

UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2013

TITLE OF PAPER : NON-PARAMETRIC ANALYSIS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

**REQUIREMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER QUESTION ONE AND ANY
THREE (3) QUESTIONS. ALL QUESTIONS
CARRY MARKS AS INDICATED WITHIN
THE PARENTHESIS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

ANSWER QUESTION ONE & ANY THREE QUESTIONS:

For all questions, clearly state the name of the test, the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, the decision & the conclusions.

QUESTION ONE.

[20 + 20 marks]

Ten students took a road test for driving skills as well as a written test on traffic rules. Their scores on both tests are as follows:

Student	Written Test	Road Test
1	25	38
2	30	36
3	42	50
4	44	45
5	58	30
6	59	78
7	75	77
8	79	85
9	87	65
10	90	76

- a. Test first (i) that the road test scores are higher than the written test scores at 5% level of significance, and then (ii) that a student's score on the road test is not different from his or her score on the written test at 10% level of significance using *an appropriate test based on the binomial distribution*.
- b. Test again either a(i) or a(ii) using another *appropriate test not based on the binomial distribution*. Also find the P-value.

QUESTION TWO.

[14 + 6 marks]

- a. The amount of damage to the soil on a farm caused by water and wind is examined for many different farms. At the same time the type of farming practiced on each farm is noted, with the following results (the values are the number of farms).

Amount Of Damage	Type of Farming			
	Minimum Tillage	Contour	Terrace	Other
No damage	17	19	4	21
Slight damage	3	10	4	42
Moderate damage	0	2	2	34
Severe damage	0	0	2	6

Use the Kruskal-Wallis test to determine whether the type of farming affects the degree of damage. Use $\alpha = 0.05$. Also calculate the P-value.

- b. If so, which types of farming practiced are significantly different?

QUESTION THREE.

[20 marks]

The emissions of nitrous oxide from last year's model of automobile have been measured for thousands of cars and found to be approximately normal with mean 5.6 and standard deviation 1.2. Twelve of this year's model automobile have been tested with the results

4.8, 6.2, 6.0, 5.9, 6.6, 5.5, 5.8, 5.9, 6.3, 6.6, 6.2, 5.0

Does this year's model appear to have the same distribution as last year's model? Find the P-value.

QUESTION FOUR.

[10 + 10 marks]

- The sergeant recalls that in the "good old days" the upper quartile for time required to complete the obstacle course was 42 minutes. He suspects that the new recruits are not as fit as the recruits in the good old days, so he times them on the obstacle course. He finds that out of 38 recruits, only ten of them complete the course within the 42 minute time period. Use the quantile test to test the hypothesis that the upper quartile is 42 minutes against the appropriate one-sided alternative. Use 5% level of significance.
- Five doctoral students took a test on current affairs. The ages of the doctoral students and their test scores are given below.

Age	Test Score
24	68
31	85
38	84
45	92
45	90

Do older students tend to get higher test scores? Use either Spearman's ρ test or Kendall's τ test with $\alpha = 0.10$.

QUESTION FIVE.

[20 marks]

The number of eggs laid by a group of insects in a laboratory is counted on an hourly basis during a 24-hur experiment, to test that the number of eggs laid tends to be minimum at 2:15pm, increasing to a maximum at 2:15am, and decreasing again until 2:15pm. The hourly counts are as follows.

Time	#Eggs	Time	#Eggs	Time	#Eggs
9am	151	5pm	83	1am	286
10am	119	6pm	166	2am	235
11am	146	7pm	143	3am	223
Noon	111	8pm	116	4am	176
1pm	63	9pm	163	5am	176
2pm	84	10pm	208	6am	174
3pm	60	11pm	283	7am	139
4pm	109	Midnight	296	8am	137

Use the Cox and Stuart test for Trend at 5% level of significance. Also calculate the P-value.

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3340
0.91	1.3406	1.3469	1.3532	1.3595	1.3650	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4110	1.4187	1.4255	1.4325	1.4395	1.4466	1.4539	1.4611	1.4684
0.93	1.4756	1.4823	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5546	1.5632	1.5718	1.5805	1.5893	1.5981	1.6072	1.6164	1.6250	1.6332
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

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*The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

<i>n</i>	<i>p</i>	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0096	0.0018	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000	0.0000
9	0.5000	0.3290	0.1861	0.0875	0.0316	0.0089	0.0016	0.0001	0.0000	0.0000	0.0000
10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000	0.0000
11	0.8204	0.6831	0.5122	0.3344	0.1810	0.0775	0.0233	0.0041	0.0003	0.0000	0.0000
12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000	0.0000
13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002	0.0002
14	0.9804	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020	0.0020
15	0.9778	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132	0.0000	0.0000
16	0.9996	0.9985	0.9945	0.9830	0.9238	0.8887	0.7631	0.5587	0.2946	0.0665	0.0000
17	1.0000	0.9998	0.9992	0.9869	0.9696	0.9690	0.9171	0.8015	0.5797	0.2453	0.0000
18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000	0.0000
10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000	0.0000
11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000	0.0000
12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000	0.0000
13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000	0.0000
14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003	0.0000
15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026	0.0000
16	0.9987	0.9951	0.9840	0.9356	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159	0.0000
17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755	0.0000
18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6883	0.2642	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

*Y has the binomial distribution with parameters n and p. The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95. For n larger than 20, the rth quantile Y_r of a binomial random variable may be approximated using $Y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the rth quantile of a standard normal random variable, obtained from Table A1.

TABLE A7 (Continued)

<i>n</i>	<i>p</i>	<i>m</i> = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
17	0.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220
	0.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235
	0.01	154	158	162	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242
	0.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253
	0.05	157	163	169	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263
	0.10	160	166	172	179	185	192	199	206	212	219	226	233	239	246	253	260	267	274
	0.001	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243
	0.005	171	174	178	183	188	193	198	203	209	214	219	225	230	236	242	247	253	259
	0.01	172	176	181	186	191	196	202	208	213	219	225	231	237	242	248	254	260	266
18	0.025	174	179	184	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278
	0.05	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288
	0.10	178	185	192	199	206	213	220	227	234	241	249	256	263	270	278	285	292	300
	0.001	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268
	0.005	191	194	198	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284
	0.01	192	195	200	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292
	0.025	193	198	204	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304
	0.05	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314
	0.10	198	205	212	219	227	234	242	249	257	264	272	280	288	295	303	311	319	326
19	0.001	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293
	0.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310
	0.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318
	0.025	213	219	225	231	238	245	251	259	266	273	280	287	294	301	309	316	323	330
	0.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341
	0.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354

For *n* or *m* greater than 20, the *p*th quantile *w_p* of the Mann-Whitney test statistic may be approximated by

$$w_p = n(N + 1)/2 + z_p \sqrt{nm(N + 1)/12}$$

where *z_p* is the *p*th quantile of a standard normal random variable, obtained from Table A1, and where *N* = *m* + *n*.

*The entries in this table are quantiles *w_p* of the Mann-Whitney test statistic *T*, given by Equation 5.1.1, for selected values of *p*. Note that *P(T < w_p)* ≤ *p*. Upper quantiles may be found from the equation

$$w_p = n(n + m + 1) - w_{1-p}$$

Critical regions correspond to values of *T* less than (or greater than) but not equal to the appropriate quantile.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes*

Sample Sizes	<i>w_{0.01}</i>	<i>w_{0.05}</i>	<i>w_{0.10}</i>
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7330	5.5758	7.1364
4, 4, 4	5.0500	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

Source: Adapted from Inan, Quade, and Alexander (1975), with permission from the American Mathematical Society.

*The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A10 Quantiles of Spearman's ρ^a

n	$p = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p \approx \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glasser and Winter (1961), with corrections, with permission from the Biometrika Trustees.

^aThe entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$p = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

										$n(n+1)$
										2
<i>n</i>	<i>w_{0.900}</i>	<i>w_{0.950}</i>	<i>w_{0.975}</i>	<i>w_{0.990}</i>	<i>w_{0.995}</i>	<i>w_{0.005}</i>	<i>w_{0.01}</i>	<i>w_{0.025}</i>	<i>w_{0.05}</i>	<i>w_{0.10}</i>
4	0	0	0	0	0	1	3	3	3	4
5	0	0	0	0	0	1	3	4	5	6
6	0	0	1	1	3	4	6	8	9	10.5
7	0	1	2	4	6	9	12	14	16	18
8	1	2	4	6	9	11	15	18	20	22.5
9	2	4	6	9	11	15	19	22	25	27.5
10	4	6	9	11	15	19	23	27	30	33
11	6	8	11	14	18	22	26	30	33	36
12	8	10	14	18	22	26	30	36	39	42
13	10	13	18	22	27	33	38	42	45.5	51
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	56	65	71	76.5	153
18	28	33	41	48	56	63	73	80	85.5	171
19	33	38	47	54	63	71	82	89	95	190
20	38	44	53	61	70	78	89	98	105	210
21	44	50	59	68	78	87	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	151	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	248	266	282	297.5	595
35	160	175	196	214	233	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	268	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903

TABLE A11 (Continued)

<i>n</i>	<i>p = 0.900</i>	<i>0.950</i>	<i>0.975</i>	<i>0.990</i>	<i>0.995</i>
18	103 (0.1465)	131 (0.1863)	155 (0.2205)	183 (0.2603)	203 (0.2888)
19	107 (0.1444)	137 (0.1849)	161 (0.2173)	191 (0.2578)	211 (0.2848)
20	110 (0.1372)	142 (0.1821)	168 (0.2154)	198 (0.2538)	220 (0.2821)
21	114 (0.1390)	146 (0.1780)	174 (0.2122)	206 (0.2512)	228 (0.2780)
22	119 (0.1382)	151 (0.1754)	181 (0.2102)	213 (0.2474)	235 (0.2729)
23	123 (0.1362)	157 (0.1739)	187 (0.2071)	221 (0.2447)	245 (0.2713)
24	128 (0.1353)	162 (0.1712)	194 (0.2051)	228 (0.2410)	252 (0.2664)
25	132 (0.1333)	168 (0.1697)	200 (0.2020)	236 (0.2383)	262 (0.2646)
26	135 (0.1304)	173 (0.1671)	207 (0.2000)	245 (0.2367)	271 (0.2618)
27	141 (0.1304)	179 (0.1656)	213 (0.1970)	253 (0.2340)	279 (0.2581)
28	144 (0.1277)	186 (0.1649)	220 (0.1950)	260 (0.2305)	288 (0.2553)
29	150 (0.1276)	190 (0.1616)	228 (0.1939)	268 (0.2279)	296 (0.2517)
30	153 (0.1249)	197 (0.1608)	233 (0.1902)	277 (0.2261)	305 (0.2490)
31	159 (0.1247)	203 (0.1592)	241 (0.1890)	285 (0.2235)	315 (0.2471)
32	162 (0.1222)	208 (0.1569)	248 (0.1870)	294 (0.2217)	324 (0.2443)
33	168 (0.1219)	214 (0.1553)	256 (0.1858)	302 (0.2192)	334 (0.2424)
34	173 (0.1209)	221 (0.1544)	263 (0.1838)	311 (0.2173)	343 (0.2397)
35	177 (0.1192)	227 (0.1529)	269 (0.1811)	319 (0.2148)	353 (0.2377)
36	182 (0.1182)	232 (0.1506)	276 (0.1792)	328 (0.2130)	362 (0.2351)
37	186 (0.1165)	240 (0.1504)	284 (0.1779)	336 (0.2105)	372 (0.2331)
38	191 (0.1155)	245 (0.1482)	291 (0.1760)	345 (0.2087)	381 (0.2305)
39	197 (0.1151)	251 (0.1467)	299 (0.1748)	355 (0.2075)	391 (0.2285)
40	202 (0.1141)	258 (0.1458)	306 (0.1729)	364 (0.2056)	402 (0.2271)

For *n* greater than 60, approximate quantiles of *T* may be obtained from

$$w_p \approx z_p \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

where *z_p* is from the standard normal distribution given by Table A11. Approximate quantiles of *T* may be obtained from

$$w_p \approx z_p \frac{\sqrt{2(2n+5)}}{3\sqrt{n(n-1)}}$$

Critical regions correspond to values of *T* greater than (or less than) but not including the appropriate quantile. Note that the median of *T* is 0. Quantiles for *T* are obtained by dividing the quantiles of *T* by $(n-1)/2$.

SOURCE: Adapted from Table I, Best (1974), with permission from the author.

TABLE A12 (Continued)

	$w_{0.05}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$n(n+1)/2$
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n + 1)/4] + z_p \sqrt{n(n + 1)(2n + 1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE. Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n + 1)/2 - w_{1-p}$$

where $n(n + 1)/2$ is given in the right hand column in the table. Note that $P(T^* < w_p) \leq p$ and $P(T^* > w_{1-p}) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^* less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic*

One-Sided Test						Two-Sided Test					
	$p = 0.90$	0.95	0.975	0.99	0.995		$p = 0.90$	0.95	0.975	0.99	0.995
	$p = 0.80$	0.90	0.95	0.98	0.99		$p = 0.80$	0.90	0.95	0.98	0.99
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252
Approximation for $n \geq 40$							1.07	1.22	1.36	1.52	1.63
\sqrt{n}							\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}

SOURCE. Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

*The entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m^a

		One-Sided Test: $p = 0.90$	0.95	0.975	0.99	0.995	Two-Sided Test: $p = 0.80$	0.90	0.95	0.975	0.99	0.995	One-Sided Test: $p = 0.90$	0.95	0.975	0.99	0.995			
		One-Sided Test:					Two-Sided Test:					Two-Sided Test:								
$N_1 = 1$	$N_2 = 9$	17/18					$N_1 = 2$	$N_2 = 3$	5/6					$N_1 = 3$	$N_2 = 4$	3/4				
		10 9/10							4 3/4							4/5 5/6				
$n = 3$	2/3	2/3					$n = 22$	7/22	8/22	8/22	10/22	10/22	$N_1 = 1$	$N_2 = 9$	17/18					
4	3/4	3/4	3/4				23	7/23	8/23	9/23	10/23	10/23	$N_1 = 2$	$N_2 = 3$	5/6					
5	3/5	3/5	4/5	4/5			24	7/24	8/24	9/24	10/24	11/24			4	3/4				
6	3/6	4/6	4/6	5/6	5/6		25	7/25	8/25	9/25	10/25	11/25			5	4/5	4/5			
7	4/7	4/7	5/7	5/7	5/7		26	7/26	8/26	9/26	10/26	11/26			6	5/6	5/6			
8	4/8	4/8	5/8	5/8	6/8		27	7/27	8/27	9/27	11/27	11/27			7	5/7	6/7			
9	4/9	5/9	5/9	6/9	6/9		28	8/28	9/28	10/28	11/28	12/28			8	3/4	7/8	7/8		
10	4/10	5/10	6/10	6/10	7/10		29	8/29	9/29	10/29	11/29	12/29			9	7/9	8/9	8/9		
11	5/11	5/11	6/11	7/11	7/11		30	8/30	9/30	10/30	11/30	12/30			10	7/10	4/5	9/10		
12	5/12	5/12	6/12	7/12	7/12		31	8/31	9/31	10/31	11/31	12/31								
13	5/13	6/13	6/13	7/13	8/13		32	8/32	9/32	10/32	12/32	12/32								
14	5/14	6/14	7/14	7/14	8/14		33	8/33	9/33	11/33	12/33	13/33								
15	5/15	6/15	7/15	8/15	8/15		34	8/34	10/34	11/34	12/34	13/34								
16	6/16	6/16	7/16	8/16	9/16		35	8/35	10/35	11/35	12/35	13/35								
17	6/17	7/17	7/17	8/17	9/17		36	9/36	10/36	11/36	12/36	13/36								
18	6/18	7/18	8/18	9/18	9/18		37	9/37	10/37	11/37	13/37	13/37								
19	6/19	7/19	8/19	9/19	9/19		38	9/38	10/38	11/38	13/38	14/38								
20	6/20	7/20	8/20	9/20	10/20		39	9/39	10/39	11/39	13/39	14/39								
21	6/21	7/21	8/21	9/21	10/21		40	9/40	10/40	12/40	13/40	14/40								
		Approximation					1.52	1.73	1.92	2.15	2.30									
		for $n > 40$:					\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}	\sqrt{n}									

