

UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2010/11

TITLE OF PAPER: QUANTITATIVE METHODS IN DEMOGRAPHY

COURSE CODE: DEM 206

TIME ALLOWED: TWO (2) HOURS

INSTRUCTIONS: ANSWER ANY THREE (3) QUESTIONS

SPECIAL REQUIREMENTS: STATISTICAL TABLES

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THE INVIGILATOR**

Question 1

One of the local daily newspapers carried an article on the number of women that would prefer to give birth in the three main hospitals of the country.

Hospital	Work status	
	Domestic Worker	House Wife
Hlatikulu Government Hosp.	110	97
Mbabane Government Hosp.	163	145
Raleigh Fitkim Memorial Hosp.	178	152

- a) Find the probability that a woman prefers Mbabane Government hospital given that she is a house wife. (3)

It is known that 50% of babies are born in the three main hospitals of the country, 20% in private clinics and 30% in government health centres and clinics. A further look shows that 6% of the parents who give birth in the three main hospitals make use of the child registration centres to make birth certificates. The corresponding percentage for private clinics is 8%. The government health centres and clinics have 15% using the registration centres for birth certificates.

- b) (i) What percentage of all born babies are registered at registration centres?
(ii) Given that a child was registered where he/she was born, what is the probability that he/she was born at a government health centre or government clinic? (7)

In the biggest company in Swaziland, 60% of the workers are skilled and 80% of the workers are full-time. Ninety percent of the skilled workers are full-time.

- c) What is the probability that an employee selected at random is a skilled worker or a full-time worker? (3)

During the exam study break, a lecturer attends to 7 students, on average per day, who come to ask for academic assistance .

- d) (i) What is the probability that on a given day, no more than 5 students will come to the lecturer for assistance?
(ii) What is the probability that 6 or 9 students will come to the lecturer on any given day? (7)

Question 2

The Chief Immigration Officer has to make a report to parliament on the number of permit applications from citizens of different countries on any particular day. He collects data using passports of countries he can see on his desk and groups them according to continents from which the applicant comes from.

Africa:	14	16	14	9	11	15
Europe:	17	13	14	13	16	12
Asia:	12	15	10	19	11	
Americas:	15	12	13	15		

- a) Conduct a test at 5% significance level to ascertain whether there is a statistical difference in the number of applicants from the different continents (10).

As an independent consultant you want to find out if union interest is the same across the four regions of Swaziland. The observed frequencies are presented in the table below.

	Hhohho	Lubombo	Manzini	Shiselweni
Interested	120	41	45	112
Not Interested	35	38	40	36
Indifferent	45	21	15	52

- b) Use a 95% confidence level to answer the question on whether union interest is the same for all four regions.

Question 3

Hotel occupancy levels are an important indicator of a healthy tourism industry. A large city in South Africa used recorded data for 30 time periods. A tourism student used this data to identify possible predictors of *hotel occupancy levels*. The student used three measures:

- Number of people arriving through the city's airport (in 10 000's).
- Number of conferences taking place in the city.
- Money spent on advertising (in millions of Rands).

The data for the first two time of the 30 periods follows below.

City Hotel Occupancy Rates				
Period	Visitors (in 10000's)	No. Of Conferences	Advertising Money (Millions of Rands)	Occupancy
1	35	10	4.6	65
2	40	6	3.1	78

The student carried out a regression analysis using SPSS and the output is shown in the Appendix (Regression Output).

- a) Write out an equation for the multiple regression model. (2)
- b) Why is there a difference between R-squared and Adjusted R-squared, which one would you report (between R-squared and Adjusted R-squared) and what does it mean? (4)
- c) Comment on the correlation of the *Occupancy* variable with each one of the other variables. (3)
- d) Formulate hypotheses for the correlation (your answer should be both in words and symbol form). (2)
- e) Define multicollinearity, and comment on it in relation to this regression analysis. (4)
- f) State whether each of the three independent variables is significant in determining *Occupancy*. Explain your answer using a 0.05 significance level. (2)
- g) Use the multiple regression model to estimate the average occupancy for period 2. (2)
- h) What can you say about the predictive ability of the overall multiple regression model? (1)

Question 4

- a) Calculate the four quarterly moving average trend of the following time series (of births in a large hospital). (6)

Year	Quarter			
	1	2	3	4
2007	5100	4600	3600	5600
2008	3600	3100	2100	3600
2009	2600	2100	1600	2600

- b) Is any significant trend evident from the data? State your answer with reasons. (1)
- c) Calculate the de-seasonalized values for each quarter. Comment on the seasonal influences per quarter. (10)
- d) How does the seasonal variation of a time series differ from the cycle? (2)
- e) Why is the error not captured in a statistical model? (1)

Question 5

- a)
- i. What is meant by statistical inference? In your answer include its function and importance? (2)
 - ii. Define the terms parameter and statistic. State the relationship between the two. (2)
 - iii. What is meant by estimation? (1)
 - iv. What is meant by hypothesis testing, and what is its function? (2)
 - v. What is meant by the 95% confidence interval? (1)

b)

You have been offered an internship by the Swaziland Central Statistical Office (CSO).

