UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2015

TITLE OF PAPER	:	INFERENTIAL STATISTICS
COURSE CODE	:	ST 220
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	THIS PAPER HAS SIX (6). ANSWER ANY THREE (3) QUESTIONS.

Question 1 [20 marks, 9+8+3]

(a) A pharmaceutical company is considering introducing new, easier to open, packaging for a drug used in the treatment of arthritis. The company seeks the views of two groups of patients. One group consists of those who have been using the existing packaging for a long time and the other group consists of new users. The preferences are shown below.

	Prefer new packaging	Prefer existing packaging
Long-term users	35	32
Recent users	25	8

Investigate, at the 5% significance level, whether there is evidence of a difference between the two groups of patients in their packaging preferences and briefly state your conclusions.

(i) Perform a χ^2 test at the 5% significance level to investigate whether there is an association between usage classification and the package preference. State your null hypothesis and report your conclusions.

(ii) Estimate the proportions who prefer the existing packaging for each of the two groups and calculate an approximate 95% confidence interval for the difference in these two proportions.

(iii) You could perform a hypothesis test to examine whether there is a difference in the proportions of those who prefer the old packaging in the two groups. Without performing this test, outline briefly how its results would relate to your answers to parts (a) and (b).

Question 2 [20 marks, 6+14]

A textile manufacturer produces long rolls of wide cotton cloth for making bed linen. The spun cotton thread he uses has occasional irregularities which produce flaws in the woven fabric. As part of a quality control exercise, a random sample of 180 ten-metre lengths of the cloth is examined carefully and the number of flaws in each length is noted. The results of this exercise are shown in the table below.

Number of flaws per ten-metre length, x	Frequency, f
0	30
1	58
2	49
3	29
4	7
5	7
>5	0

(a) Calculate the sample mean and sample variance, showing all working clearly. Comment on the sample mean and variance values in relation to the Poisson distribution.

(b) Perform a χ^2 goodness-of-fit test at the 5% significance level to investigate the null hypothesis that the number of flaws in ten-metre lengths of cloth has a Poisson distribution. Show all your working and report your conclusions.

Question 3 [20 marks, 4+5+7+3+1]

Gross mean weekly earnings (y, in £ per week) for a sample of male clerical workers of varying ages (x, in completed years) in a large company are as follows.

Earnings y	215	259	348	387	534	660	726	$\sum y = 3129$	$\sum y^2 = 1632011$
Age x	18	20	23	28	35	45	55	$\sum x = 224$	$\sum x^2 = 8312$

You are also given that $\sum xy = 116210$.

(a) Plot a scatter diagram of these data and comment on their suitability for simple linear regression analysis.

(b) Write down the models for

(i) simple linear regression of y on x,

(ii) simple linear regression of x on y.

Define your notation and explain clearly which model is better suited to fit the variables and data as defined in the table above.

(c) (i) Fit the simple linear regression model of y on x to the data above, find the equation of the fitted regression line, draw this line on your scatter diagram, and use the equation to estimate the mean weekly earnings at age 50.

(ii) You are given that the estimated standard error of the slope of the regression line is 1.128. Use this result to test for the significance of the regression slope at the 5% level, and state your conclusion clearly.

(iii) Your line manager asks you to use your model to estimate the mean weekly earnings at age 70. How would you answer him?

Question 4 [20 marks, 10+6+4]

(a) Random samples are taken from two populations with distributions $N(\mu_X, \sigma^2)$ and $N(\mu_Y, \sigma^2)$ (i.e. their variances are the same). The summary statistics for the two samples are shown in the following Table:

	Sample Size n	**Sample Mean** m	**Sample Variance** s^2
x-data	19	7.0	1.69
y-data	25	5.1	2.56

Compute a 95% confidence interval for the difference $\mu_X - \mu_Y$ between the two population means. Does the result support the view that there is no true difference between the population means? (Explain your reasoning!)

(b) A short-stay car park in a shopping area has spaces marked out for 90 cars. A local councillor notices that there are always some vacant spaces. He puts forward a plan to create a garden and seating area using part of the car park. This would reduce the number of parking spaces to 78.

(i) From a random sample of 33 users of the car park, 26 say that the car park will be too small if this plan is carried out. Carry out a test, at the 5% significance level, to determine whether more than half of the users of the car park think it will be too small.

(ii) The number of occupied spaces, x, in the car park is recorded on each of 16 randomly chosen occasions during shopping hours. The results may be summarised as follows:

$$\bar{x} = 59.9 \qquad s = 7.83$$

Construct a 95% confidence interval for the mean, μ, of the number of spaces occupied in the car park during shopping hours. Assume that the sample is drawn from a normal population.

Question 5 [20 marks, 8+12]

(a) The question of interest is whether the subject matter of the postgraduate courses is relevant to the level of female participation. The MSc courses are either finance or management oriented. The MBA is predominantly a management course. The MSc students are generally younger than the MBA students, who have work experience and are often sponsored by their employers. Subdividing the MScs into finance and management gives the following table.

	MSc (finance)	MSc (management)	MBA (management)
Male	611	105	169
Female	363	103	51

Analyse these data in a manner that allows you to comment on the relationship between the type of Master's course studied and the proportion of female students, and also to comment on any influence of the type of subject matter. Write a short report (5 or 6 sentences) to summarise your findings.

(b) A trial is undertaken to investigate the effect on fuel economy of 3 fuel additives **A**, **B** and **C**, where **A** and **B** are new and **C** is the current standard additive. The same driver drives the same car on a fixed test route during 20 working days. The additive used on each day is randomly assigned so that **A** and **B** are each used for 5 days and **C** is used for 10 days. The response variable measured each day is Y, the number of miles per gallon (mpg) achieved.

The results are shown in the following table.

Additive	y	Total
A	39, 35, 37, 36, 38	$\sum y_A = 185$
B	36, 41, 39, 40, 39	$\sum y_B = 195$
C	37, 33, 30, 34, 36, 34, 31, 36, 34, 35	$\sum y_C = 340$

You are given that the sum of squares of the observations is 26078.

Carry out an analysis of variance to test for differences between the effects on Y of the additives. State clearly your null and alternative hypotheses and present your conclusions.

Question 6 [20 marks, 3+6+3+8]

A population consists of the five values 1, 4, 9, 16 and 25.

(a) Calculate the population mean and variance.

(b) Write down all the samples of size two that may be drawn, with replacement, from this population, and calculate the sample mean of each.

(c) Let $\bar{X}$ denote the mean of a random sample of size two drawn, with replacement, from this population. Write down the expected value and variance of $\bar{X}$.

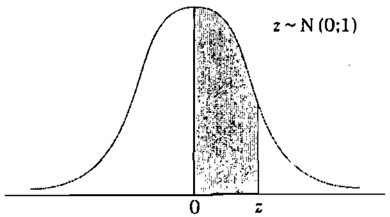
(d) For $\bar{X}$ as in part (iii), find $P(\bar{X} > 16.5)$. Find also an approximation to this quantity, using an appropriate Normal approximation [use of a continuity correction is not expected], and comment briefly on your results.

APPENDIX 1: LIST OF STATISTICAL TABLES

TABLE 1

The standard normal distribution (z)

This table gives the area under the standard normal curve between 0 and z
i.e. $P[0 < Z < z]$

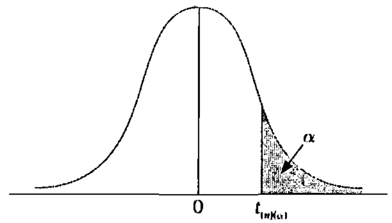


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2703	0.2734	0.2764	0.2793	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3557	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49991	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