SOURCE: Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

^aThe entries in this table are selected quantiles w_{α} of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A21 The t Distribution^a

Degrees of Freedom	$p = 0.6$	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.377	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

SOURCE. Reprinted from Vol. I of Pearson and Hartley (1976), with permission from the Biometrika Trustees.

^aThe entries in this table are quantiles w_p of the t distribution for various degrees of freedom. Quantiles w_p for $p < 0.5$ may be computed from the equation

$$w_p = -w_{1-p}$$

Note that $w_{0.50} = 0$ for all degrees of freedom.

ILE A20 (Continued)

	$p = 0.90$	0.95	0.975	0.99	0.995	
$N_1 = 8$	27/56	33/56	5/8	41/56	3/4	
9	31/63	5/9	40/63	5/7	47/63	
10	33/70	39/70	43/70	7/10	5/7	
14	3/7	1/2	4/7	9/14	5/7	
28	3/7	13/28	15/28	17/28	9/14	
$N_1 = 9$	4/9	13/24	5/8	2/3	3/4	
10	19/40	21/40	23/40	27/40	7/10	
12	11/24	1/2	7/12	5/8	2/3	
16	7/16	1/2	9/16	5/8	5/8	
32	13/32	7/16	1/2	9/16	19/32	
$N_1 = 10$	7/15	1/2	26/45	2/3	31/45	
12	4/9	1/2	5/9	11/18	2/3	
15	19/45	22/45	8/15	3/5	29/45	
18	7/18	4/9	1/2	5/9	11/18	
36	13/36	5/12	17/36	19/36	5/9	
$N_1 = 15$	2/5	7/15	1/2	17/30	19/30	
20	2/5	9/20	1/2	11/20	3/5	
40	7/20	2/5	9/20	1/2	—	
$N_1 = 15$	23/60	9/20	1/2	11/20	7/12	
16	3/8	7/16	23/48	13/24	7/12	
18	13/36	5/12	17/36	19/36	5/9	
20	11/30	5/12	7/15	31/60	17/30	
$N_1 = 20$	7/20	2/5	13/30	29/60	31/60	
16	$N_1 = 20$	27/80	31/80	17/40	19/40	41/80
the sample approximation	$1.07\sqrt{\frac{m+n}{mn}}$	$1.22\sqrt{\frac{m+n}{mn}}$	$1.36\sqrt{\frac{m+n}{mn}}$	$1.52\sqrt{\frac{m+n}{mn}}$	$1.63\sqrt{\frac{m+n}{mn}}$	

Adapted from Massey (1952), with permission from the Institute of Mathematical Statistics. Entries in this table are selected quantiles w_p of the Smirnov test statistic T for two samples, defined in sections 6.3.1, 6.3.2, and 6.3.3. To enter the table let N_1 be the smaller sample size and let N_2 be the sample size. Reject H_0 at the level α if T exceeds $w_{1-\alpha}$ as given in this table. If n and m are not covered by this table, use the large sample approximation given at the end of the table, or consult exact tables by Jennrich, which appear in Harter and Owen (1970) for $n, m \leq 100$.