

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2015

TITLE OF PAPER : NON-PARAMETRIC ANALYSIS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

REQUIREMENTS : CALCULATOR

**INSTRUCTIONS : ANSWER ANY THREE (3) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

ANSWER ANY THREE QUESTIONS

For all questions, clearly state the name of the test, the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, the decision & the conclusions.

QUESTION ONE.

[12 + 8 marks]

- Total annual precipitation is recorded yearly for 19 years. This record is examined to see if the amount of precipitation is tending to increase or decrease. The precipitation in inches was 45.25, 45.83, 41.77, 36.26, 45.37, 52.25, 35.37, 57.16, 35.37, 58.32, 41.05, 33.72, 45.73, 37.90, 41.72, 36.07, 49.83, 36.24, and 39.90. Indicate the appropriate conclusion with $\alpha = 0.01$ and also calculate the P-value.
- An item A is manufactured using certain process. Item B serves the same function as A but manufactured using a new process. The manufacturer wishes to determine whether B is preferred to A by the consumer, so she selects a random sample consisting of 10 consumers, gives each of them one A and one B and asks them to use the items for some period of time. At the end of the allotted period of time the consumers report their preferences to the manufacturer. Eight consumers preferred B to A, 1 preferred A to B, and 1 reported "no preference." Use an appropriate test to find out the manufacturer's decision at 5% level of significance. Also calculate the P-value.

QUESTION TWO.

[20 marks]

Twelve MBA graduates are studied to measure the strength of the relationship between their score on the GMAT, which they took prior to entering graduate school, and their grade point average while they were in the MBA program. Their GMAT scores and their GPAs are given below:

Student	1	2	3	4	5	6	7	8	9	10	11	12
GMAT	710	610	640	580	545	560	610	530	560	540	570	560
GPA	4.0	4.0	3.9	3.8	3.7	3.6	3.5	3.5	3.5	3.3	3.2	3.2

Use either Spearman's ρ test or Kendall's τ to test whether GMAT scores and GPAs are positively correlated. Use $\alpha = 0.05$.

QUESTION THREE.

[20 marks]

Each of the three aerospace companies has randomly selected a group of technical staff worker to participate in a training conference sponsored by a supplier firm. Three companies have sent 6, 5, and 7 employees, respectively. At the beginning of the session, a preliminary test is given, and the scores are shown in the following table. At the 0.05 level, use the Kruskal-Wallis test to determine whether evidence exists to conclude that the median scores for the three populations of technical staff workers could be the same?

Technical Staff from		
Firm 1	Firm 2	Firm 3
67	64	75
57	73	61
62	72	76
59	68	71
70	65	78
67		74
		79

QUESTION FOUR.

[20 marks]

Two computer software packages are being considered for use in the inventory control department of a small manufacturing firm. The firm has selected 12 different computing tasks that are typical of the kinds of jobs such a package would have to perform, and then recorded the number of seconds each package required to complete each task, given in the following table. Do the data present sufficient evidence to indicate that both software's performed the jobs at the same time? Analyse the data by using the Wilcoxon Signed Rank Test with $\alpha = 0.10$.

Computing Task	Time Required for Software Packages X and Y	
	X _i	Y _i
A	24.0	23.1
B	16.7	20.4
C	21.6	17.7
D	23.7	20.7
E	37.5	42.1
F	31.4	36.1
G	14.9	21.8
H	37.3	40.3
I	17.9	26.0
J	15.5	15.5
K	29.0	35.4
L	19.9	25.5