TABLE 2

The t distribution

This table gives the value of $t_{(n)(\alpha)}$
where n is the degrees of freedom
i.e. $\alpha = P[t \geq t_{(n)(\alpha)}]$

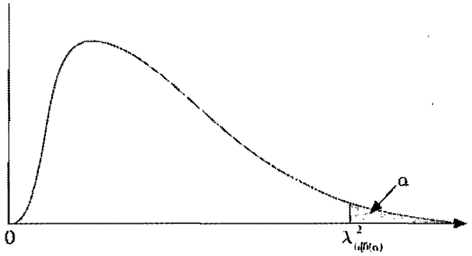


α / df	0.100	0.050	0.025	0.010	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	1.886	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.397	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.372	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.356	1.782	2.179	2.681	3.055	3.428
13	1.350	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.369	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
110	1.289	1.659	1.982	2.361	2.621	2.865
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.847
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
∞	1.282	1.645	1.960	2.327	2.576	2.807

TABLE 3

The Chi-Squared distribution (χ^2)

This table gives the value of $\chi^2_{(df)(\alpha)}$ where df is the degrees of freedom i.e. $\alpha = P[\chi^2 > \chi^2_{(df)(\alpha)}]$

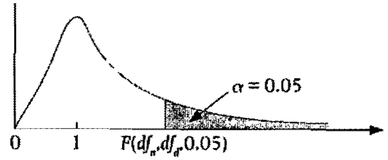


α / df	0.100	0.050	0.025	0.01	0.005	0.0025
1	2.707	3.843	5.026	6.637	7.881	9.142
2	4.605	5.991	7.378	9.210	10.597	11.983
3	6.251	7.815	9.348	11.345	12.838	14.321
4	7.779	9.488	11.143	13.277	14.860	16.424
5	9.236	11.071	12.833	15.086	16.750	18.386
6	10.645	12.592	14.449	16.812	18.548	20.249
7	12.017	14.067	16.013	18.475	20.278	22.040
8	13.362	15.507	17.535	20.090	21.955	23.774
9	14.684	16.919	19.023	21.666	23.589	25.462
10	15.987	18.307	20.483	23.209	25.188	27.112
11	17.275	19.675	21.920	24.725	26.757	28.729
12	18.549	21.026	23.337	26.217	28.300	30.318
13	19.812	22.362	24.736	27.688	29.819	31.883
14	21.064	23.685	26.119	29.141	31.319	33.426
15	22.307	24.996	27.488	30.578	32.801	34.950
16	23.542	26.296	28.845	32.000	34.267	36.456
17	24.769	27.587	30.191	33.409	35.718	37.946
18	25.989	28.869	31.526	34.805	37.156	39.422
19	27.204	30.144	32.852	36.191	38.582	40.885
20	28.412	31.410	34.170	37.566	39.997	42.336
21	29.615	32.671	35.479	38.932	41.401	43.775
22	30.813	33.924	36.781	40.289	42.796	45.204
23	32.007	35.172	38.076	41.638	44.181	46.623
24	33.196	36.415	39.364	42.980	45.558	48.034
25	34.382	37.652	40.646	44.314	46.928	49.435
26	35.563	38.885	41.923	45.642	48.290	50.829
27	36.741	40.113	43.195	46.963	49.645	52.215
28	37.916	41.337	44.461	48.278	50.993	53.594
29	39.087	42.557	45.722	49.588	52.336	54.967
30	40.256	43.773	46.979	50.892	53.672	56.332
31	44.422	44.985	48.232	52.191	55.003	57.692
32	42.585	46.194	49.480	53.486	56.328	59.046
33	43.745	47.400	50.725	54.776	57.648	60.395
34	44.903	48.602	51.966	56.061	58.964	61.738
35	46.059	49.802	53.203	57.342	60.275	63.076
36	47.212	50.998	54.437	58.619	61.581	64.410
37	48.363	52.192	55.668	59.892	62.883	65.739
38	49.513	53.384	56.896	61.162	64.181	67.063
39	50.660	54.572	58.120	62.428	65.476	68.383
40	51.805	55.758	59.342	63.691	66.766	69.699
45	57.505	61.656	65.410	69.957	73.166	76.233
50	63.167	67.505	71.420	76.154	79.490	82.664
60	74.399	79.087	83.305	88.386	91.957	95.357
70	85.529	90.537	95.031	100.432	104.222	107.812
80	96.581	101.885	106.636	112.336	116.329	120.107
90	107.568	113.151	118.144	124.125	128.307	132.262
100	118.501	124.348	129.570	135.815	140.178	144.300
110	129.388	135.487	140.925	147.423	151.958	156.238
120	146.571	152.222	157.389	163.678	168.122	172.351
140	168.618	174.659	180.174	186.875	191.604	196.099
160	190.522	196.926	202.766	209.852	214.845	219.588
180	212.310	219.056	225.200	232.647	237.890	242.866

