UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2012

| | | |
|---|---|---|
| TITLE OF PAPER | : | INFERENTIAL STATISTICS |
| COURSE CODE | : | ST 220 |
| TIME ALLOWED | : | TWO (2) HOURS |
| REQUIREMENTS | : | CALCULATOR AND STATISTICAL TABLES |
| INSTRUCTIONS | : | THIS PAPER HAS FIVE (5). ANSWER ANY THREE (3) QUESTIONS. |

## Question 1 [20 marks, 8+8+4]

(a) A manufacturer of a migraine headache drug claimed that the drug is 90% effective in relieving migraines for a period of 24 hours. In a sample of 200 people who have migraine headaches, the drug provided relief for 160 people for a period of 24 hours. Determine whether the manufacture's claim is legitimate at a level of significance of 0.05.

(b) To compare the times required by four experienced assembly workers to assemble a table, the assembling of several identical tables by these workers was timed, in minutes, and the following observations were obtained.

| Worker | | | |
|---|---|---|---|
| **Sgubhu** | **Sgcoko** | **Mafutha** | **Macandza** |
| 22.0 | 22.9 | 21.1 | 20.9 |
| 20.3 | 21.1 | 18.6 | 25.1 |
| 18.2 | 23.2 | 22.2 | 20.7 |
| 23.6 | 24.8 | 18.0 | 21.8 |
| 22.4 | | 19.9 | 23.4 |
| | | 23.0 | |

At the 0.05 level of significance, can the differences among the means of the four workers be attributed to chance?

(c) The mean of the grades of 36 freshmen is used to estimate the true average grade for the freshman class. If $\mu$ is the true mean, what is the probability that the estimate differs from the true mean by 3.6 marks if the standard deviation is known to be 24?

## Question 2 [20 marks, 8+4+4+4]

(a) After a recent National Aids Awareness Campaign, a market research company conducted a countrywide survey on behalf of the Department of National Health. The brief was to establish whether the *recall rate* of *teenagers* differed from that of *young adults* between 20 and 30 years of age.

The market research company interviewed 640 teenagers and 420 young adults countrywide. Three hundred and sixty-two teenagers recalled the Aids Awareness slogan used during the campaign, and 260 young adults were able to recall the same Aids Awareness slogan of "Aids: don't let it happen".

Test, at the 5% level of significance, the hypothesis that there is an *equal recall rate* between teenagers and young adults.

(b) An aircraft maintenance company bought equipment for detecting structural defects in aircrafts. Tests indicate that 95% of the time the equipment detects defects when they actually exist, and 1% of the time it gives a false alarm that indicates the presence of a structural defect when in fact there is none. If 2% of the aircrafts actually have structural defects, what is the probability that an aircraft actually has a structural defect given that the equipment indicates that it has a structural defect?

(c) If $\sigma_X = 1$, determine the number of observations required to ensure that at the 99% confidence level, $\bar{X} - 0.1 \leq \mu \leq \bar{X} + 0.1$ where

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

(d) A random sample of size 81 is taken from a population that has a mean of 24 and variance 324. Use the central limit theorem to determine the probability that the sample mean lies between 23.9 and 24.2.

## Question 3 [20 marks, 2+6+8+4]

(a) An air ambulance is based at a hospital. There is a probability of 0.005 that on any one day, there is a delay in the air ambulance leaving to deal with an emergency.

(i) Assuming that the probability of a delay is independent from day to day, specify a distribution that might be used to model the number of days on which a delay occurs during a period of 200 days.

(ii) Use a distributional approximation to find the probability that, during a period of 200 days, there are three or more days on which a delay occurs.

(b) A company wants to know, with a 95% level of confidence, if it can claim that the boxes of detergent that it sells contain more than 500 grams of detergent each. From past experience the company knows that the amount of detergent in the boxes is normally distributed with a standard deviation of 75 grams. A worker takes a random sample of 100 boxes and finds that the average amount of detergent in a box is 510 grams. Should the company make the claim?

(c) The probabilities that zero, one, two, or three workers will be injured in a factory during a month are 0.50, 0.30, 0.15, and 0.05 respectively. Find the mean and the variance of this distribution.

## Question 4 [20 marks, 2+6+8+4]

(a) Alpha Technologies, an electronics retail company in Manzini, has kept records of the number of ipods sold within a week of placing advertisements in the *Times of Swaziland.* The following table shows the *number of ipods sold*, and the corresponding *number of advertisements placed* in the *Times of Swaziland* for 12 randomly selected weeks over the past year.

| **Ads** | 4 | 4 | 3 | 2 | 5 | 2 | 4 | 3 | 5 | 5 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Sales** | 26 | 28 | 24 | 18 | 35 | 24 | 36 | 25 | 31 | 37 | 30 | 32 |

(i) Estimate the linear regression line ($\sum x = 44$, $\sum y = 346$, $\sum x^2 = 174$, $\sum xy = 1324$ and $\sum y^2 = 10336$).

(ii) Is the *relationship* between the number of *newspaper advertisements* placed and *ipod sales* meaningful? Use $\alpha = 0.05$.

(b) In 2001, the Supreme Court, by a vote of 8-0, struck down state laws that legalized marijuana for medicinal purposes. The Gallup Organization later conducted surveys of randomly selected

individuals (18+ years) and asked them whether they support the limited use of marijuana when prescribed by physicians to relive pain and suffering. The results of the survey by age group, are as follows:

| | Age | | |
|---|---|---|---|
| **Opinion** | **18-29** | **30-49** | **60+** |
| **For** | 172 | 313 | 258 |
| **Against** | 52 | 103 | 119 |

Is there evidence to indicate that the proportions of individuals in each age group who are for the legalization of marijuana for medicinal use if different at the $\alpha = 0.01$ level of significance.

(c) A box contains red and white balls in an unknown proportion. A random sample of 60 balls selected with replacement from the box showed that 70% were red. Find the 95% confidence limits for the actual proportion of red balls in the box.

## Question 5 [20 marks, 10+10]

(a) Following is a random sample of grades achieved on a statistics examination by nine male students in a large class and a random sample of grades achieved in the same examination by six female students:

| | |
|---|---|
| Male students | 79, 88, 64,91, 83, 66, 89, 74, 68 |
| Female students | 70, 51, 82, 72, 90, 61 |

Use the level of significance $\alpha = 0.01$ to test whether the difference between the means of these two samples is significant.

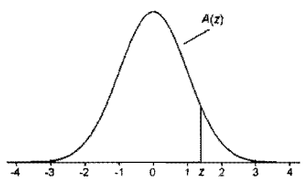
(b) An investigation is to be carried out into the effects of exercise on pulse rate. Part of the investigation will involve measuring the pulse rates of volunteers after they have spent three minutes stepping on and off a bench. Before proceeding, the investigator wishes to find out whether the height of the bench is a relevant factor. The pulse rates, in beats per minute, of 7 volunteers after they have stepped on and off benches of heights 30 cm and 40 cm are recorded in the table.

| | Height of bench | |
|---|---|---|
| **Volunteer** | **30cm** | **40 cm** |
| **Waheed** | 124 | 131 |
| **Sonny** | 118 | 126 |
| **Debbie** | 121 | 127 |
| **Marian** | 124 | 136 |
| **Dimitri** | 137 | 134 |
| **Sajid** | 129 | 138 |
| **Maha** | 142 | 141 |

Carry out a test to examine whether the height of the bench affects the mean pulse rate. Use the 10% significance level.

**TABLE A.1**

**Cumulative Standardized Normal Distribution**




$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

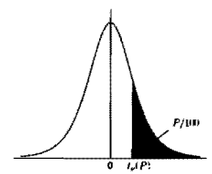
| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

# Percentage Points of the $t$-Distribution

This table gives the percentage points $t_\nu(P)$ for various values of $P$ and degrees of freedom $\nu$, as indicated by the figure to the right.

The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.

The limiting distribution of $t$ as $\nu \to \infty$ is the normal distribution with zero mean and unit variance.



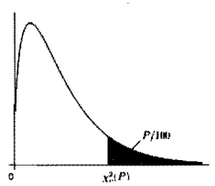

| | Percentage points $P$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $\nu$ | **10** | **5** | **2.5** | **1** | **0.5** | **0.1** | **0.05** |
| **1** | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **18** | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **21** | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **25** | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **30** | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **40** | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **50** | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **70** | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| **100** | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **∞** | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

## Percentage Points of the $\chi^2$-Distribution

This table gives the percentage points $\chi^2_\nu(P)$ for various values of $P$ and degrees of freedom $\nu$, as indicated by the figure to the right.

If $X$ is a variable distributed as $\chi^2$ with $\nu$ degrees of freedom, $P/100$ is the probability that $X \geq \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.




| | Percentage points $P$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $\nu$ | 10 | 5 | 2.5 | 1 | 0.5 | 0.1 | 0.05 |
| 1 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.828 | 12.116 |
| 2 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 13.816 | 15.202 |
| 3 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 | 16.266 | 17.730 |
| 4 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 | 18.467 | 19.997 |
| 5 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 | 22.105 |
| 6 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 | 24.103 |
| 7 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 | 26.018 |
| 8 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.124 | 27.868 |
| 9 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 | 29.666 |
| 10 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 | 31.420 |
| 11 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 31.264 | 33.137 |
| 12 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 32.909 | 34.821 |
| 13 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 34.528 | 36.478 |
| 14 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 36.123 | 38.109 |
| 15 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 37.697 | 39.719 |
| 16 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 | 39.252 | 41.308 |
| 17 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 40.790 | 42.879 |
| 18 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 42.312 | 44.434 |
| 19 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 43.820 | 45.973 |
| 20 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 45.315 | 47.498 |
| 25 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 | 52.620 | 54.947 |
| 30 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 59.703 | 62.162 |
| 40 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 73.402 | 76.095 |
| 50 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 86.661 | 89.561 |
| 80 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 | 124.839 | 128.261 |

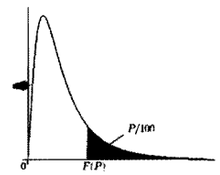


## 5 Percent Points of the $F$-Distribution

This table gives the percentage points $F_{\nu_1,\nu_2}(P)$ for $P = 0.05$ and degrees of freedom $\nu_1, \nu_2$, as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1,\nu_2}(P)$ such that the probability that $F \leq F'_{\nu_1,\nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1,\nu_2}(P) = 1/F_{\nu_1,\nu_2}(P)$$




| | $\nu_1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\nu_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 12 | 24 | $\infty$ |
| 2 | 18.513 | 19.000 | 19.164 | 19.247 | 19.296 | 19.330 | 19.413 | 19.454 | 19.496 |
| 3 | 10.128 | 9.552 | 9.277 | 9.117 | 9.013 | 8.941 | 8.745 | 8.639 | 8.526 |
| 4 | 7.709 | 6.944 | 6.591 | 6.388 | 6.256 | 6.163 | 5.912 | 5.774 | 5.628 |
| 5 | 6.608 | 5.786 | 5.409 | 5.192 | 5.050 | 4.950 | 4.678 | 4.527 | 4.365 |
| 6 | 5.987 | 5.143 | 4.757 | 4.534 | 4.387 | 4.284 | 4.000 | 3.841 | 3.669 |
| 7 | 5.591 | 4.737 | 4.347 | 4.120 | 3.972 | 3.866 | 3.575 | 3.410 | 3.230 |
| 8 | 5.318 | 4.459 | 4.066 | 3.838 | 3.687 | 3.581 | 3.284 | 3.115 | 2.928 |
| 9 | 5.117 | 4.256 | 3.863 | 3.633 | 3.482 | 3.374 | 3.073 | 2.900 | 2.707 |
| 10 | 4.965 | 4.103 | 3.708 | 3.478 | 3.326 | 3.217 | 2.913 | 2.737 | 2.538 |
| 11 | 4.844 | 3.982 | 3.587 | 3.357 | 3.204 | 3.095 | 2.788 | 2.609 | 2.404 |
| 12 | 4.747 | 3.885 | 3.490 | 3.259 | 3.106 | 2.996 | 2.687 | 2.505 | 2.296 |
| 13 | 4.667 | 3.806 | 3.411 | 3.179 | 3.025 | 2.915 | 2.604 | 2.420 | 2.206 |
| 14 | 4.600 | 3.739 | 3.344 | 3.112 | 2.958 | 2.848 | 2.534 | 2.349 | 2.131 |
| 15 | 4.543 | 3.682 | 3.287 | 3.056 | 2.901 | 2.790 | 2.475 | 2.288 | 2.066 |
| 16 | 4.494 | 3.634 | 3.239 | 3.007 | 2.852 | 2.741 | 2.425 | 2.235 | 2.010 |
| 17 | 4.451 | 3.592 | 3.197 | 2.965 | 2.810 | 2.699 | 2.381 | 2.190 | 1.960 |
| 18 | 4.414 | 3.555 | 3.160 | 2.928 | 2.773 | 2.661 | 2.342 | 2.150 | 1.917 |
| 19 | 4.381 | 3.522 | 3.127 | 2.895 | 2.740 | 2.628 | 2.308 | 2.114 | 1.878 |
| 20 | 4.351 | 3.493 | 3.098 | 2.866 | 2.711 | 2.599 | 2.278 | 2.082 | 1.843 |
| 25 | 4.242 | 3.385 | 2.991 | 2.759 | 2.603 | 2.490 | 2.165 | 1.964 | 1.711 |
| 30 | 4.171 | 3.316 | 2.922 | 2.690 | 2.534 | 2.421 | 2.092 | 1.887 | 1.622 |
| 40 | 4.085 | 3.232 | 2.839 | 2.606 | 2.449 | 2.336 | 2.003 | 1.793 | 1.509 |
| 50 | 4.034 | 3.183 | 2.790 | 2.557 | 2.400 | 2.286 | 1.952 | 1.737 | 1.438 |
| 100 | 3.936 | 3.087 | 2.696 | 2.463 | 2.305 | 2.191 | 1.850 | 1.627 | 1.283 |
| $\infty$ | 3.841 | 2.996 | 2.605 | 2.372 | 2.214 | 2.099 | 1.752 | 1.517 | 1.002 |