TABLE A1 Normal Distribution^a

<i>p</i>	Selected values									
	<i>Z</i> _{0.001} = -3.7190	<i>Z</i> _{0.005} = -3.2905	<i>Z</i> _{0.05} = -1.9600	<i>Z</i> _{0.95} = -1.6449	<i>Z</i> _{0.995} = 3.7190	<i>Z</i> _{0.999} = 3.2905	<i>Z</i> _{0.9995} = 1.9600	<i>Z</i> _{0.9999} = 1.6449	<i>Z</i> _{0.001} = -3.7190	<i>Z</i> _{0.005} = -3.2905
0.00	-3.0902	-2.8782	-2.7478	-2.6521	-2.5758	-2.5121	-2.4573	-2.4089	-2.3656	
0.01	-2.3263	-2.2904	-2.2571	-2.2262	-2.1973	-2.1701	-2.1444	-2.1201	-2.0969	-2.0749
0.02	-2.0537	-2.0335	-2.0141	-1.9954	-1.9774	-1.9600	-1.9431	-1.9268	-1.9110	-1.8957
0.03	-1.8808	-1.8663	-1.8522	-1.8384	-1.8250	-1.8119	-1.7991	-1.7866	-1.7744	-1.7624
0.04	-1.7507	-1.7392	-1.7279	-1.7169	-1.7060	-1.6954	-1.6849	-1.6747	-1.6646	-1.6546
0.05	-1.6449	-1.6352	-1.6258	-1.6164	-1.6072	-1.5982	-1.5893	-1.5805	-1.5718	-1.5632
0.06	-1.5548	-1.5464	-1.5382	-1.5301	-1.5220	-1.5141	-1.5063	-1.4985	-1.4909	-1.4833
0.07	-1.4758	-1.4684	-1.4611	-1.4538	-1.4466	-1.4395	-1.4325	-1.4255	-1.4187	-1.4118
0.08	-1.4051	-1.3984	-1.3917	-1.3852	-1.3787	-1.3722	-1.3658	-1.3595	-1.3532	-1.3469
0.09	-1.3408	-1.3346	-1.3285	-1.3225	-1.3165	-1.3106	-1.3047	-1.2988	-1.2930	-1.2873
0.10	-1.2816	-1.2759	-1.2702	-1.2646	-1.2591	-1.2536	-1.2481	-1.2426	-1.2372	-1.2319
0.11	-1.2265	-1.2212	-1.2160	-1.2107	-1.2055	-1.2004	-1.1952	-1.1901	-1.1850	-1.1800
0.12	-1.1750	-1.1700	-1.1650	-1.1601	-1.1552	-1.1503	-1.1455	-1.1407	-1.1359	-1.1311
0.13	-1.1264	-1.1217	-1.1170	-1.1123	-1.1077	-1.1031	-1.0985	-1.0939	-1.0893	-1.0848
0.14	-1.0803	-1.0758	-1.0714	-1.0669	-1.0625	-1.0581	-1.0537	-1.0494	-1.0450	-1.0407
0.15	-1.0364	-1.0322	-1.0279	-1.0237	-1.0194	-1.0152	-1.0110	-1.0069	-1.0027	-0.9986
0.16	-0.9945	-0.9904	-0.9863	-0.9822	-0.9782	-0.9741	-0.9701	-0.9661	-0.9621	-0.9581
0.17	-0.9542	-0.9502	-0.9463	-0.9424	-0.9385	-0.9346	-0.9307	-0.9269	-0.9230	-0.9192
0.18	-0.9154	-0.9116	-0.9078	-0.9040	-0.9002	-0.8965	-0.8927	-0.8890	-0.8853	-0.8816
0.19	-0.8779	-0.8742	-0.8705	-0.8669	-0.8633	-0.8596	-0.8560	-0.8524	-0.8488	-0.8452
0.20	-0.8416	-0.8381	-0.8345	-0.8310	-0.8274	-0.8239	-0.8204	-0.8169	-0.8134	-0.8099
0.21	-0.8064	-0.8030	-0.7995	-0.7961	-0.7926	-0.7892	-0.7858	-0.7824	-0.7790	-0.7756
0.22	-0.7722	-0.7688	-0.7655	-0.7621	-0.7588	-0.7554	-0.7521	-0.7488	-0.7454	-0.7421
0.23	-0.7388	-0.7356	-0.7323	-0.7290	-0.7257	-0.7225	-0.7192	-0.7160	-0.7128	-0.7095
0.24	-0.7063	-0.7031	-0.6999	-0.6967	-0.6935	-0.6903	-0.6871	-0.6840	-0.6808	-0.6776

TABLE A1 (Continued)