TABLE 4 (a)

F distribution ($\alpha = 0.05$)

The entries in this table are critical values of F for which the area under the curve to the right is equal to 0.05.



Degrees of freedom for denominator	Degrees of freedom for numerator 1	2	3	4	5	6	7	8	9	10
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

TABLE 4 (a) continued

F distribution ($\alpha = 0.05$)

Degrees of freedom for denominator	Degrees of freedom for numerator: 12	15	20	24	30	40	60	120	∞
1	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

APPENDIX 2: LIST OF KEY FORMULAE

MEASURES OF CENTRAL LOCATION

Arithmetic mean *Ungrouped data*

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad 3.1$$

Grouped data

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{n}$$

Mode *Grouped data*

$$M_o = O_{mo} + \frac{c(f_m - f_{m-1})}{2f_m - f_{m-1} - f_{m+1}} \qquad 3.3$$

Median *Grouped data*

$$M_e = O_{me} + \frac{c\left[\frac{n}{2} - f(<)\right]}{f_{me}} \qquad 3.2$$

Lower quartile *Grouped data*

$$Q_1 = O_{q1} + \frac{c\left(\frac{n}{4} - f(<)\right)}{f_{q1}} \qquad 3.7$$

Upper quartile *Grouped data*

$$Q_3 = O_{q3} + \frac{c\left(\frac{3n}{4} - f(<)\right)}{f_{q3}} \qquad 3.8$$

Geometric mean *Ungrouped data*

$$GM = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n} \qquad 3.4$$

Weighted arithmetic mean *Grouped data*

$$\text{weighted } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad 3.5$$

MEASURES OF DISPERSION AND SKEWNESS

Range Range = Maximum value − Minimum value + 1 $= x_{max} - x_{min} + 1$ 3.9

Variance *Mathematical – ungrouped data*

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{(n-1)} \quad 3.10$$

Computational – ungrouped data

$$s^2 = \frac{\Sigma x_i^2 - n\bar{x}^2}{(n-1)} \quad 3.11$$

Standard deviation $s = \sqrt{s^2}$ 3.12

Coefficient of variation $CV = \frac{s}{\bar{x}} \times 100\%$ 3.13

Pearson's coefficient of skewness

$$sk_p = \frac{n\Sigma(x_i - \bar{x})^3}{(n-1)(n-2)s^3} \quad 3.14$$

$$sk_p = \frac{3\ (\text{Mean} - \text{Median})}{\text{Standard deviation}} \quad \text{(approximation)} \quad 3.15$$

PROBABILITY CONCEPTS

Conditional probability $P(A/B) = \frac{P(A \cap B)}{P(B)}$ 4.2

Addition rule *Non-mutually exclusive events*
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 4.3

Mutually exclusive events
$P(A \cup B) = P(A) + P(B)$ 4.4

Multiplication rule *Statistically dependent events*
$P(A \cap B) = P(A/B) \times P(B)$ 4.5

