

UNIVERSITY OF SWAZILAND



SUPPLEMENTARY EXAMINATION PAPER 2014

TITLE OF PAPER	: TOPICS IN STATISTICS (STATISTICAL MODELLING)
COURSE CODE	: ST 405
TIME ALLOWED	: 3 HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	: ANSWER ANY FIVE QUESTIONS

Question 1

Derive the deviance as a function of the estimated mean for the Normal, Poisson, Binomial, and Gamma distributions.

(20 Marks)

Question 2

The following data were collected after a food poisoning outbreak. It is suspected that the potato salad, the crab salad or both were the cause. The contingency table below shows the results of a random survey of 304 diners: whether they were sick (food-poisoned) and the food that they ate.

	Potato Salad		No Potato Salad	
	Crab Salad	No Crab Salad	Crab Salad	No Crab Salad
Not Sick	80	24	31	23
Sick	120	22	4	0

(a) What is a generalized linear model? What is the saturated model in the context of generalized linear models?

(4 Marks)

(b) A log-linear generalized linear model with a Poisson distribution was fitted to the data. The computer output below shows the analysis of deviance table for these data. Each row of the table refers to a model containing the terms given in the left-hand column of that row and all the rows above it.

	Deviance	Change in Deviance
intercept	295.253	
sick	294.779	0.474
potato	169.664	125.115
crab	73.871	95.793
potato:crab	63.196	10.676
sick:potato	6.482	56.714
sick:crab	2.743	3.739
sick:potato:crab	4.123e-10	2.743

Find a suitable model for these data and give an interpretation. What can be concluded about the likely cause of the outbreak?

(6 Marks)

(c) How is a Pearson residual defined in this model?

(2 Marks)

(d) Calculate the Pearson residuals for your fitted model. Do they indicate an adequate fit to the data?

(4 Marks)

(e) A colleague suggests that a logistic regression model with sickness as response would be more appropriate for these data than the log-linear model. Describe briefly the different aims of these two approaches, and discuss whether your colleague's suggestion is a good one.

(4 Marks)

Question 3

If we write the probability density function (p.d.f.) for the GLM in the form

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$$f(y_i|\theta_i, \phi) = \exp \left[\frac{y_i \theta_i - m(\theta_i)}{h(\phi)} \right] + n(y_i, \phi)$$

then prove that $E(Y_i) = m'(\theta_i)$ and $Var(Y_i) = m''(\theta_i) h(\phi)$.
(20 Marks)

Question 4

If the random variable Y_i follows a Gamma distribution, with scale parameter θ and shape parameter ϕ , then it has a p.d.f.

$$f(y_i|\theta_i, \phi) = \frac{y_i^{\phi-1} \theta_i^{\phi} e^{-y_i \theta_i}}{\Gamma(\phi)}$$

Show that the distribution is a member of the exponential family, and find $E(Y_i)$ and $Var(Y_i)$.

(20 Marks)

Question 5

A doctor is investigating the effect of a woman's age on the success of an IVF (in vitro fertilisation) procedure. She has randomly selected 10 women aged under 35 and 10 women aged at least 35. From hospital records she has obtained the following data, which record the numbers of eggs obtained from the women and the numbers that were fertilised during one IVF procedure. She wants to investigate the effect of the woman's age on the probability of an egg being successfully fertilised. She calls this probability the "fertilisation rate".

Women aged under 35		Women aged at least 35	
<i>Number of eggs</i>	<i>Number of fertilised</i>	<i>Number of eggs</i>	<i>Number of fertilised</i>
10	9	7	6
9	7	10	7
7	5	9	5
5	3	8	4
10	9	6	4
7	7	5	1
9	5	7	4
8	8	6	4
7	2	5	2
7	5	7	5

- (a) Carry out a suitable exploratory analysis to see whether the fertilisation rate might depend on the woman's age.
(4 Marks)
- (b) Let n_i denote the number of eggs and x_i the number of fertilised eggs for the i th woman. Let π_i denote the fertilisation rate for the i th woman.

i) Explain why a binomial distribution may be valid to model the data.
(2 Marks)

- ii) Write down the expression for the log likelihood of the observed data, assuming a binomial distribution with different fertilisation rates for each woman. Identify the logit function in your expression. (2 Marks)
- (c) The data are analysed using a generalised linear model, with the logit link. The model assumes constant fertilisation rate within each age group, so contains a constant and age as a covariate. Age is coded as 1 for older women, and 0 for younger women. Part of the output from a computer program is given below.

```

Deviance = 28.26      (1/df) Scaled Deviance = 1.57
Variance function:  V(u) = u*(1-u/eggs)      [Binomial]
Link function      :  g(u) = log(u/(eggs-u))   [Logit]

```

- i) Explain why the highlighted value 1.57 is useful, and how it is derived from the other numerical value in the output. (2 Marks)
- ii) Explain what the highlighted expressions $V(u)$ and $g(u)$ are and how their formulae are obtained. (2 Marks)
- iii) The estimated value of the coefficient for age in the generalised linear model is -0.744 and the estimate of the constant is 1.150 . Obtain estimates of the predicted success rates for the two types of women. (4 Marks)
- iv) For the model which contains only the constant (i.e. does not take age into account), the value for the scaled deviance is 32.65 . State, with reasoning, whether the effect of woman's age is statistically significant. (2 Marks)
- v) Someone else has modelled these data but coded younger women as age = 1 and older women as age = 0. Explain how the results and estimates would be different from those given above. (2 Marks)

Question 6

Many of the wells used for drinking water in South Asian countries are contaminated with natural arsenic, affecting an estimated 100 million people. Arsenic is a cumulative poison, and exposure increases the risk of cancer and other diseases. A research team measured all wells in an area and labelled them with their arsenic level as well as a characterization as “safe” or “unsafe”, depending on whether the arsenic level was above or below the national standard of 50 micrograms per litre (50 $\mu\text{g/L}$).

People with unsafe wells were encouraged to switch to nearby private or community wells, or to new wells of their own construction. The amount of water needed for drinking is low enough that adding users to a well would not exhaust its capacity.

A few years later the researchers returned to see who had switched wells and found that 57.5% of the 3020 households with unsafe wells had switched. The team performed a series of GLM analyses to understand the factors predictive of well switching among users of unsafe wells.

A preliminary analysis focused on switching and distance to the nearest safe well, then an obvious variable of interest was the arsenic level of the existing well. To explore the

extent to which distance to a safe well might be less of a deterrent when the existing well has high arsenic levels, the research team added an interaction term. The final model adds a social predictor, years of education of the well user. Using the results in APPENDIX A:

- (a) For the first model interpret the estimated coefficient of distance, and Estimate the probability of switching for households that have to travel 100 meters to find a safe well and for households that have a safe well right at hand (say, at zero meters), and describe the effect of distance in the probability scale.
(7 Marks)
- (b) Interpret the coefficient of log-arsenic in terms of the relative effect of arsenic level on of switching wells. Compare families who have to travel 100 meters to find a safe well when the level of arsenic is 50 and 100.
(6Marks)
- (c) Explain briefly the effect of education on well-switching, including how it varies for different types of respondents. For simplicity describe your results in terms of odds, although translation to probabilities would be useful for a general audience.
(7 Marks)

Question 7

A cross-sectional subsample from a Socio-Economic Panel, which collected data on doctor visits before and after a major health care reform that took place in 1997. The reform increased the copayments for prescription drugs by up to 200% and imposed upper limits on the reimbursement of physicians by the state insurance. The full panel dataset was analyzed, and the outcome is the number of doctor visits in a three month period. The predictors of interest are reform, a dummy variable that takes the value 1 after the reform and 0 before, age in years, education in years, a dummy variable for bad health, and the log of income. Three models were fitted to the data as follows;

- A Poisson regression model to estimate the effect of reform. Linear terms age, education and logincome, and the dummy for bad health were used as controls.
- A negative binomial model with the same predictors.
- A zero-inflated Poisson model using the same predictors.

Considering the results obtained (See APPENDIX B) and bearing in mind parsimony and goodness of fit, which of the models used here provides the best description of the data? Make sure you provide a clear justification of your choice.

APPENDIX A

```
. logit switch distnearest
Iteration 0: log likelihood = -2059.0496
Iteration 1: log likelihood = -2038.1212
Iteration 2: log likelihood = -2038.1189
Iteration 3: log likelihood = -2038.1189

Logistic regression
Log likelihood = -2038.1189
Number of obs = 3020
LR chi2(1) = 41.86
Prob > chi2 = 0.0000
Pseudo R2 = 0.0102
```

switch	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
distnearest	-.0062188	.0009743	-6.38	0.000	-.0081283	-.0043093
_cons	.6059594	.0603102	10.05	0.000	.4877535	.7241652

```
. estimates store linear
. logit switch distne logas
Iteration 0: log likelihood = -2059.0496
Iteration 1: log likelihood = -1949.5561
Iteration 2: log likelihood = -1949.1836
Iteration 3: log likelihood = -1949.1836

Logistic regression
Log likelihood = -1949.1836
Number of obs = 3020
LR chi2(2) = 219.73
Prob > chi2 = 0.0000
Pseudo R2 = 0.0534
```

switch	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
distnearest	-.0097957	.0010587	-9.25	0.000	-.0118709	-.0077206
logas	.8759499	.0684786	12.79	0.000	.7417343	1.010165
_cons	-3.507408	.323903	-10.83	0.000	-4.142247	-2.87257

```
. di exp(_b[logas])
2.401155
. logit switch distne logas ed edxdist
Iteration 0: log likelihood = -2059.0496
Iteration 1: log likelihood = -1932.6038
Iteration 2: log likelihood = -1932.3102
Iteration 3: log likelihood = -1932.3102

Logistic regression
Log likelihood = -1932.3102
Number of obs = 3020
LR chi2(4) = 253.48
Prob > chi2 = 0.0000
Pseudo R2 = 0.0616
```

switch	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
distnearest	-.0089869	.0010956	-8.20	0.000	-.0111343	-.0068395
logas	.9046483	.069253	13.06	0.000	.7689149	1.040382
ed	.0456681	.0096902	4.71	0.000	.0266757	.0646606
edxdist	.0009318	.0002568	3.63	0.000	.0004284	.0014351
_cons	-3.90513	.334618	-11.67	0.000	-4.560969	-3.249291

```
. di exp(_b[ed])
1.046727
. di exp(100*_b[edxdist])
1.0976541
. di exp(_b[ed] + 100*_b[edxdist])
1.1489441
```

APPENDIX B

Poisson regression				Number of obs = 1518		
				LR chi2(5) = 1185.59		
Log likelihood = -4195.7433				Prob > chi2 = 0.0000		
				Pseudo R2 = 0.1238		
numvisit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
reform	-.2273629	.0315274	-7.21	0.000	-.2891554	-.1655704
age	.0049815	.0014734	3.38	0.001	.0020937	.0078693
educ	-.0006806	.0068736	-0.10	0.921	-.0141526	.0127915
loginc	.1119396	.0427084	2.62	0.009	.0282327	.1956465
badh	1.172635	.035256	33.26	0.000	1.103535	1.241736
_cons	-.1742048	.316784	-0.55	0.582	-.7950901	.4466804
. estimates store poisson						
. estat gof						
Goodness-of-fit chi2 = 5298.561						
Prob > chi2(1512) = 0.0000						
. scalar sigma2 = exp(_b[/lnalpha])						
. local v = sigma2						
. glm numvisit reform age educ loginc badh, family(nb `v') nolog						
Generalized linear models				No. of obs = 1518		
Optimization : ML				Residual df = 1512		
				Scale parameter = 1		
Deviance = 1645.595797				(1/df) Deviance = 1.088357		
Pearson = 1844.682834				(1/df) Pearson = 1.220028		
Variance function: V(u) = u+(1.0075)u^2				[Neg. Binomial]		
Link function : g(u) = ln(u)				[Log]		
Log likelihood = -3154.591778				AIC = 4.164153		
				BIC = -9430.029		
numvisit	Coef.	OIM Std. Err.	z	P> z	[95% Conf. Interval]	
reform	-.2153372	.062054	-3.47	0.001	-.3369608	-.0937136
age	.0066236	.0028882	2.29	0.022	.0009628	.0122845
educ	.0092555	.0136676	0.68	0.498	-.0175324	.0360434
loginc	.0766135	.08572	0.89	0.371	-.0913946	.2446215
badh	1.166646	.0882387	13.22	0.000	.9937011	1.33959
_cons	-.0817257	.6410911	-0.13	0.899	-1.338241	1.17479
. di e(deviance), chi2tail(e(df), e(deviance))						
1645.5958 .00881315						
. zip numvis reform age educ loginc badh, ///						
> inflate(reform age educ loginc badh) nolog						
Zero-inflated Poisson regression				Number of obs = 1518		
				Nonzero obs = 1073		
				Zero obs = 445		
Inflation model = logit				LR chi2(5) = 810.48		
Log likelihood = -3814.717				Prob > chi2 = 0.0000		
numvisit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
numvisit						
reform	-.1849748	.0334925	-5.52	0.000	-.2506189	-.1193307
age	.004652	.0015403	3.02	0.003	.001633	.0076709
educ	-.0242949	.0074959	-3.24	0.001	-.0389866	-.0096033
loginc	.0774144	.0443247	1.75	0.081	-.0094604	.1642891
badh	.9475226	.0363537	26.06	0.000	.8762708	1.018774
_cons	.6827736	.3311059	2.06	0.039	.033818	1.331729
inflate						
reform	.163969	.1328714	1.23	0.217	-.0964541	.4243922
age	-.0018933	.0061404	-0.31	0.758	-.0139283	.0101418
educ	-.1246792	.033992	-3.67	0.000	-.1913024	-.0580561
loginc	-.1241885	.1791205	-0.69	0.488	-.4752583	.2268812
badh	-1.222295	.2534564	-4.82	0.000	-1.719061	-.7255296

_cons		1.401565	1.329329	1.05	0.292	-1.203872	4.007003

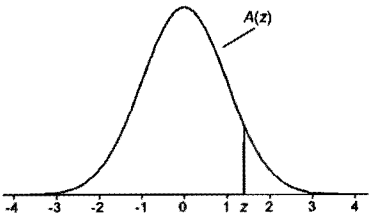
. estimates store zip							
. di exp(_b[inflate:educ])-1,exp(_b[inflate:badh])-1							
-.11721997 -.70544663							

STATISTICAL TABLES

- Cumulative normal distribution
- Critical values of the *t* distribution
- Critical values of the *F* distribution
- Critical values of the chi-squared distribution

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2
t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: One-tailed test:	Significance level					
		10% 5%	5% 2.5%	2% 1%	1% 0.5%	0.2% 0.1%	0.1% 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

TABLE A.3

F Distribution: Critical Values of F (5% significance level)																
v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	
v_2	1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58	1.58

TABLE A.3 (continued)

F Distribution: Critical Values of F (5% significance level)

v_1	25	30	35	40	50	60	75	100	150	200
v_2										
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
v_2															
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6142.67	6170.10	6191.53	6208.73
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.44	99.45
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.92	26.83	26.75	26.69
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.25	14.15	14.08	14.02
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.77	9.68	9.61	9.55
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.60	7.52	7.45	7.40
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.36	6.28	6.21	6.16
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.56	5.48	5.41	5.36
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	5.01	4.92	4.86	4.81
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.60	4.52	4.46	4.41
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.29	4.21	4.15	4.10
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.05	3.97	3.91	3.86
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.86	3.78	3.72	3.66
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.70	3.62	3.56	3.51
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.56	3.49	3.42	3.37
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.45	3.37	3.31	3.26
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.35	3.27	3.21	3.16
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.27	3.19	3.13	3.08
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.19	3.12	3.05	3.00
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.13	3.05	2.99	2.94
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.07	2.99	2.93	2.88
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.02	2.94	2.88	2.83
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.97	2.89	2.83	2.78
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.93	2.85	2.79	2.74
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.89	2.81	2.75	2.70
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.86	2.78	2.72	2.66
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.82	2.75	2.68	2.63
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.79	2.72	2.65	2.60
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.77	2.69	2.63	2.57
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.74	2.66	2.60	2.55
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.64	2.56	2.50	2.44
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.56	2.48	2.42	2.37
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.46	2.38	2.32	2.27
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.39	2.31	2.25	2.20
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.35	2.27	2.20	2.15
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.31	2.23	2.17	2.12
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.29	2.21	2.14	2.09
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.27	2.19	2.12	2.07
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.23	2.15	2.09	2.03
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.20	2.12	2.06	2.00
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.27	2.17	2.09	2.03	1.97
250	6.74	4.69	3.86	3.40	3.09	2.87	2.71	2.58	2.48	2.39	2.26	2.15	2.07	2.01	1.95
300	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38	2.24	2.14	2.06	1.99	1.94
400	6.70	4.66	3.83	3.37	3.06	2.85	2.68	2.56	2.45	2.37	2.23	2.13	2.05	1.98	1.92
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.22	2.12	2.04	1.97	1.92
600	6.68	4.64	3.81	3.35	3.05	2.83	2.67	2.54	2.44	2.35	2.21	2.11	2.03	1.96	1.91
750	6.67	4.63	3.81	3.34	3.04	2.83	2.66	2.53	2.43	2.34	2.21	2.11	2.02	1.96	1.90
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.20	2.10	2.02	1.95	1.90

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)											
v_1	25	30	35	40	50	60	75	100	150	200	
v_2											
1	6239.83	6260.65	6275.57	6286.78	6302.52	6313.03	6323.56	6334.11	6344.68	6349.97	
2	99.46	99.47	99.47	99.47	99.48	99.48	99.49	99.49	99.49	99.49	
3	26.58	26.50	26.45	26.41	26.35	26.32	26.28	26.24	26.20	26.18	
4	13.91	13.84	13.79	13.75	13.69	13.65	13.61	13.58	13.54	13.52	
5	9.45	9.38	9.33	9.29	9.24	9.20	9.17	9.13	9.09	9.08	
6	7.30	7.23	7.18	7.14	7.09	7.06	7.02	6.99	6.95	6.93	
7	6.06	5.99	5.94	5.91	5.86	5.82	5.79	5.75	5.72	5.70	
8	5.26	5.20	5.15	5.12	5.07	5.03	5.00	4.96	4.93	4.91	
9	4.71	4.65	4.60	4.57	4.52	4.48	4.45	4.41	4.38	4.36	
10	4.31	4.25	4.20	4.17	4.12	4.08	4.05	4.01	3.98	3.96	
11	4.01	3.94	3.89	3.86	3.81	3.78	3.74	3.71	3.67	3.66	
12	3.76	3.70	3.65	3.62	3.57	3.54	3.50	3.47	3.43	3.41	
13	3.57	3.51	3.46	3.43	3.38	3.34	3.31	3.27	3.24	3.22	
14	3.41	3.35	3.30	3.27	3.22	3.18	3.15	3.11	3.08	3.06	
15	3.28	3.21	3.17	3.13	3.08	3.05	3.01	2.98	2.94	2.92	
16	3.16	3.10	3.05	3.02	2.97	2.93	2.90	2.86	2.83	2.81	
17	3.07	3.00	2.96	2.92	2.87	2.83	2.80	2.76	2.73	2.71	
18	2.98	2.92	2.87	2.84	2.78	2.75	2.71	2.68	2.64	2.62	
19	2.91	2.84	2.80	2.76	2.71	2.67	2.64	2.60	2.57	2.55	
20	2.84	2.78	2.73	2.69	2.64	2.61	2.57	2.54	2.50	2.48	
21	2.79	2.72	2.67	2.64	2.58	2.55	2.51	2.48	2.44	2.42	
22	2.73	2.67	2.62	2.58	2.53	2.50	2.46	2.42	2.38	2.36	
23	2.69	2.62	2.57	2.54	2.48	2.45	2.41	2.37	2.34	2.32	
24	2.64	2.58	2.53	2.49	2.44	2.40	2.37	2.33	2.29	2.27	
25	2.60	2.54	2.49	2.45	2.40	2.36	2.33	2.29	2.25	2.23	
26	2.57	2.50	2.45	2.42	2.36	2.33	2.29	2.25	2.21	2.19	
27	2.54	2.47	2.42	2.38	2.33	2.29	2.26	2.22	2.18	2.16	
28	2.51	2.44	2.39	2.35	2.30	2.26	2.23	2.19	2.15	2.13	
29	2.48	2.41	2.36	2.33	2.27	2.23	2.20	2.16	2.12	2.10	
30	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.13	2.09	2.07	
35	2.35	2.28	2.23	2.19	2.14	2.10	2.06	2.02	1.98	1.96	
40	2.27	2.20	2.15	2.11	2.06	2.02	1.98	1.94	1.90	1.87	
50	2.17	2.10	2.05	2.01	1.95	1.91	1.87	1.82	1.78	1.76	
60	2.10	2.03	1.98	1.94	1.88	1.84	1.79	1.75	1.70	1.68	
70	2.05	1.98	1.93	1.89	1.83	1.78	1.74	1.70	1.65	1.62	
80	2.01	1.94	1.89	1.85	1.79	1.75	1.70	1.65	1.61	1.58	
90	1.99	1.92	1.86	1.82	1.76	1.72	1.67	1.62	1.57	1.55	
100	1.97	1.89	1.84	1.80	1.74	1.69	1.65	1.60	1.55	1.52	
120	1.93	1.86	1.81	1.76	1.70	1.66	1.61	1.56	1.51	1.48	
150	1.90	1.83	1.77	1.73	1.66	1.62	1.57	1.52	1.46	1.43	
200	1.87	1.79	1.74	1.69	1.63	1.58	1.53	1.48	1.42	1.39	
250	1.85	1.77	1.72	1.67	1.61	1.56	1.51	1.46	1.40	1.36	
300	1.84	1.76	1.70	1.66	1.59	1.55	1.50	1.44	1.38	1.35	
400	1.82	1.75	1.69	1.64	1.58	1.53	1.48	1.42	1.36	1.32	
500	1.81	1.74	1.68	1.63	1.57	1.52	1.47	1.41	1.34	1.31	
600	1.80	1.73	1.67	1.63	1.56	1.51	1.46	1.40	1.34	1.30	
750	1.80	1.72	1.66	1.62	1.55	1.50	1.45	1.39	1.33	1.29	
1000	1.79	1.72	1.66	1.61	1.54	1.50	1.44	1.38	1.32	1.28	

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)																	
v_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20		
v_2																	
1	4.05e05	5.00e05	5.40e05	5.62e05	5.76e05	5.86e05	5.93e05	5.98e05	6.02e05	6.06e05	6.11e05	6.14e05	6.17e05	6.19e05	6.21e05		
2	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39	999.40	999.42	999.43	999.44	999.44	999.45		
3	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	129.25	128.32	127.64	127.14	126.74	126.42		
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.95	46.60	46.32	46.10		
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92	26.42	26.06	25.78	25.57	25.39		
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	17.99	17.68	17.45	17.27	17.12		
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.43	13.23	13.06	12.93		
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54	11.19	10.94	10.75	10.60	10.48		
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.33	9.15	9.01	8.90		
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75	8.45	8.22	8.05	7.91	7.80		
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.41	7.24	7.11	7.01		
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.79	6.63	6.51	6.40		
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.31	6.16	6.03	5.93		
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.13	5.93	5.78	5.66	5.56		
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.62	5.46	5.35	5.25		
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81	5.55	5.35	5.20	5.09	4.99		
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.13	4.99	4.87	4.78		
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.94	4.80	4.68	4.59		
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.78	4.64	4.52	4.43		
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.64	4.49	4.38	4.29		
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.51	4.37	4.26	4.17		
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.40	4.26	4.15	4.06		
23	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.30	4.16	4.05	3.96		
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.21	4.07	3.96	3.87		
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56	4.31	4.13	3.99	3.88	3.79		
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	4.06	3.92	3.81	3.72		
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.99	3.86	3.75	3.66		
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.93	3.80	3.69	3.60		
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.88	3.74	3.63	3.54		
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.82	3.69	3.58	3.49		
35	12.90	8.47	6.79	5.88	5.30	4.89	4.59	4.36	4.18	4.03	3.79	3.62	3.48	3.38	3.29		
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.47	3.34	3.23	3.14		
50	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	3.67	3.44	3.27	3.14	3.04	2.95		
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.32	3.15	3.02	2.91	2.83		
70	11.80	7.64	6.06	5.20	4.66	4.28	3.99	3.77	3.60	3.45	3.23	3.06	2.93	2.83	2.74		
80	11.67	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53	3.39	3.16	3.00	2.87	2.76	2.68		
90	11.57	7.47	5.91	5.06	4.53	4.15	3.87	3.65	3.48	3.34	3.11	2.95	2.82	2.71	2.63		
100	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	3.30	3.07	2.91	2.78	2.68	2.59		
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.85	2.72	2.62	2.53		
150	11.27	7.24	5.71	4.88	4.35	3.98	3.71	3.49	3.32	3.18	2.96	2.80	2.67	2.56	2.48		
200	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26	3.12	2.90	2.74	2.61	2.51	2.42		
250	11.09	7.10	5.59	4.77	4.25	3.88	3.61	3.40	3.23	3.09	2.87	2.71	2.58	2.48	2.39		
300	11.04	7.07	5.56	4.75	4.22	3.86	3.59	3.38	3.21	3.07	2.85	2.69	2.56	2.46	2.37		
400	10.99	7.03	5.53	4.71	4.19	3.83	3.56	3.35	3.18	3.04	2.82	2.66	2.53	2.43	2.34		
500	10.96	7.00	5.51	4.69	4.18	3.81	3.54	3.33	3.16	3.02	2.81	2.64	2.52	2.41	2.33		
600	10.94	6.99	5.49	4.68	4.16	3.80	3.53	3.32	3.15	3.01	2.80	2.63	2.51	2.40	2.32		
750	10.91	6.97	5.48	4.67	4.15	3.79	3.52	3.31	3.14	3.00	2.78	2.62	2.49	2.39	2.31		
1000	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	2.99	2.77	2.61	2.48	2.38	2.30		

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

v_1	25	30	35	40	50	60	75	100	150	200
1	6.24e05	6.25e05	6.28e05	6.29e05	6.30e05	6.31e05	6.32e05	6.33e05	6.35e05	6.35e05
2	999.46	999.47	999.47	999.47	999.48	999.48	999.49	999.49	999.49	999.49
3	125.84	125.45	125.17	124.96	124.66	124.47	124.27	124.07	123.87	123.77
4	45.70	45.43	45.23	45.09	44.88	44.75	44.61	44.47	44.33	44.26
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.80	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.96	7.93
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.37	5.30	5.24	5.17	5.10	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.70	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	4.00	3.93	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.86	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.43	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.18	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.21	2.11	2.05	1.98	1.91	1.83	1.79
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.03	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.80	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.45	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

Degrees of freedom	Significance level		
	5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588