<i>p</i>	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.25	-0.6745	-0.6713	-0.6682	-0.6651	-0.6620	-0.6588	-0.6557	-0.6526	-0.6495	-0.6464
0.26	-0.6433	-0.6403	-0.6372	-0.6341	-0.6311	-0.6280	-0.6250	-0.6219	-0.6189	-0.6158
0.27	-0.6128	-0.6098	-0.6068	-0.6038	-0.6008	-0.5978	-0.5948	-0.5918	-0.5888	-0.5858
0.28	-0.5828	-0.5799	-0.5769	-0.5740	-0.5710	-0.5681	-0.5651	-0.5622	-0.5592	-0.5563
0.29	-0.5534	-0.5505	-0.5476	-0.5446	-0.5417	-0.5388	-0.5359	-0.5330	-0.5302	-0.5273
0.30	-0.5244	-0.5215	-0.5187	-0.5158	-0.5129	-0.5101	-0.5072	-0.5044	-0.5015	-0.4987
0.31	-0.4959	-0.4930	-0.4902	-0.4874	-0.4845	-0.4817	-0.4789	-0.4761	-0.4733	-0.4705
0.32	-0.4677	-0.4649	-0.4621	-0.4593	-0.4565	-0.4538	-0.4510	-0.4482	-0.4454	-0.4427
0.33	-0.4399	-0.4372	-0.4344	-0.4316	-0.4289	-0.4261	-0.4234	-0.4207	-0.4179	-0.4152
0.34	-0.4125	-0.4097	-0.4070	-0.4043	-0.4016	-0.3989	-0.3961	-0.3934	-0.3907	-0.3880
0.35	-0.3853	-0.3826	-0.3799	-0.3772	-0.3745	-0.3719	-0.3692	-0.3665	-0.3638	-0.3611
0.36	-0.3585	-0.3558	-0.3531	-0.3505	-0.3478	-0.3451	-0.3425	-0.3398	-0.3372	-0.3345
0.37	-0.3319	-0.3292	-0.3266	-0.3239	-0.3213	-0.3186	-0.3160	-0.3134	-0.3107	-0.3081
0.38	-0.3055	-0.3029	-0.3002	-0.2976	-0.2950	-0.2924	-0.2898	-0.2871	-0.2845	-0.2819
0.39	-0.2793	-0.2767	-0.2741	-0.2715	-0.2689	-0.2663	-0.2637	-0.2611	-0.2585	-0.2559
0.40	-0.2532	-0.2508	-0.2482	-0.2456	-0.2430	-0.2404	-0.2378	-0.2353	-0.2327	-0.2301
0.41	-0.2275	-0.2250	-0.2224	-0.2198	-0.2172	-0.2147	-0.2121	-0.2096	-0.2070	-0.2045
0.42	-0.2019	-0.1993	-0.1968	-0.1942	-0.1917	-0.1891	-0.1866	-0.1840	-0.1815	-0.1789
0.43	-0.1764	-0.1738	-0.1713	-0.1687	-0.1662	-0.1637	-0.1611	-0.1586	-0.1560	-0.1535
0.44	-0.1510	-0.1484	-0.1459	-0.1434	-0.1408	-0.1383	-0.1358	-0.1332	-0.1307	-0.1282
0.45	-0.1257	-0.1231	-0.1206	-0.1181	-0.1156	-0.1130	-0.1105	-0.1080	-0.1055	-0.1030
0.46	-0.1004	-0.0979	-0.0954	-0.0929	-0.0904	-0.0878	-0.0853	-0.0828	-0.0803	-0.0778
0.47	-0.0753	-0.0728	-0.0702	-0.0677	-0.0652	-0.0627	-0.0602	-0.0577	-0.0552	-0.0527
0.48	-0.0502	-0.0476	-0.0451	-0.0426	-0.0401	-0.0376	-0.0351	-0.0326	-0.0301	-0.0276
0.49	-0.0251	-0.0226	-0.0201	-0.0175	-0.0150	-0.0125	-0.0100	-0.0075	-0.0050	-0.0025
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231

Table A1 (Continued)

<i>p</i>	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

<i>p</i>	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3656	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4116	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4756	1.4823	1.4908	1.4985	1.5063	1.5141	1.5220	1.5301	1.5383	1.5464
0.94	1.5546	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

SOURCE: Generated by R. L. Iman. Used with permission.

*The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

TABLE A2 Chi-Squared Distribution^a

	<i>p</i> = 0.750	0.900	0.950	0.975	0.990	0.995	0.999
<i>k</i> = 1	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	4.108	6.251	7.815	9.348	11.34	12.84	16.27
4	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	28.24	33.20	36.42	39.37	42.98	45.56	51.18
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.58	101.9	106.6	112.3	116.3	124.8
90	98.65	107.6	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.4
<i>z_p</i>	0.675	1.282	1.645	1.960	2.326	2.576	3.090

For *k* > 100 use the approximation *w_p* = (*i*)*(z_p + √(2*k* - 1))²*, or the more accurate *w_p* =

$$k \left(1 - \frac{2}{9k} + z_p \sqrt{\frac{2}{9k}} \right)^2, \text{ where } z_p \text{ is the value from the standardized normal distribution shown in the bottom of the table.}$$

SOURCE: Abridged from Table 8, Vol. I of Pearson and Hartley (1976), with permission from the Biometrika Trustees.

*The entries in this table are quantiles *w_p* of a chi-squared random variable *W* with *k* degrees of freedom, selected so *P(W ≤ w_p) = p* and *P(W > w_p) = 1 - p*.

TABLE A3 Binomial Distribution^a

<i>n</i>	<i>y</i>	<i>p</i> = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
1	0	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7182	0.6480	0.5748
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024
8	0	0.6622	0.4943	0.3262	0.2050	0.1364	0.0847	0.0532	0.0316	0.0164
	1	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083
9	0	0.6100	0.4998	0.3988	0.2953	0.1971	0.0972	0.0444	0.0207	0.0071
	1	0.9999	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9463
10	0	0.5600	0.4600	0.3600	0.2600	0.1600	0.0600	0.0200	0.0000	0.0000
	1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9994	0.9984	0.9963

TABLE A3 (Continued)

TABLE A3
(Continued)

TABLE A3 (Continued)

TABLE A3
(Continued)

TABLE A3
(Continued)

TABLE A1 (Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

<i>n</i>	<i>y</i>	<i>p</i> = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0022	0.0005	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.3239	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000	0.0000
9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000	0.0000
10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000	0.0000
11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000	0.0000
12	0.9165	0.8273	0.6191	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000	0.0000
13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002	0.0000
14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020	0.0000
15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132	0.0000
16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7831	0.5887	0.2946	0.0645	0.0000
17	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453	0.0000
18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0207	0.0064	0.0116	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.2517	0.1308	0.0565	0.0198	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000	0.0000
10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000	0.0000
11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000	0.0000
12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000	0.0000
13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000	0.0000
14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003	0.0000
15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0076	0.0000
16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159	0.0000
17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755	0.0000
18	1.0000	0.9999	0.9995	0.9975	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642	0.0000
19	1.0000	1.0000	0.9996	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415	0.2453	0.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* γ has the binomial distribution with parameters n and p . The entries are the values of $P(\gamma \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95. For n larger than 20, the r th quantile γ_r of a binomial random variable may be approximated using $\gamma_r = np + z_r \sqrt{np(1-p)}$, where z_r is the r th quantile of a standard normal random variable, obtained from Table A1.

TABLE A7 (Continued)

<i>n</i>	<i>p</i>	<i>m</i> = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
17	0.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220	224
	0.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235	240
	0.01	154	158	162	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242	247
	0.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253	259
	0.05	157	163	169	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263	269
	0.10	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274	281
18	0.001	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243	248
	0.005	171	174	178	183	188	193	198	203	209	214	219	225	230	236	242	247	253	259	264
	0.01	172	176	181	186	191	196	202	208	213	219	225	231	237	242	248	254	260	266	272
	0.025	174	179	184	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278	284
	0.05	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288	295
	0.10	178	185	192	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300	307
19	0.001	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268	273
	0.005	191	194	198	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284	290
	0.01	192	195	200	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292	298
	0.025	193	198	204	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304	310
	0.05	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314	321
	0.10	198	205	212	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326	334
20	0.001	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293	299
	0.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310	316
	0.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318	325
	0.025	213	219	225	231	238	245	251	259	266	273	280	287	294	301	309	316	323	330	338
	0.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341	349
	0.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354	362

For *n* or *m* greater than 20, the *p*th quantile *w_p* of the Mann-Whitney test statistic may be approximated by

$$w_p = n(N + 1)/2 + z_{p/2} \sqrt{nm(N + 1)/12}$$

where *z_p* is the *p*th quantile of a standard normal random variable, obtained from Table A1, and where *N* = *m* + *n*.

