UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2013

TITLE OF PAPER	:	**PROBABILITY THEORY**
COURSE CODE	:	**ST 201**
TIME ALLOWED	:	**THREE (3) HOURS**
INSTRUCTIONS	:	**ANSWER ANY FIVE QUESTIONS.**
REQUIREMENTS	:	**SCIENTIFIC CALCULATOR AND STATISTICAL TABLES.**

Question 1

(a) A random car is chosen among all those passing through Matsapha Traffic Circle on a certain day. The probability that the car is yellow is 3/100: the probability that the driver is female is 1/5; and the probability that the car is yellow and the driver is female is 1/50. Find the conditional probability that the driver is blonde given that the car is yellow.

(8 Marks)

(b) 2% of the population have a certain blood disease in a serious form; 10% have it in a mild form; and 88% don't have it at all. A new blood test is developed; the probability of testing positive is 9/10 if the subject has the serious form, 6/10 if the subject has the mild form, and 1/10 if the subject doesn't have the disease. A person has just tested positive. What is the probability that (s)he has the serious form of the disease?

(12 Marks)

Question 2

Suppose X has the following pdf, where A is a constant to be determined:

$$f_X(u) = \begin{cases} A(1-u^2) & -1 \leq u \leq 1 \\ 0 & \text{else.} \end{cases}$$

Find A, P{0.5 < X < 1:5}, $F_X(u)$, μ_X, Var(X), and σ_X.

(4+4+4+6+2 Marks)

Question 3

(a) Telephone calls enter the UNISWA switchboard on the average of two every 3 minutes. What is the probability of five or more calls arriving in a 9-minute period?

(6 Marks)

(b) A manufacturer of Christmas tree bulbs knows that 2% of its bulbs are defective. What is the probability that a box of 100 of these bulbs contains at most three defective bulbs?

(6 Marks)

(c) Suppose that an average of 30 customers per hour is connected to electricity by Swaziland Electricity (SEC). What is the probability that SEC will wait more than 5 minutes before both of the first two customers are connected?

(8 Marks)

Question 4

Let X ~Gamma(α,λ). Prove that;

(a) Derive the $M_X(t)$

(3 Marks)

(b) Use $M_X(t)$ Prove that $E(X) = \alpha/\lambda$

(5 Marks)

(c) Use $M_X(t)$ Prove that $Var(X) = \alpha/\lambda^2$

(6 Marks)

(d) Use $M_X(t)$ Prove that $\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$

(6 Marks)

Question 5

Suppose that X and Y are independent random variables with the same probability density function (pdf) f (x).

(a) Write down, without proof, a formula for the pdf of X + Y.

(2 Marks)

(b) Suppose that f (x) = x/2 for $0 < x < 2$ (and f (x) = 0 elsewhere). Find the pdf of W = X + Y for $0 < w < 2$ and for $2 < w < 4$.

(12 Marks)

(c) Find the pdf of $V = (X-1)^2$.

(6 Marks)

Question 5

a) The continuous random variable X has probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi x}} \exp\left\{-\frac{x}{2}\right\}, \qquad x > 0.$$

Show that X has moment generating function (mgf)

$$M_X(t) = \frac{1}{\sqrt{(1-2t)}}, \qquad t < \tfrac{1}{2}.$$

Hence find the expected value and variance of X.

(10 Marks)

b) Suppose that the discrete random variable X has the probability function

$$P(X = x) = (1-\theta)^{x-1}\theta, \qquad x = 1, 2, \ldots.$$

Show that X has moment generating function (mgf)

$$M_X(t) = \frac{e^t\theta}{1-e^t(1-\theta)}, \qquad t < -\ln(1-\theta).$$

Hence find the expected value and variance of X.

(10 Marks)

Question 7

Suppose the continuous random variables (X, Y, Z) have joint p.d.f

$$f(x, y, z) = Kxyz^2, \qquad 0 \leq x, y \leq 1, \quad 0 \leq z \leq 3.$$

(a) Show that the constant K = 4/9.

(4 Marks)

(b) Find the marginal probability density function of Y and hence show that E(Y) = 2/3.

(4 Marks)

(c) Show that the marginal joint probability density function of (X, Z) is

$$f_{X,Z}(x, z) = \frac{2}{9}xz^2, \qquad 0 \leq x \leq 1, \quad 0 \leq z \leq 3.$$

(4 Marks)

(d) Find the conditional distribution of Y given X = 1/2 , Z = 1 and hence find $\mathrm{E}(Y|X = \frac{1}{2}, Z = 1)$.

(4 Marks)

TABLE 1. STANDARD NORMAL DISTRIBUTION. The table gives the area under the standard normal curve between 0 and z, i.e. $\Pr[0 < Z < z]$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

TABLE 2. t-DISTRIBUTION. The table gives the value of $t_n^{(P)}$ where n is the degrees of freedom, i.e. $P = \Pr[t_n > t_n^{(P)}]$.

P	0.4	0.3	0.2	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	0.727	1.376	3.078	6.314	12.71	31.82	63.65	127.4	318.3	637.9
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	7.453	10.22	12.92
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	0.256	0.530	0.853	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	0.255	0.530	0.853	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	0.255	0.530	0.853	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	0.255	0.529	0.852	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	0.255	0.529	0.852	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36	0.255	0.529	0.852	1.306	1.688	2.028	2.434	2.719	2.990	3.333	3.582
37	0.255	0.529	0.851	1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574
38	0.255	0.529	0.851	1.304	1.686	2.024	2.429	2.712	2.980	3.319	3.566
39	0.255	0.529	0.851	1.304	1.685	2.023	2.426	2.708	2.976	3.313	3.558
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
45	0.255	0.528	0.850	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
50	0.255	0.528	0.849	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.254	0.527	0.847	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.254	0.526	0.846	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.254	0.526	0.846	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.254	0.526	0.845	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
110	0.254	0.526	0.845	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
140	0.254	0.526	0.844	1.288	1.656	1.977	2.353	2.611	2.852	3.149	3.361
160	0.254	0.525	0.844	1.287	1.654	1.975	2.350	2.607	2.846	3.142	3.352
180	0.254	0.525	0.844	1.286	1.653	1.973	2.347	2.603	2.842	3.136	3.345
200	0.254	0.525	0.843	1.286	1.653	1.972	2.345	2.601	2.839	3.131	3.340
z	0.253	0.524	0.841	1.282	1.645	1.960	2.326	2.576	2.807	3.091	3.291

TABLE 3. CHI-SQUARED DISTRIBUTION. The table gives the value of $\chi_n^{2(P)}$ where n is the degrees of freedom, i.e. $P = \Pr[\chi_n^2 > \chi_n^{2(P)}]$, for $P > 0.5$.

P	0.9995	0.999	0.9975	0.995	0.99	0.975	0.95	0.9	0.8	0.6
1	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.016	0.064	0.275
2	0.001	0.002	0.005	0.010	0.020	0.051	0.103	0.211	0.446	1.022
3	0.015	0.024	0.045	0.072	0.115	0.216	0.352	0.584	1.005	1.869
4	0.064	0.091	0.145	0.207	0.297	0.484	0.711	1.064	1.649	2.753
5	0.158	0.210	0.307	0.412	0.554	0.831	1.145	1.610	2.343	3.655
6	0.299	0.381	0.527	0.676	0.872	1.237	1.635	2.204	3.070	4.570
7	0.485	0.598	0.794	0.989	1.239	1.690	2.167	2.833	3.822	5.493
8	0.710	0.857	1.104	1.344	1.646	2.180	2.733	3.490	4.594	6.423
9	0.972	1.152	1.450	1.735	2.088	2.700	3.325	4.168	5.380	7.357
10	1.265	1.479	1.827	2.156	2.558	3.247	3.940	4.865	6.179	8.295
11	1.587	1.834	2.232	2.603	3.053	3.816	4.575	5.578	6.989	9.237
12	1.934	2.214	2.661	3.074	3.571	4.404	5.226	6.304	7.807	10.182
13	2.305	2.617	3.112	3.565	4.107	5.009	5.892	7.042	8.634	11.129
14	2.697	3.041	3.582	4.075	4.660	5.629	6.571	7.790	9.467	12.078
15	3.108	3.483	4.070	4.601	5.229	6.262	7.261	8.547	10.307	13.030
16	3.536	3.942	4.573	5.142	5.812	6.908	7.962	9.312	11.152	13.983
17	3.980	4.416	5.092	5.697	6.408	7.564	8.672	10.085	12.002	14.937
18	4.439	4.905	5.623	6.265	7.015	8.231	9.390	10.865	12.857	15.893
19	4.912	5.407	6.167	6.844	7.633	8.907	10.117	11.651	13.716	16.850
20	5.398	5.921	6.723	7.434	8.260	9.591	10.851	12.443	14.578	17.809
21	5.896	6.447	7.289	8.034	8.897	10.283	11.591	13.240	15.445	18.768
22	6.405	6.983	7.865	8.643	9.542	10.982	12.338	14.041	16.314	19.729
23	6.924	7.529	8.450	9.260	10.196	11.689	13.091	14.848	17.187	20.690
24	7.453	8.085	9.044	9.886	10.856	12.401	13.848	15.659	18.062	21.652
25	7.991	8.649	9.646	10.520	11.524	13.120	14.611	16.473	18.940	22.616
26	8.538	9.222	10.256	11.160	12.198	13.844	15.379	17.292	19.820	23.579
27	9.093	9.803	10.873	11.808	12.879	14.573	16.151	18.114	20.703	24.544
28	9.656	10.391	11.497	12.461	13.565	15.308	16.928	18.939	21.588	25.509
29	10.227	10.986	12.128	13.121	14.256	16.047	17.708	19.768	22.475	26.475
30	10.804	11.588	12.765	13.787	14.953	16.791	18.493	20.599	23.364	27.442
31	11.389	12.196	13.407	14.458	15.655	17.539	19.281	21.434	24.255	28.409
32	11.979	12.811	14.056	15.134	16.362	18.291	20.072	22.271	25.148	29.376
33	12.576	13.431	14.709	15.815	17.074	19.047	20.867	23.110	26.042	30.344
34	13.179	14.057	15.368	16.501	17.789	19.806	21.664	23.952	26.938	31.313
35	13.788	14.688	16.032	17.192	18.509	20.569	22.465	24.797	27.836	32.282
36	14.401	15.324	16.700	17.887	19.233	21.336	23.269	25.643	28.735	33.252
37	15.020	15.965	17.373	18.586	19.960	22.106	24.075	26.492	29.635	34.222
38	15.644	16.611	18.050	19.289	20.691	22.878	24.884	27.343	30.537	35.192
39	16.273	17.262	18.732	19.996	21.426	23.654	25.695	28.196	31.441	36.163
40	16.906	17.916	19.417	20.707	22.164	24.433	26.509	29.051	32.345	37.134
45	20.137	21.251	22.900	24.311	25.901	28.366	30.612	33.350	36.884	41.995
50	23.461	24.674	26.464	27.991	29.707	32.357	34.764	37.689	41.449	46.864
60	30.341	31.738	33.791	35.534	37.485	40.482	43.188	46.459	50.641	56.620
70	37.468	39.036	41.332	43.275	45.442	48.758	51.739	55.329	59.898	66.396
80	44.791	46.520	49.043	51.172	53.540	57.153	60.391	64.278	69.207	76.188
90	52.276	54.155	56.892	59.196	61.754	65.647	69.126	73.291	78.558	85.993
100	59.896	61.918	64.857	67.328	70.065	74.222	77.930	82.358	87.945	95.808
110	67.631	69.789	72.922	75.550	78.458	82.867	86.792	91.471	97.362	105.632
120	75.467	77.755	81.072	83.852	86.923	91.573	95.705	100.624	106.806	115.465
140	91.392	93.926	97.591	100.655	104.034	109.137	113.659	119.029	125.758	135.149
160	107.597	110.360	114.350	117.679	121.346	126.870	131.756	137.546	144.783	154.856
180	124.033	127.011	131.306	134.884	138.820	144.741	149.969	156.153	163.868	174.580
200	140.661	143.843	148.426	152.241	156.432	162.728	168.279	174.835	183.003	194.320

TABLE 3, continued. CHI-SQUARED DISTRIBUTION. The table gives the value of $\chi_n^{2(P)}$ where n is the degrees of freedom, i.e. $P = \Pr[\chi_n^2 > \chi_n^{2(P)}]$, for $P < 0.5$.

P	0.4	0.2	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.708	1.642	2.706	3.841	5.024	6.635	7.879	9.141	10.828	12.116
2	1.833	3.219	4.605	5.991	7.378	9.210	10.597	11.983	13.816	15.202
3	2.946	4.642	6.251	7.815	9.348	11.345	12.838	14.320	16.266	17.730
4	4.045	5.989	7.779	9.488	11.143	13.277	14.860	16.424	18.467	19.997
5	5.132	7.289	9.236	11.070	12.833	15.086	16.750	18.386	20.515	22.105
6	6.211	8.558	10.645	12.592	14.449	16.812	18.548	20.249	22.458	24.103
7	7.283	9.803	12.017	14.067	16.013	18.475	20.278	22.040	24.322	26.018
8	8.351	11.030	13.362	15.507	17.535	20.090	21.955	23.774	26.125	27.868
9	9.414	12.242	14.684	16.919	19.023	21.666	23.589	25.462	27.877	29.666
10	10.473	13.442	15.987	18.307	20.483	23.209	25.188	27.112	29.588	31.420
11	11.530	14.631	17.275	19.675	21.920	24.725	26.757	28.729	31.264	33.136
12	12.584	15.812	18.549	21.026	23.337	26.217	28.300	30.318	32.910	34.821
13	13.636	16.985	19.812	22.362	24.736	27.688	29.819	31.883	34.528	36.478
14	14.685	18.151	21.064	23.685	26.119	29.141	31.319	33.426	36.123	38.109
15	15.733	19.311	22.307	24.996	27.488	30.578	32.801	34.950	37.697	39.719
16	16.780	20.465	23.542	26.296	28.845	32.000	34.267	36.456	39.252	41.308
17	17.824	21.615	24.769	27.587	30.191	33.409	35.718	37.946	40.790	42.879
18	18.868	22.760	25.989	28.869	31.526	34.805	37.156	39.422	42.312	44.434
19	19.910	23.900	27.204	30.144	32.852	36.191	38.582	40.885	43.820	45.973
20	20.951	25.038	28.412	31.410	34.170	37.566	39.997	42.336	45.315	47.498
21	21.991	26.171	29.615	32.671	35.479	38.932	41.401	43.775	46.797	49.010
22	23.031	27.301	30.813	33.924	36.781	40.289	42.796	45.204	48.268	50.511
23	24.069	28.429	32.007	35.172	38.076	41.638	44.181	46.623	49.728	52.000
24	25.106	29.553	33.196	36.415	39.364	42.980	45.559	48.034	51.179	53.479
25	26.143	30.675	34.382	37.652	40.646	44.314	46.928	49.435	52.620	54.947
26	27.179	31.795	35.563	38.885	41.923	45.642	48.290	50.829	54.052	56.407
27	28.214	32.912	36.741	40.113	43.195	46.963	49.645	52.215	55.476	57.858
28	29.249	34.027	37.916	41.337	44.461	48.278	50.993	53.594	56.892	59.300
29	30.283	35.139	39.087	42.557	45.722	49.588	52.336	54.967	58.301	60.735
30	31.316	36.250	40.256	43.773	46.979	50.892	53.672	56.333	59.703	62.162
31	32.349	37.359	41.422	44.985	48.232	52.191	55.003	57.692	61.098	63.582
32	33.381	38.466	42.585	46.194	49.480	53.486	56.328	59.046	62.487	64.995
33	34.413	39.572	43.745	47.400	50.725	54.776	57.648	60.395	63.870	66.402
34	35.444	40.676	44.903	48.602	51.966	56.061	58.964	61.738	65.247	67.803
35	36.475	41.778	46.059	49.802	53.203	57.342	60.275	63.076	66.619	69.198
36	37.505	42.879	47.212	50.998	54.437	58.619	61.581	64.410	67.985	70.588
37	38.535	43.978	48.363	52.192	55.668	59.893	62.883	65.739	69.346	71.972
38	39.564	45.076	49.513	53.384	56.896	61.162	64.181	67.063	70.703	73.351
39	40.593	46.173	50.660	54.572	58.120	62.428	65.476	68.383	72.055	74.725
40	41.622	47.269	51.805	55.758	59.342	63.691	66.766	69.699	73.402	76.094
45	46.761	52.729	57.505	61.656	65.410	69.957	73.166	76.223	80.077	82.876
50	51.892	58.164	63.167	67.505	71.420	76.154	79.490	82.664	86.661	89.560
60	62.135	68.972	74.397	79.082	83.298	88.379	91.952	95.344	99.607	102.695
70	72.358	79.715	85.527	90.531	95.023	100.425	104.215	107.808	112.317	115.577
80	82.566	90.405	96.578	101.879	106.629	112.329	116.321	120.102	124.839	128.261
90	92.761	101.054	107.565	113.145	118.136	124.116	128.299	132.256	137.208	140.782
100	102.946	111.667	118.498	124.342	129.561	135.807	140.169	144.293	149.449	153.167
110	113.121	122.250	129.385	135.480	140.917	147.414	151.948	156.230	161.581	165.435
120	123.289	132.806	140.233	146.567	152.211	158.950	163.648	168.082	173.617	177.602
140	143.604	153.854	161.827	168.613	174.648	181.840	186.847	191.566	197.451	201.683
160	163.898	174.828	183.310	190.516	196.915	204.530	209.824	214.809	221.019	225.480
180	184.173	195.743	204.704	212.304	219.044	227.056	232.620	237.855	244.371	249.048
200	204.433	216.609	226.021	233.994	241.058	249.445	255.264	260.735	267.541	272.422