DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2012/13

COURSE TITLE:	OPERATIONS RESEARCH I
COURSE CODE:	ST 307
TIME ALLOWED:	TWO (2) HOURS
INSTRUCTION:	1. ANSWER QUESTION ONE AND CHOOSE TWO QUESTIONS FROM THE REMAINING QUESTIONS. 2. QUESTION ONE CARRIES 40 MARKS AND THE REMAINING QUESTIONS CARRY 30 MARKS EACH.
SPECIAL REQUIREMENTS:	SCIENTIFIC CALCULATORS AND GRAPH PAPER

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Question 1

(a) Define the following terms in the context of linear programming:

(i) Decision variables **(3 marks)**
(ii) Objective function **(3 marks)**
(iii) Functional constraints **(4 marks)**

(b) A manufacturing firm has discontinued production of certain unprofitable product line. This created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; products 1,2 and 3. The available capacity on the machines that might limit output is summarised in the following table:

Machine type	Available time (in machine hours per week
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is as follows:

Machine type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that sales potential for product 1 and 2 exceeds the maximum production rate and that sales potential for product 3 is 20 units per week. The unit profit for products 1, 2 and 3 would be E30, E12 and E15.

Formulate the linear programming model for determining how much of each product the firm should produce to maximise profit. **(10 marks)**

(c) Consider the dual problem:

Minimise $Z = 4y_1 + 12y_2 + 18y_3$

Subject to $y_1 + 3y_3 \geq 3$

$2y_2 + 2y_3 \geq 5$

$y_1, y_2, y_3 \geq 0.$

(i) Find the primal problem. **(5 marks)**
(ii) Find the optimal solution of the primal problem using the simplex method. **(8 marks)**
(iii) Use information in the final simplex tableau in (b) above to find the optimal solution for the dual problem. **(3 marks)**
(iv) Verify the dual theorem using the optimal solutions in (b) and (c) above. **(4 marks)**

Question 2

Consider a production problem with the following initial and final simplex tableaus. The objective of the company is to maximize profits subject to three resource constraints: raw material, machine hours and labour days.

Initial tableau

C			4	5	6	0	0	0
	SOL	RHS	X_1	X_2	X_3	S_1	S_2	S_3
0	S_1	60	2	3	4	1	0	0
0	S_2	40	1	1	1	0	1	0
0	S_3	50	2	3	0	0	0	1
	Z_j	0	0	0	0	0	0	0
		C_j-Z_j	4	5	6	0	0	0

Final tableau

C			4	5	6	0	0	0
	SOL	RHS	X_1	X_2	X_3	S_1	S_2	S_3
6	X_3	5/2	0	0	1	1/4	0	-1/4
0	S_2	25/2	0	-1/2	0	-1/4	1	-1/4
4	X_1	25	1	3/2	0	0	0	1/2
	Z_j	115	4	6	6	3/2	0	1/2
		C_j-Z_j	0	-1	0	-3/2	0	-1/2

(a) Write both the primal and the dual problems. **(10 marks)**

(b) Give solutions to both problems, interpreting clearly the values of each variable in the context of the problem i.e. provide an economic interpretation. **(10 marks)**

(c) Give the range over which the objective function coefficient for X_1 can vary without changing the optimal combination. **(10 marks)**

Question 3

A farm cooperative in Makhungutsha in the Manzini region wishes to distribute its products to four large storage depots for exportation to neighbouring countries in the Southern African region. It has produced the following:

Corn	150,000 bushels
Wheat	750,000 bushels
Oats	1,500,000 bushels
Soybeans	400,000 bushels

The demand requirements for these four crops are as follows:

Storage Depot 1	400,000 bushels
Storage Depot 2	850,000 bushels
Storage Depot 3	400,000 bushels
Storage Depot 1	1,200,000 bushels

The distribution costs for the supply-demand situation are as follows:

Product	Storage Depots			
	Storage Depot 1	Storage Depot 2	Storage Depot 3	Storage Depot 4
Corn	30	25	40	25
Wheat	45	25	45	35
Oats	35	20	30	20
Soybean	50	40	50	50

Determine the minimum cost shipping plan. **(30 marks)**

Question 4

Janice Phillips is the manager of the computer services bureau at a large corporation. She currently has a backlog of six disk copying jobs which she wishes to assign to six technicians. The time requirements, by disk copying job and by technician, are as follows:

	Disk Copying Job					
Technician	1	2	3	4	5	6
Jan	6	4	3	5	9	7
Bill	2	6	11	8	3	10
Sue	4	7	13	9	4	9
Tom	5	3	2	8	7	4
Mary	3	6	9	5	2	6
Joe	5	7	3	8	4	6

Formulate and solve as an assignment problem. **(30 marks)**

END OF EXAM!!