*The entries in this table are quantiles *w_p* of the Mann-Whitney test statistic *T*, given by Equation 5.1.1, for selected values of *p*. Note that *P(T < w_p)* ≤ *p*. Upper quantiles may be found from the equation

$$w_p = n(n + m + 1) - w_{1-p}$$

Critical regions correspond to values of *T* less than (or greater than) but not equal to the appropriate quantile.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes*

Sample Sizes	<i>W_{0.90}</i>	<i>W_{0.95}</i>	<i>W_{0.99}</i>
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.1643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1657
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.9500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.1211	5.5152	6.9818
5, 4, 1	3.9600	4.6600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6300	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.9363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

SOURCE: Adapted from Iman, Quadra, and Alexander (1975), with permission from the American Mathematical Society.

*The null hypothesis may be rejected at the level *α* if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A7 Quantiles of the Mann-Whitney Test Statistic^a

<i>n</i>	<i>p</i>	<i>m</i> = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0.001	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	0.005	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4
	0.01	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4	5	5
	0.025	3	3	3	3	3	3	3	3	4	4	4	5	5	5	5	5	6	6	6
	0.05	3	3	3	4	4	4	5	5	5	5	5	6	6	6	6	6	7	7	7
	0.10	3	4	4	5	5	5	6	6	6	7	7	8	8	9	9	9	9	10	10
	0.001	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	7	7
	0.005	6	6	6	6	6	6	6	7	7	7	8	8	9	9	9	9	9	10	10
	0.01	6	6	6	6	6	6	7	7	8	8	9	9	10	10	10	11	11	11	12
	0.025	6	6	6	7	8	8	9	9	10	10	11	11	12	12	12	13	13	13	14
3	0.05	6	7	7	8	9	9	10	11	11	11	12	12	13	14	14	15	16	16	17
	0.10	7	8	8	9	10	11	12	12	13	14	15	16	17	17	18	19	20	21	22
	0.001	10	10	10	10	10	10	10	10	11	11	11	12	12	12	13	13	14	14	14
	0.005	10	10	10	11	11	12	12	13	13	14	14	14	15	16	16	17	17	18	19
	0.01	10	10	10	11	12	12	13	14	14	15	15	16	16	17	18	18	19	20	21
	0.025	10	10	11	12	13	14	15	15	16	17	18	19	20	21	22	23	25	26	27
	0.05	10	11	12	13	14	15	16	17	18	19	20	21	23	24	26	27	28	31	32
	0.10	11	12	14	15	16	17	18	20	21	22	23	24	26	27	28	29	31	32	33
	0.001	15	15	15	15	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
	0.005	15	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
4	0.01	15	15	16	17	18	19	21	22	23	24	25	27	28	29	30	31	33	34	35
	0.025	15	16	17	18	19	21	22	24	25	27	28	29	31	32	34	35	36	38	39
	0.05	16	17	18	20	21	22	24	25	27	28	29	31	33	34	36	38	39	41	44
	0.10	17	18	20	21	23	24	26	28	29	31	33	34	36	38	39	41	43	46	46
	0.001	21	21	21	21	21	23	24	25	25	26	26	27	28	29	30	31	32	33	34
	0.005	21	21	22	23	24	25	26	28	29	30	31	32	33	34	35	37	38	39	40
	0.01	21	21	23	24	25	26	28	30	31	33	34	35	37	38	40	41	42	44	44
	0.025	21	23	24	25	27	28	30	32	33	35	36	38	39	41	43	44	46	47	49
	0.05	22	24	25	27	29	30	32	34	36	38	39	41	43	45	47	49	51	53	58
	0.10	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	56	58	60
5	0.001	28	28	28	29	30	30	31	32	34	35	36	37	38	39	40	42	43	44	45
	0.005	28	28	29	30	32	33	35	36	38	39	41	42	44	45	47	48	50	51	53
	0.01	28	29	30	32	33	35	36	38	40	41	43	45	46	48	50	52	53	55	57
	0.025	28	30	32	34	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
	0.05	29	31	33	35	37	39	42	44	46	48	50	53	55	57	59	62	64	66	68
	0.10	30	33	35	37	40	42	45	47	50	52	55	57	60	62	65	67	70	72	75
	0.001	36	36	36	37	38	39	41	42	43	45	46	48	49	51	52	54	55	57	58
	0.005	36	36	38	39	41	43	44	46	48	50	52	54	55	57	59	61	63	65	67
	0.01	36	37	39	41	43	44	46	48	50	52	54	56	59	61	63	65	67	69	71
	0.025	37	39	41	43	45	47	50	52	54	56	58	60	63	65	68	70	73	75	78
6	0.05	38	40	42	45	47	50	52	55	57	60	63	65	68	70	73	76	78	81	84
	0.10	39	42	44	47	50	53	56	59	61	64	67	70	73	76	79	82	85	88	91

TABLE A7 (Continued)

<i>n</i>	<i>p</i>	<i>m</i> = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
9	0.001	45	45	45	47	48	49	51	53	54	56	58	60	61	63	65	67	69	71	72
	0.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	73	75	77	79	82
	0.01	45	47	49	51	53	55	57	60	62	64	67	69	72	74	77	79	82	84	86
	0.025	46	48	50	53	56	58	61	63	66	69	72	74	77	80	83	85	88	91	94
	0.05	47	50	52	55	58	61	64	67	70	73	76	79	82	85	88	90	94	97	100
	0.10	48	51	55	58	61	64	68	71	74	77	81	84	87	91	94	98	101	104	108
	0.001	55	55	56	57	59	61	62	64	66	68	70	73	75	77	79	81	83	85	88
	0.005	55	56	58	60	62	65	67	69	72	74	77	80	82	85	87	90	93	95	98
	0.01	55	57	59	62	64	67	69	72	75	78	80	83	86	89	92	94	97	100	103
	0.025	56	59	61	64	67	70	73	76	79	82	85	89	92	95	98	101	104	111	118
10	0.05	57	60	63	67	70	73	76	80	83	87	90	93	97	100	104	107	111	114	118
	0.10	59	62	66	69	73	77	80	84	88	92	95	99	103	107	110	114	118	122	126
	0.001	66	66	67	69	71	73	75	77	79	82	84	87	89	91	94	96	99	101	104
	0.005	66	67	69	72	74	77	80	83	85	88	91	94	97	100	103	106	109	112	115
	0.01	66	68	71	74	76	79	82	85	89	92	95	98	101	104	108	111	114	117	120
	0.025	67	70	73	76	80	83	86	90	93	97	100	104	107	111	114	118	122	125	129
	0.05	68	72	75	79	83	86	90	94	98	101	105	109	113	117	121	124	128	132	136
	0.10	70	74	78	82	86	90	94	98	103	107	111	115	119	124	128	132	136	140	145
	0.001	78	78	79	81	83	86	88	91	93	96	98	102	104	106	110	113	116	118	121
	0.005	78	80	82	85	88	91	94	97	100	103	107	110	114	117	121	125	128	132	139
11	0.01	78	81	84	87	90	93	96	100	103	107	110	114	117	121	125	128	132	135	139
	0.025	80	83	86	90	93	97	101	105	108	112	116	120	124	128	132	136	140	144	148
	0.05	81	84	88	92	96	100	105	109	111	117	121	126							

TABLE A10 Quantiles of Spearman's ρ^*

<i>n</i>	0.900	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For *n* greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p = \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glasser and Winter (1961), with corrections, with permission from the Biometrika Trustees.

*The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

<i>n</i>	0.900	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A11 (Continued)

<i>n</i>	<i>p</i> = 0.900	0.950	0.975	0.990	0.995
18	103 (0.1465)	131 (0.1863)	155 (0.2205)	183 (0.2603)	203 (0.2888)
19	107 (0.1444)	137 (0.1849)	161 (0.2173)	191 (0.2578)	211 (0.2848)
20	110 (0.1372)	142 (0.1821)	168 (0.2154)	198 (0.2538)	220 (0.2821)
21	114 (0.1390)	146 (0.1780)	174 (0.2122)	206 (0.2512)	228 (0.2780)
22	119 (0.1382)	151 (0.1754)	181 (0.2102)	213 (0.2474)	235 (0.2729)
23	123 (0.1362)	157 (0.1739)	187 (0.2071)	221 (0.2447)	245 (0.2713)
24	128 (0.1353)	162 (0.1712)	194 (0.2051)	228 (0.2410)	252 (0.2664)
25	132 (0.1333)	168 (0.1697)	200 (0.2020)	236 (0.2383)	262 (0.2646)
26	135 (0.1304)	173 (0.1671)	207 (0.2000)	245 (0.2367)	271 (0.2618)
27	141 (0.1304)	179 (0.1656)	213 (0.1970)	253 (0.2340)	279 (0.2581)
28	144 (0.1277)	186 (0.1649)	220 (0.1950)	260 (0.2305)	288 (0.2553)
29	150 (0.1276)	190 (0.1616)	228 (0.1939)	268 (0.2279)	296 (0.2517)
30	153 (0.1249)	197 (0.1608)	233 (0.1902)	277 (0.2261)	305 (0.2490)
31	159 (0.1247)	203 (0.1592)	241 (0.1890)	285 (0.2235)	315 (0.2471)
32	162 (0.1222)	208 (0.1569)	248 (0.1870)	294 (0.2217)	324 (0.2443)
33	168 (0.1219)	214 (0.1553)	256 (0.1858)	302 (0.2192)	334 (0.2424)
34	173 (0.1209)	221 (0.1544)	263 (0.1838)	311 (0.2173)	343 (0.2397)
35	177 (0.1192)	227 (0.1529)	269 (0.1811)	319 (0.2148)	353 (0.2377)
36	182 (0.1182)	232 (0.1506)	276 (0.1792)	328 (0.2130)	362 (0.2351)
37	186 (0.1165)	240 (0.1504)	284 (0.1779)	336 (0.2105)	372 (0.2331)
38	191 (0.1155)	245 (0.1482)	291 (0.1760)	345 (0.2087)	381 (0.2305)
39	197 (0.1151)	251 (0.1467)	299 (0.1748)	355 (0.2075)	391 (0.2285)
40	202 (0.1141)	258 (0.1458)	306 (0.1729)	364 (0.2056)	402 (0.2271)

For *n* greater than 60, approximate quantiles of *T* may be obtained from

$$w_p \approx z_p \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

where z_p is from the standard normal distribution given by Table A1. Approximate quantiles of τ may be obtained from

$$w_p \approx z_p \frac{\sqrt{2(2n+5)}}{3\sqrt{n(n-1)}}$$

Critical regions correspond to values of *T* greater than (or less than) but not including the appropriate quantile. Note that the median of *T* is 0. Quantiles for τ are obtained by dividing the quantiles of *T* by $(n-1)/2$.

SOURCE: Adapted from Table I, Best (1974), with permission from the author.

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

<i>n</i>	$w_{0.001}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$\frac{n(n+1)}{2}$
4	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	11	13	14	16	36
9	2	4	6	9	11	13	15	18	20	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	56	65	71	76.5	153
18	28	33	41	48	55	63	73	80	85.5	171
19	33	38	47	54	63	71	82	89	95	190
20	38	44	53	61	70	79	91	98	105	210
21	44	50	59	68	78	87	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	163	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	240	266	282	297.5	595
35	160	175	196	214	236	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	266	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903

TABLE A12 (Continued)

									$n(n+1)/2$	
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n + 1)/4] + z_p \sqrt{n(n + 1)(2n + 1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE. Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n + 1)/2 - w_{1-p}$$

where $n(n + 1)/2$ is given in the right hand column in the table. Note that $P(T^* < w_p) \leq p$ and $P(T^* > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^* less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic*

One-Sided Test						Two-Sided Test						
$p = 0.90$		0.95	0.975	0.99	0.995	$p = 0.80$		0.90	0.95	0.975	0.99	0.995
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344	
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337	
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330	
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323	
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317	
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311	
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305	
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300	
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295	
10	0.323	0.369	0.409	0.457	0.499	30	0.190	0.218	0.242	0.270	0.290	
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266	0.285	
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281	
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277	
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273	
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269	
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265	
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262	
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258	
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255	
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252	
Approximation for $n > 40$							1.07	1.22	1.36	1.52	1.63	
							\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}	

SOURCE. Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

*The entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.