

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2012/13

COURSE TITLE: OPERATIONS RESEARCH I

COURSE CODE: ST 408

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER QUESTION ONE AND ANY TWO QUESTIONS

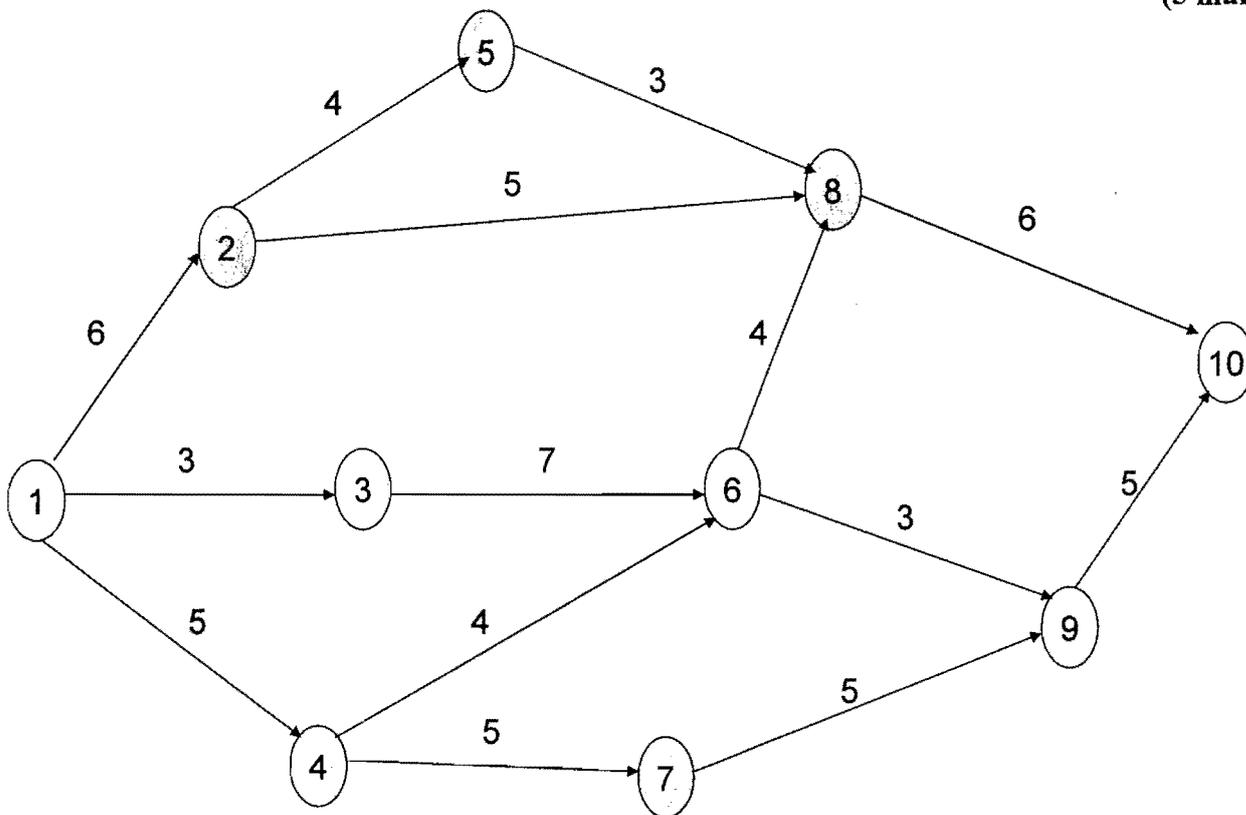
SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS AND GRAPH PAPER

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Question 1

(a) Consider the network shown in the figure below. The activity times in days are given along the arrows. Calculate:

- (i) The earliest starting and finishing times (5 marks)
- (ii) The latest starting and finishing times (5 marks)
- (iii) The slack for the events and determine the critical path. Put the calculations in tabular form. (5 marks)



(b) Consider the following precedence table showing the probable activity times (in weeks) associated with staging a conference of University Professors of Management Science

| Activity | Optimistic | Most Likely | Pessimistic |
|----------|------------|-------------|-------------|
| 1-2 | 3 | 4 | 6 |
| 1-3 | 1 | 2 | 4 |
| 2-3 | 1 | 3 | 5 |
| 2-4 | 6 | 8 | 12 |
| 3-4 | 2 | 3 | 5 |
| 3-5 | 1 | 2 | 4 |
| 4-6 | 4 | 5 | 9 |
| 5-6 | 4 | 6 | 11 |

Determine;

- (i) The expected activity time and variance of the expected activity time for each activity (10 marks)
- (ii) The critical path for this project. (2 marks)
- (iii) Assume that the conference planners would like to complete all their planning activities within 10 weeks. What is the likelihood of this occurring? (3 marks)

