UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2014

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 2+3+8+4+3]

(a) In 2005, researchers in a Government Department wished to examine the degree of absenteeism amongst employees. As it was time consuming to interrogate the Department's personnel system, the researchers took a stratified random sample of personnel records, with the stratification being by pay grade (A is the highest grade). The results are summarised below.

Pay grade	**Number of employees**	**Sample size**	**Mean number of days of absence**	**Variance**
h	N_h	n_h	$\bar{y}_h$	s_h^2
A	1200	40	4.0	1.5
B	5000	40	6.5	2.5
C	3800	40	8.0	3.0
Total	10000	120		

(i) Estimate the mean number of days of absence for all employees.

(ii) Estimate the standard error of the mean number of days of absence for each of the three pay grades.

(iii) Estimate the standard error of the mean number of days of absence for all employees. Hence calculate an approximate 95% confidence interval for the mean number of days of absence for all employees.

(b) In 2007, researchers are to repeat the investigation to determine whether the absenteeism rate has improved. It has been suggested that the stratification for the 2007 survey could be improved by using optimal allocation. This would use information from the 2005 study, and a total sample size of 120.

(i) In optimal allocation, stratum sample sizes nh are proportional to $N_h s_h$ (using the notation of part (a)). Find the values of n_h that give optimal allocation, and explain why such allocation might be beneficial in this instance.

(ii) Find the values of n_h given by proportional allocation and comment on whether the precision of the estimated mean is likely to be much affected by which method is used.

Question 2 [20 marks, 7+5+4+4]

(a) An academic researcher conducted a survey of the 800 students within her faculty. The objective of the study was to examine student debt within the faculty. A simple random sample of 50 of these 800 students was selected, and a questionnaire was sent to each of them. Questionnaires were completed and returned to the researcher by 25 of these students.

In these responses, the mean size of student loan was SZL8000 and the standard deviation was SZL3000.

(i) Calculate an approximate 95% confidence interval for the mean student loan amongst students in the faculty.

The finite population correction factor is $1 - f$, where f is the sampling fraction. Explain the circumstances under which it should be used. Would it make much difference in this situation?

(ii) The researcher was hoping to estimate the mean student loan to within $\pm$SZL1000. Estimate the smallest achieved sample size that would have been necessary to do so with 95% confidence.

(b) At the same time as this research was being carried out, an academic colleague conducted a similar survey in a large London college. In a bid to increase response rates, he offered all respondents a SZL10 gift token for a local store. He received 40 completed questionnaires out of 50.

One of the key variables considered was the proportion of students who had taken out a student loan. From the 40 respondents, 25 had taken out a student loan.

Use a suitable approximation to calculate a 95% confidence interval for the proportion of students at the London college who had taken out a student loan.

(c) Discuss any concerns you might have about the planning or conduct of these surveys, especially regarding possible biases.

Question 3 [20 marks, 6+6+4+4]

(a) A shipment of 100 boxes of frozen food (each box contains 8 separate packages of food) was allowed to thaw during transit. The shipper was worried that some of the packages could be spoiled. He took a random sample of 5 boxes and checked all the packages in each box. In 2 of the boxes there were 3 spoiled packages, in 1 of the boxes there was 2 spoiled packages and in 2 of the boxes there was no spoiled packages.

(i) Under this sampling plan estimate the total number of spoiled packages in the entire shipment and give its estimated variance.

(ii) Suppose instead that the sampling plan took a simple random sample without replacement of size 40 from the population of 800 packages and 8 spoiled packages were found in the sample. Under this sampling plan estimate the total number of spoiled packages in the entire shipment and give its estimated variance.

(b) A company with 900 employees randomly selects 20 each month to ask them about working conditions.

(i) What is the probability that no one from a 4 person office gets included in the survey this year.

(ii) How many months would a person need to work at the company before they have a probability of at least 0.6 of being included in the monthly survey.

Question 4 [20 marks, 3+4+6+7]

A small survey was carried out in a country to estimate the total number of bunches of bananas produced in a region during a given growing period. The region was divided into 289 primary units such that each unit had about 500-1000 banana pits. Each pit may produce 0, 1 or more bunches of bananas. The total number of banana pits for the whole district was known to be 181 336. A simple random sample of 20 primary units was selected from the 289 units, and for each unit the number of banana pits (x) and the total number of banana bunches (y) were obtained. The results are summarised below.

	Sample ($n = 20$)	
	Mean	SD
Number of banana pits per unit (x)	644.35	115.9025
Total number of banana bunches per unit (y)	901.70	221.8112

The correlation is 0.7737 between the number of banana pits and the total number of banana bunches.

(a) Explain why a ratio estimator is appropriate for these data.

(b) Estimate the total number of banana bunches for the district, and obtain a standard error of your estimate, using

(i) the simple random sample mean,

(ii) the ratio estimator.

(c) Consider a study area for wildebeest in East Africa that consists of 100 strips of varying length, and where the probability of selection is proportional to length of the strip. Three strips are selected for the sample. Three strips selected in sample:

y	Length	p
60	5	0.05
14	2	0.02
1	1	0.01

Estimate the population total of wildebeest and its margin of error.

Question 5 [20 marks, 2+6+6+6]

A simple random sample of 10 hospitals was selected from a population of 33 hospitals that had received state funding to upgrade their emergency medical services. Within each of the selected hospitals, the records of all patients hospitalised in the past 12 months for traumatic injuries (i.e. accidents, poisonings, violence, burns, etc.) were examined. The numbers of patients hospitalised for trauma conditions and the numbers who died for the selected hospitals are given below.

Hospital	Number of patients hospitalised for trauma conditions	Number with trauma conditions who died
1	560	4
2	190	4
3	260	2
4	370	4
5	190	4
6	130	0
7	170	9
8	170	2
9	60	0
10	110	1

(a) Explain why this design may be considered as a cluster sample. What are the first-stage and second-stage units?

(b) Obtain a point estimate and an approximate 95% confidence interval for the total number of persons hospitalised for trauma conditions for the 33 hospitals. State the properties of your estimator.

(c) Obtain a point estimate of the proportion of persons who died among those hospitalised for trauma conditions for the 33 hospitals, using the cluster totals. Hence calculate an approximate 95% confidence interval for this proportion, and comment on the validity of the assumptions necessary for this calculation.

(d) Give reasons why, for this survey, cluster sampling might be preferred to stratified random sampling. What might be the drawbacks of cluster sampling? Discuss, with reasons, any improvements you might make if another survey was being planned on the same topic.

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{\mathrm{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{\mathrm{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathrm{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n} \qquad \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad \hat{\mathrm{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N} \qquad \hat{\mathrm{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathrm{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i} \qquad \hat{\mathrm{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N} \qquad \hat{\mathrm{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathrm{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{\mathrm{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x \qquad \hat{\mathrm{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x \qquad \hat{\mathrm{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x \qquad \hat{\mathrm{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L \qquad \hat{\mathrm{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h \qquad \hat{\mathrm{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{\mathrm{V}}(\hat{\tau}_{str}) = N^2\hat{\mathrm{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h \qquad \hat{\mathrm{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h \qquad \hat{\mathrm{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_hs_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{V}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{V}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{V}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{V}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y}-\hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{V}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{V}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{V}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

$$n \text{ for } \mu \text{ SRS} \quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$$

$$n \text{ for } \tau \text{ SRS} \quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$$

$$n \text{ for } p \text{ SRS} \quad n = \frac{Np(1-p)}{(N-1)(d^2/z^2)+p(1-p)}$$

$$n \text{ for } \mu \text{ SYS} \quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$$

$$n \text{ for } \tau \text{ SYS} \quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$$

$$n \text{ for } \mu \text{ STR} \quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$$

$$n \text{ for } \tau \text{ STR} \quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^L N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^L N_k\sigma_k\sqrt{c_k}\right)\left(\sum_{k=1}^L N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^L N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^L N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^L N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^L N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^L N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^L N_k\sigma_k^2}$$

Allocations for STR τ:

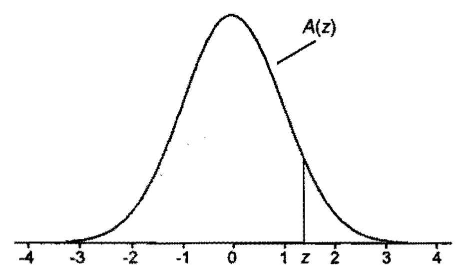
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^L N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^L N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^L N_k p_k(1-p_k)}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: / One-tailed test:	Significance level 10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291