You are told that the office has been tasked with the responsibility of conducting a survey that will investigate sexual behavior amongst school going pupils in Swaziland. Part of your job is to help the CSO sample at least 10000 pupils to be part of the study.

List and describe the probability based sampling methods you would use. You should also include what the sampling units for this survey would be and how you would choose these sampling units. (12)

END OF EXAMINATION QUESTIONS

Model Summary

Model	R	Adjusted R : Square	Std. Error of the Estimate	Change Statistics				
				R Square Change	F Change	df1	df2	Sig. F Change
1	.749 ^a	.561	5.92075	.561	11.061	3	26	.000

a. Predictors: (Constant), Airport_No, Conferences, Advertising

ANOVA^b

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	1163.229	3	387.743	11.061	.000 ^a
Residual	911.438	26	35.055		
Total	2074.667	29			

a. Predictors: (Constant), Airport_No, Conferences, Advertising

b. Dependent Variable: Occupancy

Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Collinearity Statistics	
	B	Std. Error	Beta			Lower Bound	Upper Bound	Tolerance	VIF
1	(Constant)	33.877	8.316	4.074	.000	16.784	50.971		
	Conferences	1.587	.370	4.289	.000	.826	2.347	.912	1.096
	Advertising	-2.646	1.334	-1.984	.058	-5.388	.096	.462	2.165
	Airport No	.898	.290	3.097	.005	.302	1.494	.468	2.137

a. Dependent Variable: Occupancy

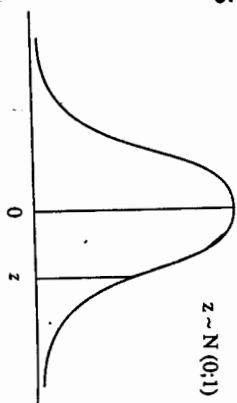
Correlations

	Occupancy	Conferences	Advertising	Airport_No
Pearson Correlation	Occupancy	.630	.215	.466
	Conferences	1.000	.284	.263
	Advertising	.215	1.000	.727
	Airport_No	.466	.727	1.000
Sig. (1-tailed)	Occupancy	.000	.127	.005
	Conferences	.000	.064	.080
	Advertising	.127	.064	.000
	Airport_No	.005	.000	.
N	Occupancy	30	30	30
	Conferences	30	30	30
	Advertising	30	30	30
	Airport_No	30	30	30

APPENDIX 1 - LIST OF STATISTICAL TABLES

TABLE 1
The standard normal distribution (z)

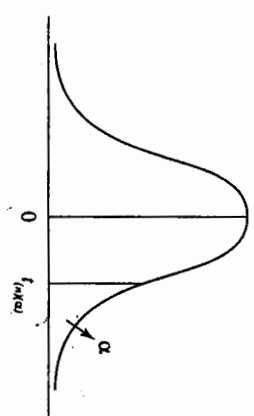
This table gives the area under the standard normal curve between 0 and z, i.e. $P[0 < Z < z]$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2703	0.2734	0.2764	0.2793	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4517	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49991	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997

TABLE 2
The t distribution

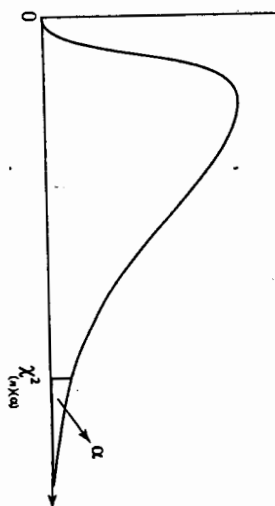
This table gives the value of $t_{(n)}(\alpha)$ where n is the degrees of freedom i.e. $\alpha = P[t \geq t_{(n)}(\alpha)]$



α df	0.100	0.050	0.025	0.010	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	2.308	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.387	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.377	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.350	1.782	2.179	2.681	3.055	3.428
13	1.340	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.369	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
110	1.289	1.659	1.982	2.361	2.621	2.865
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.847
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
1.285	1.652	1.971	2.344	2.600	2.600	2.838

TABLE 3
The Chi-square distribution (χ^2)

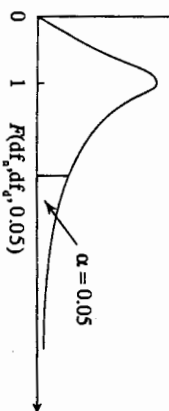
This table gives the value of $\chi^2_{(n)(\alpha)}$ where n is the degrees of freedom i.e. $\chi^2_{(n)(\alpha)} = P[\chi^2 > \chi^2_{(n)(\alpha)}]$



α	0.100	0.050	0.025	0.01	0.005	0.0025
1	2.707	3.843	5.026	6.637	7.881	9.142
2	4.605	5.991	7.378	9.210	10.597	11.983
3	6.251	7.879	9.348	11.345	12.838	14.164
4	7.779	9.488	11.143	13.277	14.860	16.421
5	9.236	11.071	12.833	15.086	16.750	18.366
6	10.645	12.592	14.449	16.812	18.548	20.249
7	12.017	14.067	16.013	18.475	20.278	22.040
8	13.362	15.507	17.535	20.090	21.955	23.774
9	14.684	16.919	19.023	21.666	23.589	25.462
10	15.987	18.307	20.483	23.029	25.188	27.112
11	17.275	19.675	21.920	24.725	26.757	28.779
12	18.549	21.026	23.337	26.217	28.300	30.318
13	19.812	22.362	24.736	27.688	29.819	31.883
14	21.064	23.685	26.119	29.141	31.319	33.428
15	22.307	24.996	27.488	30.578	32.801	34.950
16	23.542	26.296	28.845	32.000	34.267	36.456
17	24.769	27.587	30.191	33.409	35.718	37.946
18	25.989	28.869	31.526	34.805	37.156	39.422
19	27.204	30.144	32.852	36.191	38.582	40.885
20	28.412	31.410	34.170	37.566	39.997	42.336
21	29.615	32.671	35.479	38.932	41.401	43.775
22	30.813	33.924	36.781	40.289	42.796	45.204
23	32.007	35.172	38.076	41.638	44.181	46.623
24	33.196	36.415	39.364	42.980	45.558	48.034
25	34.382	37.652	40.646	44.314	46.928	49.435
26	35.563	38.885	41.923	45.642	48.290	50.829
27	36.741	40.113	43.195	46.963	49.645	52.215
28	37.916	41.337	44.461	48.278	50.993	53.594
29	39.087	42.557	45.722	49.588	52.336	54.967
30	40.256	43.773	46.979	50.892	53.672	56.332
31	41.422	44.985	48.232	52.191	55.003	57.692
32	42.585	46.194	49.480	53.486	56.328	59.046
33	43.745	47.400	50.725	54.776	57.648	60.395
34	44.903	48.602	51.966	56.061	58.964	61.738
35	46.059	49.802	53.203	57.342	60.275	63.076
36	47.212	50.998	54.437	58.619	61.581	64.410
37	48.363	52.182	55.668	59.892	62.883	65.739
38	49.513	53.384	56.896	61.162	64.181	67.063
39	50.660	54.572	58.120	62.428	65.476	68.383
40	51.805	55.758	59.342	63.691	66.766	69.699
45	57.505	61.656	65.410	69.957	73.166	76.233
50	63.167	67.505	71.420	76.154	79.490	82.367
60	74.399	79.087	83.305	88.386	91.957	95.357
70	85.529	90.537	95.031	100.432	104.212	107.812
80	96.581	101.885	106.656	112.336	116.329	120.107
90	107.568	113.151	118.144	124.125	128.307	132.262
100	118.501	124.348	129.570	135.815	140.178	144.300
110	129.568	135.487	140.925	147.423	151.958	156.238
120	140.671	146.571	152.222	158.923	163.678	167.925
140	168.618	174.659	180.174	186.875	191.604	196.099
160	190.522	196.926	202.766	209.852	214.845	219.588
180	212.310	219.056	225.200	232.647	237.890	242.866

TABLE 4(a)
F distribution ($\alpha = 0.05$)

The entries in this table are critical values of F for which the area under the curve to the right is equal to 0.05.



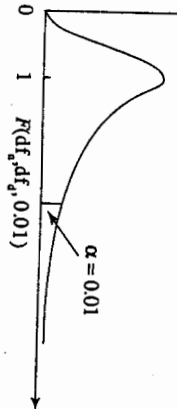
		Degrees of Freedom for Numerator									
		1	2	3	4	5	6	7	8	9	10
Degrees of Freedom for Denominator	1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9
	2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
	3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
	17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
	30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
	120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
	∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

TABLE 4(a) (continued)
F distribution ($\alpha = 0.05$)

Degrees of Freedom for Numerator											Degrees of Freedom for Denominator										
	12	15	20	24	30	40	60	120	∞		12	15	20	24	30	40	60	120	∞		
1	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3	2	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5		
2	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53		
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63		
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37		
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37	6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67		
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23		
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93		
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71		
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54		
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40		
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30		
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21		
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13		
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07		
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01		
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96		
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92		
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88		
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84		
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81		
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78		
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76		
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73		
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71		
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62		
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51		
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39		
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25		
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00		

TABLE 4(b)
F distribution ($\alpha = 0.01$)

The entries in the table are critical values of F for which the area under the curve to the right is equal to 0.01.



Degrees of Freedom for Denominator																				
Degrees of Freedom for Numerator																				
1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	1	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	2	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	3	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	4	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	5	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	6									
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	7									
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	8									
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	9									
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	10									
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	11									
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	12									
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	13									
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	14									
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	15									
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	16									
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	17									
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	18									
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	19									
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	20									
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	21									
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	22									
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	23									
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	24									
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	25									
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	30									
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	40									
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	60									
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	120									
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	∞									

TABLE 4(b) (continued)
F distribution ($\alpha = 0.01$)

[illegible]