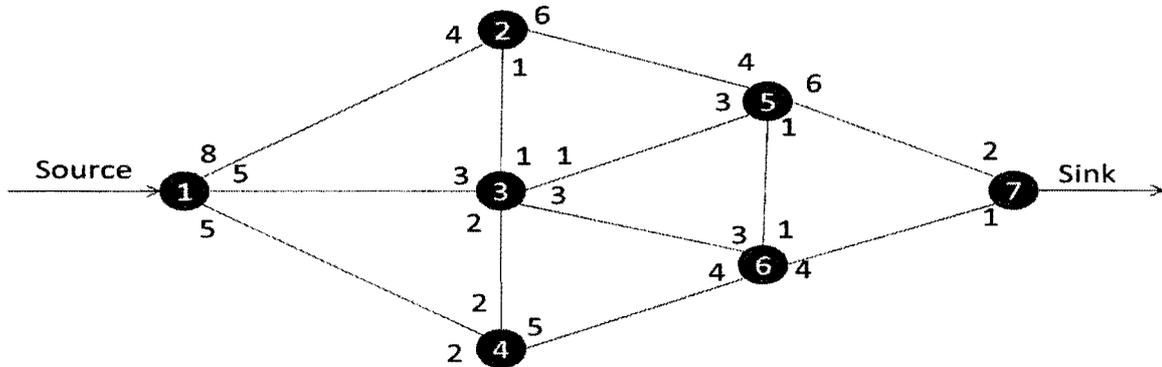
Question 2

(a) Suppose we wish to establish a communication network for the oil field, intermediate facilities and refinery, so that communication lines connect all locations. The cost associated with establishing communications between any two of the facilities is a function of the distance between any two facilities. The distances between facilities is as given below:

| Connection | Distance (km) | Connection | Distance (km) |
|---------------|---------------|--------------|---------------|
| Oil field → A | 105 | D → G | 140 |
| Oil field → B | 115 | D → E | 143 |
| Oil field → C | 125 | E → G | 121 |
| A → B | 135 | E → F | 113 |
| A → D | 120 | F → H | 109 |
| B → D | 100 | G → Refinery | 101 |
| B → E | 114 | G → H | 119 |
| C → B | 122 | H → Refinery | 121 |
| C → F | 117 | | |

- (i) Draw the network (3 marks)
- (ii) Determine the shortest route from the oil field to the refinery. (7 marks)

(b) For the following pipeline network, find the maximum natural gas flow from source to sink. (10 marks)



Question 3

(a) A mail-order house uses 18, 000 boxes a year. Carrying costs are 60 cents per box per year, and ordering costs are E96. If the following price schedule applies; determine:

- (i) Determine the optimal order quantity that will minimize total cost. **(7 marks)**
 (ii) The number of orders per year. **(3 mark)**

| Number of boxes | Price per box |
|-----------------|---------------|
| 1,000 – 1,999 | E12.50 |
| 2,000 – 4,999 | E12.00 |
| 5,000 – 9,999 | E11.50 |
| 10,000 or more | E11.00 |

(b) Demand for Café 87's whipped-cream double-layer cake at a local pastry shop can be approximated using a Poisson distribution with a mean of six per day. The manager estimates that it costs E50 to prepare each cake. Fresh cakes sell for E90 whilst day-old cakes sell for E75 each. What stocking level is appropriate if one-half of the day-old cakes are sold and the rest thrown out? **(10 marks)**

Question 4

(a) Chrissilda Swaziland currently employs one worker whose job is to load bricks on outgoing company trucks. An average of 24 trucks per day or 3 per hour, arrive at the loading bay, according to a Poisson distribution. The worker loads them at a rate of 4 trucks per hour, following an approximately exponential distribution in his service times.

Chrissilda believes that adding a second brick loader will substantially improve the firm's productivity. He estimates that a two-person crew at the loading Bay will double the loading rate from 4 trucks per hour to 8 trucks per hour.

- (i) Analyse the effect on the queue of such a change and compare the results to those achieved with one worker. **(10 marks)**

(b) Truck drivers working for Chrissilda earn an average of E350 per hour. Brick loaders receive about E200 per hour. Truck drivers waiting in the queue or at the loading bay are drawing a salary but are productively idle and unable to generate revenue during that time. What would be the hourly cost savings to the firm if it employs 2 loaders instead of 1?

(10 marks)

Question 5

An oil company has recently acquired rights in a certain area to conduct surveys and test drillings to lead to lifting oil where it is found in commercial exploitable quantities. The area is already considered to have good potential for finding oil in commercial quantities. At the onset, the company has the choice to conduct further geological tests or to carry out a drilling programme immediately. On the known conditions, the company estimates that there is a 65:35 chance of further tests showing a success.

Whether the tests show the possibility of ultimate success or not or even if no tests are undertaken at all, the company could still pursue its drilling programme or alternatively consider selling its rights to drill in the area. If the drilling programme is carried out, the likelihood of final success or failure depends on the foregoing stages. Thus:

- If successful test has been carried out, the expectation of success in drilling is given as 75:25
- If tests indicate failure, then the expectation of success in drilling is given as 25:75
- If no tests have been conducted, the expectation of success in drilling is given as 55:45.

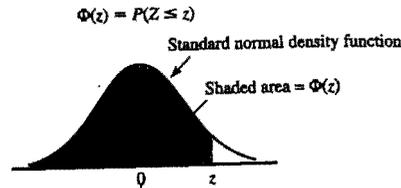
Costs and revenues have been estimated for all possible outcomes and the net receipt of each is given below:

| Outcome | Net receipt (Emillions) |
|------------------------------|--------------------------------|
| Success: | |
| With prior tests | 105 |
| Without prior tests | 125 |
| Failure: | |
| With prior tests | -55 |
| Without prior tests | -45 |
| Sale of exploitation rights: | |
| Prior tests show success | 65 |
| Prior tests shoe failure | 15 |
| Without prior tests | 45 |

- (a) Draw a decision tree to represent the above information. (5 marks)
- (b) Evaluate the tree and advise the management of the company on its best course of action. (15 marks)

END OF EXAM!!

Table A.3 Standard Normal Curve Areas



| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0038 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3482 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

(continued)

Table A.3 Standard Normal Curve Areas (cont.)

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9278 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |