UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2013

TITLE OF PAPER	:	MATHEMATICS FOR STATISTICIANS
COURSE CODE	:	ST 202
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR
INSTRUCTIONS	:	THIS PAPER HAS FIVE (5) QUESTIONS. ANSWER ANY THREE (3) QUESTIONS.

Question 1 **[20 marks, 4+4+8+4]**

(a) Let A and B be 3×3 matrices, with $\det(A) = 4$ and $\det(A^2B^{-1}) = -8$. Use properties of determinants to compute:

  (i) $\det(2A)$

  (ii) $\det(B)$

(b) The weekly output (in units) of a factory depends on the amount of capital and labour it employs as follows: if it uses k units of capital and l of labour then its output is $Q(k,l) = \sqrt{k}\sqrt{l}$ units. The cost to the firm of each unit of capital is 1 dollar, and the cost of each unit of labour is 4 dollars. Use the method of Lagrange multipliers to find the minimum weekly cost of producing a quantity of 200 units.

(c) Find the geometric series that has second term equal to 5 and sum to infinity equal to 20

Question 2 **[20 marks, 2+2+2+6+8]**

(a) Suppose

$$B = \left[\begin{array}{cc|c} 1 & 2 & k \\ 3 & h & 8 \end{array}\right]$$

  (i) What is required of h and k so that the system has no solutions?

  (ii) What is required of h and k so that the system has a unique solution?

  (iii) What is required of h and k so that the system has infinitely many solutions?

(b) Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$, and $\vec{a}_3 = \begin{bmatrix} 2 \\ h \\ 9 \end{bmatrix}$.

  (i) Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3]$, i.e., A is the matrix with $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ as its columns. Find the row echelon form (REF) (**NOT** RREF) of A. Show all the calculations by hand.

  (ii) Find all possible value(s) of h such that the set $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ is linearly independent.

Question 3 **[20 marks, 4+4+8+8]**

(a) Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

  (i) Are these vectors linearly independent? Explain.

  (ii) Are these any of vectors orthogonal? Explain.

  (iii) Let $W = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Find an orthogonal basis for W.

(b) A travel company is the only provider of holidays (of one weeks duration) to two private island resorts, X and Y . The demand equations for such holidays are given by

$$x = 200 - p_X$$
$$y = 100 - p_Y,$$

where x and y are the numbers of week-long holidays at X and Y demanded (respectively) and p_X, p_Y are (respectively) the prices of these holidays. The company's joint total cost function (that is, the cost of providing x holidays in X and y holidays in Y) is

$$2x^2 + y^2 + 2xy.$$

Find an expression in terms of x and y for the profit the company obtains from selling these holidays. Determine the numbers x and y of holidays in resorts X and Y that will maximise the companys profit.

Question 4 [20 marks, 12+8]

(a) Determine if A can be diagonalized. If so, find P and D such that $A = PDP^{-1}$.

$$C = \begin{bmatrix} -4 & -3 & -3 \\ 0 & -1 & 0 \\ 6 & 6 & 5 \end{bmatrix}$$

(b) The function f is given, for some number a, by

$$f(x, y) = 2xy + x^{2a}y^a$$

Find, in terms of x, y and a, the partial derivatives

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}.$$

Now suppose we know that f satisfies

$$x^2\frac{\partial^2 f}{\partial x^2} - 2y^2\frac{\partial^2 f}{\partial y^2} - 18f + 36xy = 0.$$

Determine the possible values of a.

Question 5 [20 marks, 6+6+2+6]

(a) The production costs per week for producing x widgets is given by,

$$C(x) = 500 + 350x - 0.09x^2, \qquad 0 \le x \le 1000$$

Answer each of the following questions.

(i) What is the cost to produce the 301st widget?

(ii) What is the rate of change of the cost at $x = 300$?

(b) Consider the system of equations:

$$\begin{aligned} x + 6y + 2z - 5u - 2v &= -4 \\ 2z - 8u - v &= 3 \\ v &= 7 \end{aligned}$$

(i) Write the system as a matrix equation.

(ii) Solve the system and use vector parameter form for your solution.