UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2015

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|---|---|---|
| TITLE OF PAPER | : | SAMPLE SURVEY THEORY |
| COURSE CODE | : | ST306 |
| TIME ALLOWED | : | TWO (2) HOURS |
| REQUIREMENTS | : | CALCULATOR AND STATISTICAL TABLES |
| INSTRUCTIONS | : | ANSWER ANY THREE QUESTIONS |

## Question 1 [20 marks, 4+3+5+8]

A researcher selects a simple random sample of 2055 farms from the 75 308 farms in a large region in a developing country, and the number of cattle ($y$) and the total area under cattle ($x$) were recorded for each farm. The results were as follows.

| | |
|---|---|
| Sample total number of cattle, $\sum y_i$ | 25 751 |
| Sample total area (hectares), $\sum x_i$ | 62 989 |

The sum of the squares is $\sum y_i^2 = 2596737$. The total area under cattle in this region is 2 353 365.

(a) Using the mean of the simple random sample, estimate the total number of cattle in the region, and the standard error of your estimator.

(b) The researcher seeks your advice on how the supplementary information on the area under cattle in the region might be used to estimate the total number of cattle in the region.

(c) Discuss briefly why either a ratio or regression estimator could be appropriate for these data. Explain how you would decide whether to use a ratio or regression estimator.

(d) The researcher decides to use a ratio estimator, and asks you to comment on his results compared with those obtained in part (i). You may assume that the ratio estimate of the total number of cattle in the region is 962 055, and its estimated standard error is 14 020.7.

Comment on the relative standard errors. If it was suggested to you that the ratio estimate should not be used because it is biased, how would you reply?

(e) Explain how and why stratification and clustering might be useful in such a survey, and what practical problems they could help to overcome.

## Question 2 [20 marks, 3+7+4+6]

A simple random sample of 10 hospitals was selected from a population of 33 hospitals that had received state funding to upgrade their emergency medical services. Within each of the selected hospitals, the records were examined for all patients hospitalised in the past 12 months for trauma, which is defined as body wounds or shock produced by sudden physical injury due to accident or violence. The numbers of patients hospitalised for trauma, and the numbers of patients with trauma who were discharged dead, for the selected hospitals are given below.

| Hospital | Number of patients hospitalised for trauma | Number with trauma discharged dead |
|---|---|---|
| 1 | 560 | 4 |
| 2 | 190 | 4 |
| 3 | 260 | 2 |
| 4 | 370 | 4 |
| 5 | 190 | 4 |
| 6 | 130 | 0 |
| 7 | 170 | 9 |
| 8 | 170 | 2 |
| 9 | 60 | 0 |
| 10 | 110 | 1 |

(a) Explain why this design may be considered as a cluster sample. What are the first-stage and second-stage units?

(b) (i) Estimate the total number of people hospitalised for trauma for the 33 hospitals. State a property of your estimator. Give an approximate 95% confidence interval for this total number.

(ii) Let $p$ be the true proportion of people suffering from trauma discharged dead. Explain why an estimator of $p$ based on the above cluster sample is a ratio estimator. You are given that the 95% confidence interval for $p$ based on use of a ratio estimator is (0.006, 0.021). Explain what this confidence interval shows.

(c) Give reasons why, for this survey, cluster sampling might be preferred to stratified random sampling using each of the 33 hospitals as a stratum. What might be the drawbacks of cluster sampling?

Discuss any improvements you might make if another survey was being planned on the same topic.

## Question 3 [20 marks, 6+6+8]

(a) Consider a population of farms on a $25 \times 25$ grid of varying sizes and shapes. If we randomly select a single square on this grid, then letting $x_i$ = the area of farm $i$ and $A = 625$ total units, the probability that farm $i$ is selected is: $p_i = \frac{x_i}{A} = \frac{x_i}{625}$.

| $y_i$ =Workers | $p_i = \frac{x_i}{A} = \frac{\text{Size of Farm}}{\text{Total Area}}$ |
|---|---|
| 2 | 5/625 |
| 8 | 28/625 |
| 4 | 12/625 |
| 8 | 14/625 |

The table above shows a replacement sample of 5 farms selected with probability-proportional-to-size (PPS). Compute:

(i) The estimated number of workers (and associated standard errors).

(ii) The estimated number of farms (and associated standard errors).

using the Horvitz-Thompson estimator.

(b) For a health survey of a large population, estimates are wanted for two proportions, each measuring the yearly incidence of a disease? For designing the sample, we guess that one occurs with a frequency of 50 percent and the other with a frequency of only 1 percent. To obtain the same standard error of $\frac{1}{2}$ percent, how large an srs is needed for each disease. The large difference in the needed $n$ causes a re-evaluation of the requirements. Now the same coefficient of variation of 0.05 is declared desirable for each disease; how large a sample is needed for each disease?

## Question 4 [20 marks, 5+5+6+4]

(a) A survey is planned to study family income in a mixed urban and rural population. Discuss any practical difficulties that might arise in defining "income", in defining "family", and in combining information from rural and urban areas.

(b) A survey organisation defines the 'true level of business confidence' for a particular sector of economic activity as the proportion of managing directors of all companies in that sector who expect prospects for their company to improve in the next six months.

In a pilot survey in the light engineering sector, the managing directors of 67 out of a random sample of 125 companies stated that they expected prospects for their company to improve in the next six months.

(i) Using this information, obtain an approximate 95% confidence interval for $p$, the proportion of companies expecting an improvement. Explain what this confidence interval shows. You may assume that N, the number of companies in this sector, is much larger than 125.