Statistically independent events
$P(A \cap B) = P(A) \times P(B)$ 4.6

$n! = n$ factorial $n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$ 4.8

Permutations ${}_nP_r = \frac{n!}{(n-r)!}$ 4.10

Combinations ${}_nC_r = \frac{n!}{r!\,(n-r)!}$ 4.11

PROBABILITY DISTRIBUTIONS

Binomial distribution $P(x) = {}_nC_x\, p^x(1-p)^{(n-x)}$ for $x = 0, 1, 2, 3, \ldots, n$ 5.1

$P(x\ successes) = \frac{n!}{x!\,(n-x)!} p^x(1-p)^{(n-x)}$ for $x = 0, 1, 2, 3, \ldots, n$

Binomial descriptive measures Mean $\mu = np$
Standard deviation $\sigma = \sqrt{np(1-p)}$ 5.2

Poisson distribution $P(x) = \frac{e^{-a} a^x}{x!}$ for $x = 0, 1, 2, 3 \ldots$ 5.3

Poisson descriptive measures Mean $\mu = a$
Standard deviation $\sigma = \sqrt{a}$ 5.4

Standard normal probability $z = \frac{x - \mu}{\sigma}$ 5.6

CONFIDENCE INTERVALS

Single mean n *large; variance known*

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z\frac{\sigma}{\sqrt{n}} \quad 7.1$$

(lower limit) (upper limit)

n *small; variance unknown*

$$\bar{x} - t_{(n-1)}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{(n-1)}\frac{s}{\sqrt{n}} \quad 7.2$$

(lower limit) (upper limit)

Single proportion

$$p - z\sqrt{\frac{p(1-p)}{n}} \le \pi \le p + z\sqrt{\frac{p(1-p)}{n}} \quad 7.3$$

(lower limit) (upper limit)

HYPOTHESES TESTS

Single mean *Variance known*

10

$$z\text{-}stat = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad 8.1$$

Variance unknown; n *small*

$$t\text{-}stat = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad 8.2$$

Single proportion

$$t\text{-}stat = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \quad 8.3$$

Difference between two means *Variances known*

$$z\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad 9.1$$

Variances unknown; n_1 and n_2 small

$$t\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \quad 9.2$$

Paired *t*-test

$$t\text{-}stat = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}} \quad 9.5$$

where $\mu_d = (\mu_1 - \mu_2)$

and $s_d = \sqrt{\frac{\Sigma(x_d - \bar{x}_d)^2}{n - 1}}$

Differences between two proportions

$$z\text{-}stat = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\hat{\pi}(1 - \hat{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } \hat{\pi} = \frac{x_1 + x_2}{n_1 + n_2};\ p_1 = \frac{x_1}{n_1};\ p_2 = \frac{x_2}{n_2} \quad 9.8$$

Chi-Squared

$$\chi^2\text{-}stat = \Sigma\frac{(f_o - f_e)^2}{f_e} \quad 10.1$$

Overall mean

$$\bar{\bar{x}} = \frac{\Sigma\Sigma x_{ij}}{N} \quad 11.2$$

Total sum of squares (SSTotal)

$$= \sum_i \sum_j (x_{ij} - \bar{\bar{x}})^2 \quad 11.3$$

SST

$$= \sum_j^k n_j(\bar{x}_j - \bar{\bar{x}})^2 \quad 11.4$$

SSE

$$= \sum_j \sum_i (x_{ij} - \bar{x}_j)^2 \quad 11.5$$

SSTotal

$$= \text{SST} + \text{SSE} \quad 11.6$$

MSTotal

$$= \frac{\text{SSTotal}}{N - 1} \quad 11.7$$

MST

$$= \frac{\text{SST}}{k - 1} \quad 11.8$$

MSE

$$= \frac{\text{SSE}}{N - k} \quad 11.9$$

***F*-stat**

$$= \frac{\text{MST}}{\text{MSE}} \quad 11.10$$