ 0.95 0.975 0.99 0.995					One-Sided Test: $p = 0.90$ 0.95 0.975 0.99 0.995				
Two-Sided Test: $p = 0.80$ 0.90 0.95 0.98 0.99					Two-Sided Test: $p = 0.80$ 0.90 0.95 0.98 0.99				
$n = 3$	2/3	2/3			$n = 22$	7/22	8/22	8/22	10/22
4	3/4	3/4	3/4			23	7/23	8/23	9/23
5	3/5	3/5	4/5	4/5		24	7/24	8/24	9/24
6	3/6	4/6	4/6	5/6		25	7/25	8/25	9/25
7	4/7	4/7	5/7	5/7		26	7/26	8/26	9/26
8	4/8	4/8	5/8	5/8		27	7/27	8/27	9/27
9	4/9	5/9	5/9	6/9		28	8/28	9/28	10/28
10	4/10	5/10	6/10	6/10		29	8/29	9/29	10/29
11	5/11	5/11	6/11	7/11		30	8/30	9/30	10/30
12	5/12	5/12	6/12	7/12		31	8/31	9/31	10/31
13	5/13	6/13	6/13	7/13		32	8/32	9/32	10/32
14	5/14	6/14	7/14	7/14		33	8/33	9/33	11/33
15	6/15	7/15	8/15	8/15		34	8/34	10/34	11/34
16	6/16	7/16	8/16	9/16		35	8/35	10/35	11/35
17	6/17	7/17	7/17	9/17		36	9/36	10/36	11/36
18	6/18	7/18	8/18	9/18		37	9/37	10/37	11/37
19	6/19	7/19	8/19	9/19		38	9/38	10/38	11/38
20	6/20	7/20	8/20	9/20		39	9/39	10/39	11/39
21	6/21	7/21	8/21	9/21		40	9/40	10/40	12/40
Approximation for $n > 40$:					1.52	1.73	1.92	2.15	2.30
					\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}

SOURCE. Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

^aThe entries in this table are selected quantiles w_n of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m^a

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 1$	$N_2 = 9$	17/18				
	10	9/10				
$N_1 = 2$	$N_2 = 3$	5/6				
	4	3/4				
	5	4/5				
	6	5/6				
	7	5/7				
	8	3/4				
	9	7/9				
	10	7/10				
		4/5				
		9/10				
$N_1 = 3$	$N_2 = 4$	3/4	3/4			
	5	2/3	4/5			
	6	2/3	2/3			
	7	2/3	5/7			
	8	5/8	3/4			
	9	2/3	2/3			
	10	3/5	7/10			
	12	7/12	2/3			
		3/4	5/6			
			11/12			
$N_1 = 4$	$N_2 = 5$	3/5	3/4	4/5	4/5	
	6	7/12	2/3	3/4	5/6	
	7	17/28	5/7	3/4	6/7	
	8	5/8	5/8	7/8	7/8	
	9	5/9	2/3	3/4	7/9	
	10	11/20	13/20	7/10	4/5	
	12	7/12	2/3	3/4	5/6	
	16	9/16	5/8	11/16	3/4	
					13/16	
$N_1 = 5$	$N_2 = 6$	3/5	2/3	2/3	5/6	5/6
	7	4/7	23/35	5/7	29/35	6/7
	8	11/20	5/8	27/40	4/5	4/5
	9	5/9	3/5	31/45	7/9	4/5
	10	1/2	3/5	7/10	7/10	4/5
	15	8/15	3/5	2/3	11/15	11/15
	20	1/2	11/20	3/5	7/10	3/4
$N_1 = 6$	$N_2 = 7$	23/42	4/7	29/42	5/7	5/6
	8	1/2	7/12	2/3	3/4	3/4
	9	1/2	5/9	2/3	13/18	7/9
	10	1/2	17/30	19/30	7/10	11/15
	12	1/2	7/12	7/12	2/3	3/4
	18	4/9	5/9	11/18	2/3	13/18
	24	11/24	1/2	7/12	5/8	2/3

