

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2011

TITLE OF PAPER : DESIGN AND ANALYSIS OF EXPERIMENTS
COURSE CODE : ST404
TIME ALLOWED : TWO (2) HOURS
REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS : ANSWER ANY THREE QUESTIONS

Question 1**[20 marks, 12+8]**

A pilot study was undertaken on the interaction effects of two drugs to stimulate growth in girls who are of short stature because of a particular syndrome. Each drug was known to be modestly effective singly, but the combination of the two drugs had never been investigated. Blocking by both subject and time period was desired whereby repeated measures for different treatments applied to the same subject are obtained. A 4 by 4 latin square design, shown below, was utilized for four subjects, four time periods, and four treatments. The four time periods consisted of one month each separated by an intervening month during which no treatment was given. The four treatments were *A*: no treatment (placebo); *B*: drug *X* alone; *C*: drug *Y* alone; *D*: both drug *X* and *Y*. The response was the difference in the growth rates (in centimetres per month) during the treatment period and the base period before the experiment began. The results of the study follow.

Subject (i)	Period (j)			
	k = 1	k = 2	k = 3	k = 4
i = 1	A = 0.02	B = 0.15	D = 0.45	C = 0.18
i = 2	B = 0.27	C = 0.24	A = -0.01	D = 0.58
i = 3	C = 0.11	D = 0.35	B = 0.14	A = -0.03
i = 4	D = 0.48	A = 0.04	C = 0.18	B = 0.22

- (a) Obtain the analysis of variance table. Interpret your results.

- (b) Estimate the interaction contrast

$$L = \left(\frac{\mu_{++2} + \mu_{++3}}{2} - \mu_{++1} \right) - \left(\mu_{++4} - \frac{\mu_{++2} + \mu_{++3}}{2} \right)$$

using a 90 percent confidence interval. Interpret your results.

Question 2**[20 marks, 8+4+8]**

- (a) A random sample of 10 students was selected from the senior class at each of four large high schools, and the score of each of these 40 students on certain mathematics examination was observed. Suppose that for the 10 students from each school, the sample mean and the sample variance of the scores are given in table 1. Test the hypothesis that the senior classes at all four high schools would perform equally well on this examination. Discuss carefully the assumptions that you are making to carry out this test.

Table 1: Sample parameters for the four schools

School	Sample mean	Sample variance
1	105.7	30.3
2	102.0	54.4
3	93.5	25.0
4	110.8	36.33

- (b) A consumer organization studies the effect of age of automobile owner on size of cash offer for a used car by utilizing 12 persons in each of three age groups (young, middle, elderly) who acted as

the owner of the used car. A medium price, six-year-old car was selected for the experiment, and the "owners" solicited cash offers for this car from 36 dealers selected at random from the dealers in the region. Randomization was used in assigning dealers to the "owners". The offer in (thousands of Emalangeni) follow.

Young	23, 25, 21, 22, 21, 22, 20, 23, 19, 22, 19, 21
Middle	28, 27, 27, 29, 26, 29, 27, 30, 28, 27, 26, 29
Elderly	23, 20, 25, 21, 22, 23, 21, 20, 19, 20, 22, 21

- (i) How would you check whether an ANOVA model is appropriate.
- (ii) Assuming an ANOVA is applicable conduct an F test for equality of factor level means.

Question 3

[20 marks, 2+6+6+6]

A researcher studied the effects of three experimental diets with varying fat contents on the total lipid (fat) level in the plasma. Total lipid is a widely used predictor of coronary heart disease. Fifteen males were grouped into five blocks according to age. Within each block, the three experimental diets were randomly assigned to the three subjects. Data on reduction in lipid level (in grams per litre) after the subjects were on a diet for a fixed period of time follow:

Block <i>i</i>	Fat Content of Diet		
	$j = 1$	$j = 2$	$j = 3$
	Extremely Low	Fairly Low	Moderately Low
Ages 15 – 24	0.73	0.67	0.15
Ages 24 – 34	0.86	0.75	0.21
Ages 35 – 44	0.94	0.81	0.26
Ages 45 – 54	1.40	1.32	0.75
Ages 55 – 64	1.62	1.41	0.78

- (a) Why do you think that age was used as a blocking variable?
- (b) Obtain the analysis of variance table.
- (c) Test whether or not blocking effects are present; use $\alpha = 0.05$. State the alternatives, decision rule, and conclusion.
- (d) Estimate the contrast

$$D = \mu_{+2} - \frac{\mu_{+1} + \mu_{+3}}{2}$$

using the Tukey procedure with a 95 percent family confidence coefficient. State your findings.

Question 4

[20 marks, 10+10]

- (a) **Bread crustiness.** The effects of baking temperature on the crustiness of bread are contained in Table 1. The data are scores from 1 to 20.

		Temperature			
Low		Medium		High	
Batch	1	Batch	2	Batch	1
4	12	14	9	14	16
7	8	13	10	17	19
5	10	11	12	15	18

Use an ANOVA to test for temperature effects; use $\alpha = 0.05$. State the alternatives, decision rule, and conclusion.

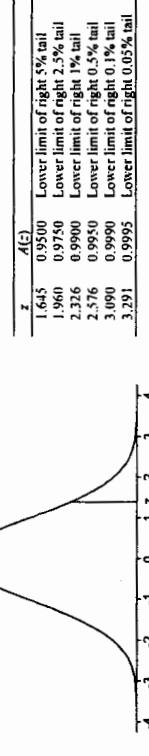
- (b) An experiment involving the case hardening of lightweight shafts machined from alloys bars of an alloy was run to study the effects of the amount of a chemical agent added to the alloy in a molten state (factor A), the temperature of the hardening process (factor B), and the time duration of the hardening process (factor C) on the outside hardness shaft. All factors were at 2 levels (1:low, 2:high), and the number of rods tested for each treatment was $n = 3$. The data on hardness (in Brinell units) follow.

		$k = 1$		$k = 2$	
		$j = 1$	$j = 2$	$j = 1$	$j = 2$
$i = 1$	39.9	53.5	56.0	70.9	
	32.3	50.7	56.9	73.7	
	36.3	52.8	56.6	71.6	
$i = 2$	45.2	63.3	69.4	82.9	
	48.0	65.5	66.6	85.2	
	47.5	63.6	68.8	82.3	

Test for three-factor interactions; use $\alpha = 0.05$. State the alternatives, decision rule, and conclusions.

TABLE A.1
Cumulative Standardized Normal Distribution

$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:



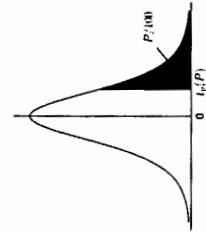
This table gives the percentage points $t_\nu(P)$ for various values of P and degrees of freedom ν , as indicated by the figure to the right.

The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.

The limiting distribution of t as $\nu \rightarrow \infty$ is the normal distribution with zero mean and unit variance.

Percentage Points of the t -Distribution

ν	Percentage points P						
	10	5	2.5	1	0.5	0.1	0.05
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.569
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.470	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.385	4.501	5.041
9	1.383	1.833	2.282	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.035	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
70	1.294	1.667	1.994	2.381	2.648	3.211	3.435
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291



Percentage points P

Lower percentage points are given

by symmetry as $-t_\nu(P)$,

and the probabil-

ity that $|t| \geq t_\nu(P)$ is $2P/100$.

The limiting distribution of t as $\nu \rightarrow \infty$

is the normal distribution with zero mean

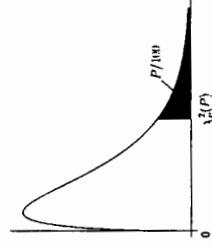
and unit variance.

Percentage Points of the χ^2 -Distribution

This table gives the percentage points $\chi_{\alpha}^2(P)$ for various values of P and degrees of freedom ν , as indicated by the figure to the right.

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geq \chi_{\alpha}^2(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



5 Percent Points of the F -Distribution

This table gives the percentage points $F_{\nu_1, \nu_2}(P)$ for $P = 0.05$ and degrees of freedom ν_1, ν_2 , as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1, \nu_2}(P)$ such that the probability that $F \leq F'_{\nu_1, \nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1, \nu_2}(P) = 1/F_{\nu_1, \nu_2}(P)$$



ν	Percentage points P					
	10	5	2.5	1	0.5	0.1
1	2.706	3.841	5.024	6.635	7.879	10.828
2	4.605	5.991	7.378	9.210	10.597	13.816
3	6.251	7.815	9.348	11.345	12.838	16.266
4	7.779	9.488	11.143	13.277	14.860	18.467
5	9.236	11.070	12.833	15.086	16.750	20.515
6	10.645	12.592	14.449	16.812	18.548	22.458
7	12.017	14.067	16.013	18.475	20.278	24.322
8	13.362	15.507	17.535	20.090	21.955	26.124
9	14.684	16.919	19.023	21.666	23.589	27.877
10	15.987	18.307	20.483	23.209	26.188	29.588
11	17.275	19.675	21.920	24.725	26.757	31.264
12	18.549	21.026	23.337	26.217	28.300	32.909
13	19.812	22.302	24.736	27.688	29.819	34.528
14	21.064	23.685	26.119	29.141	31.319	36.123
15	22.307	24.996	27.488	30.578	32.801	37.697
16	23.542	26.296	28.845	32.000	34.267	39.252
17	24.769	27.587	30.191	33.409	35.718	40.790
18	25.989	28.869	31.526	34.805	37.156	42.312
19	27.204	30.144	32.852	36.191	38.582	43.820
20	28.412	31.410	34.170	37.566	39.997	45.315
25	34.382	37.652	40.646	44.314	46.928	52.620
30	40.256	43.773	46.979	50.892	53.672	59.703
40	51.805	55.758	59.342	63.691	66.766	73.402
50	63.167	67.505	71.420	76.154	79.490	86.661
80	96.578	101.879	106.629	112.329	124.839	128.261

ν_2	ν_1						
	1	2	3	4	5	6	12
2	18.513	19.000	19.164	19.247	19.330	19.413	19.496
3	10.128	9.552	9.277	9.117	9.013	8.911	8.745
4	7.709	6.944	6.591	6.388	6.256	6.163	5.912
5	6.608	5.786	5.409	5.192	5.050	4.950	4.678
10	4.965	4.103	3.708	3.478	3.326	3.217	2.913
11	4.844	3.982	3.587	3.357	3.204	3.095	2.788
12	4.747	3.885	3.490	3.259	3.106	2.996	2.687
13	4.667	3.806	3.411	3.179	3.025	2.915	2.604
14	4.600	3.739	3.344	3.112	2.958	2.848	2.534
15	4.543	3.682	3.287	3.056	2.901	2.790	2.475
16	4.494	3.634	3.239	3.007	2.852	2.741	2.425
17	4.451	3.592	3.197	2.965	2.810	2.689	2.381
18	4.414	3.553	3.160	2.928	2.773	2.661	2.342
19	4.381	3.522	3.127	2.895	2.740	2.628	2.308
20	4.351	3.493	3.098	2.866	2.711	2.599	2.278
25	4.242	3.385	2.991	2.759	2.603	2.490	2.165
30	4.171	3.316	2.922	2.690	2.534	2.421	2.092
40	4.085	3.232	2.839	2.606	2.449	2.336	2.003
50	4.034	3.183	2.790	2.557	2.400	2.286	1.952
100	3.936	3.087	2.696	2.463	2.305	2.191	1.850
∞	3.841	2.996	2.605	2.372	2.214	2.099	1.752

10 Percent Points of the F -Distribution

This table gives the percentage points $F_{\nu_1, \nu_2}(P)$ for $P = 0.10$ and degrees of freedom ν_1, ν_2 , as indicated by the figure to the right.

The lower percentage points, that is the values $F_{\nu_1, \nu_2}^{(1)}(P)$ such that the probability that $F \leq F_{\nu_1, \nu_2}^{(1)}(P)$ is equal to $P/100$, may be found using the formula

$$F_{\nu_1, \nu_2}^{(1)}(P) = 1/F_{\nu_2, \nu_1}(P)$$

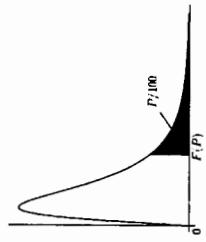


TABLE A.4 Studentized Range Statistic

		$K = \text{Number of Means or Number of Steps Between Ordered Means}$									
		Error									
		$(df \text{ within})$									
ν_2	ν_1	1	2	3	4	5	6	12	24	48	∞
2	8.526	9.000	9.162	9.243	9.293	9.326	9.408	9.450	9.491	9.531	—
3	5.538	5.462	5.391	5.343	5.309	5.285	5.216	5.176	5.134	5.097	—
4	4.545	4.325	4.191	4.107	4.051	4.010	3.896	3.831	3.761	3.696	—
5	4.060	3.780	3.619	3.520	3.453	3.405	3.268	3.191	3.105	3.035	—
6	3.776	3.463	3.289	3.181	3.108	3.055	2.905	2.818	2.722	2.635	—
7	3.589	3.257	3.074	2.961	2.883	2.827	2.668	2.575	2.471	2.375	—
8	3.458	3.113	2.924	2.806	2.726	2.668	2.502	2.404	2.293	2.185	—
9	3.360	3.006	2.813	2.693	2.611	2.551	2.379	2.277	2.159	2.034	—
10	3.285	2.924	2.728	2.605	2.522	2.461	2.284	2.178	2.056	1.919	—
11	3.226	2.860	2.660	2.536	2.451	2.389	2.209	2.100	1.972	1.836	—
12	3.177	2.807	2.606	2.480	2.394	2.331	2.147	2.036	1.904	1.765	—
13	3.136	2.763	2.560	2.434	2.347	2.283	2.097	1.983	1.846	1.696	—
14	3.102	2.726	2.522	2.395	2.307	2.243	2.054	1.938	1.797	1.619	—
15	3.073	2.695	2.490	2.361	2.273	2.208	2.017	1.889	1.755	1.574	—
16	3.048	2.668	2.462	2.353	2.244	2.178	1.985	1.866	1.718	1.518	—
17	3.026	2.645	2.437	2.308	2.218	2.152	1.958	1.836	1.686	1.456	—
18	3.007	2.624	2.416	2.286	2.196	2.130	1.933	1.810	1.657	1.424	—
19	2.990	2.606	2.397	2.266	2.176	2.109	1.912	1.787	1.631	1.395	—
20	2.975	2.580	2.380	2.240	2.158	2.091	1.892	1.767	1.607	1.351	—
25	2.918	2.528	2.317	2.184	2.092	2.024	1.820	1.689	1.518	1.217	—
30	2.881	2.489	2.276	2.142	2.049	1.980	1.773	1.638	1.456	1.124	—
40	2.835	2.440	2.226	2.091	1.997	1.927	1.715	1.574	1.377	1.024	—
50	2.809	2.412	2.197	2.061	1.966	1.895	1.680	1.536	1.327	0.982	—
100	2.756	2.356	2.139	2.002	1.906	1.834	1.612	1.460	1.214	0.856	—
∞	2.706	2.303	2.084	1.945	1.847	1.774	1.546	1.383	1.002	0.696	—