(ii) A business analyst wants to calculate an approximate 95% confidence interval for $p$. What sample size would be required to produce a 95% confidence interval which has a width of 0.08?

(c) Smart meters are to be installed in every home in the Swaziland from 2022. These devices will show exactly how much gas and electricity is used at any specific time, which could help households to be more energy efficient and cut their bills. There is limited evidence of how much and for how long Swazi consumers' behaviour might change after installing these meters. You have been asked to design a survey to assess this. Explain briefly the difference between longitudinal and cross-sectional surveys, and how the distinction might be relevant in this situation. What might be the drawbacks of a longitudinal survey here?

## Question 5 [20 marks, 4+7+4+5]

The formula for the variance of the estimator of a population mean based on a stratified (random) sample is

$$V = \sum_{h=1}^{L} W_h^2 \frac{s_h^2}{n_h}\left(1 - \frac{n_h}{N_h}\right).$$

Define the terms $N_h$, $S_h$, $n_h$ and $W_h$ in the above formula. Explain the conditions under which stratified sampling may be superior to simple random sampling.

The Chief Education Officer for a region wishes to estimate the total number of children who have played truant in the past week (that is, who have been absent from lessons without good reason). The region is divided into four education authorities (strata) and a random sample of ten schools is taken from each education authority. The results are as follows.

| Education authority $h$ | Total number of schools | Number of children who have played truant ($y$) in schools selected | Sample mean | Sample standard deviation |
|---|---|---|---|---|
| 1 | 141 | 4, 8, 10, 0, 1, 4, 0, 12, 1, 0 | 4.0 | 4.50 |
| 2 | 471 | 5, 15, 6, 9, 8, 15, 17, 10, 6, 16 | 10.7 | 4.62 |
| 3 | 256 | 23, 26, 11, 23, 14, 17, 33, 0, 6, 22 | 17.5 | 9.92 |
| 4 | 1499 | 2, 3, 3, 3, 4, 0, 3, 1, 2, 3 | 2.4 | 1.17 |

(a) Estimate the total number of children who have played truant in the past week and obtain an approximate 95% confidence interval for this total.

(b) The Officer wishes to report estimates of the total number of children who have played truant in the past week for each of the four education authorities, as supporting information. Obtain a point estimate and an approximate 95% confidence interval for this total number for education authority 1.

(c) The Officer is planning a new survey, and is intending to sample an equal number of schools from each authority in the region. Giving reasons, suggest another allocation method that might be preferred to compute the sample sizes in each authority. Use this method to compute the stratum sample sizes for a sample of 40 schools.

# Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n} \qquad \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i} \qquad \hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_i y_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x \qquad \hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L \qquad \hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h \qquad \hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_h s_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{\mathsf{V}}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{\mathsf{V}}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y}-\hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{p}_c M_i)^2}{n-1}$$

To estimate $\tau$, multiply $\hat{\mu}_{c(\cdot)}$ by $M$. To get the estimated variances, multiply $\hat{\mathsf{V}}(\hat{\mu}_{c(\cdot)})$ by $M^2$. If $M$ is not known, substitute $M$ with $Nm/n$. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

$$n \text{ for } \mu \text{ SRS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$$

$$n \text{ for } \tau \text{ SRS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$$

$$n \text{ for } p \text{ SRS} \qquad n = \frac{Np(1-p)}{(N-1)(d^2/z^2)+p(1-p)}$$

$$n \text{ for } \mu \text{ SYS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$$

$$n \text{ for } \tau \text{ SYS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$$

$$n \text{ for } \mu \text{ STR} \qquad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$$

$$n \text{ for } \tau \text{ STR} \qquad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$$

where $w_h = \frac{n_h}{n}$.

Allocations for STR $\mu$:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR $\tau$:

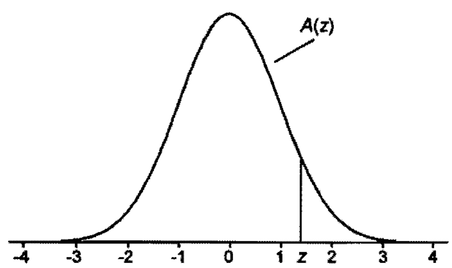
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR $p$:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

**TABLE A.1**

## Cumulative Standardized Normal Distribution




$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

### Table A.2

**t Distribution: Critical Values of t**

| Degrees of freedom | | *Significance level* | | | | | |
|---|---|---|---|---|---|---|---|
| | *Two-tailed test:* | 10% | 5% | 2% | 1% | 0.2% | 0.1% |
| | *One-tailed test:* | 5% | 2.5% | 1% | 0.5% | 0.1% | 0.05% |
| **1** | | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** | | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** | | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** | | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** | | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** | | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** | | 1.894 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** | | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** | | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** | | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** | | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** | | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** | | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** | | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** | | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** | | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **17** | | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| **18** | | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **19** | | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| **20** | | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| **21** | | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **22** | | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| **23** | | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| **24** | | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| **25** | | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **26** | | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| **27** | | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| **28** | | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| **29** | | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| **30** | | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **32** | | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| **34** | | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| **36** | | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| **38** | | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| **40** | | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **42** | | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| **44** | | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| **46** | | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| **48** | | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| **50** | | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **60** | | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| **70** | | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| **80** | | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| **90** | | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| **100** | | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **120** | | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| **150** | | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| **200** | | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| **300** | | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| **400** | | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| **500** | | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| **600** | | 1.647 | 1.964 | 2.333 | 2.584 | 3.104 | 3.307 |
| $\infty$ | | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |