UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2010

TITLE OF PAPER : SAMPLE SURVEY THEORY

COURSE CODE : ST 306

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : I .CALCULATOR AND STATISTICAL TABLES
II. FORMULA SHEET IS ATTACHED

INSTRUCTIONS : ANSWER ANY THREE QUESTIONS

Question 1

National income from manufacturing industries is to be estimated for 2010 from a sample of 6 of the 19 industrial categories that reported figures early for that year. Incomes from all 19 industries are known for 1995 and the total is E674 Million.

	EMillion	
Industry	1980	1989
Lumber and wood products	21	26
Electric and electronic equipment	63	91
Motor vehicles and equipment	35	47
Food and kindred products	60	70
Textile mill products	16	17
Chemical and allied products	50	76

(a) Estimate *total* national income from manufacturing in 2010 using the following:

i. a simple random sample estimator,
ii. a ratio estimator,
iii. a regression estimator

(6 Marks)

(b) Which of the three methods (i), (ii) or (iii) above do you consider to be most appropriate in this case? Why?

(5 Marks)

(c) Estimate and compare the relative efficiencies of your estimators. Do the results support your answer to part (b) above?

(9 Marks)

Question 2

In 2009, a large organisation surveyed its staff in three different divisions as part of a "Work/Life Balance" initiative.

Among the questions asked was "How many hours did you spend at work last week?" The results are shown below.

Division h	*Number in division* N_h	*Sample size* n_h	*Mean number of hours worked* $\bar{y}_h$	*Standard error of mean number of hours worked* $SE(\bar{y}_h)$
Information Management	300	50	41	1.5
Health Statistics	400	50	40	2.0
Internal Communications	100	20	46	2.5
ALL	800	120		

(a) Construct a 95% confidence interval for the mean number of hours worked in the Internal Communications Division.

(2 Marks)

(b) The organisation has a target that the mean working week should be 40 hours. Based upon your confidence interval, is this target being met in that Division?

(2 Marks)

(c) Construct a 95% confidence interval for the mean number of hours worked by all employees.

(5 Marks)

(d) Based upon your confidence interval, is the organisation's target of a 40-hour mean working week being met for all employees?

(1 Marks)

(e) Comment on the results found in parts (a) and (b).

(1 Mark)

(f) Identify two key purposes of stratification. How do these apply in this survey?

(3 Marks)

(g) The survey will be repeated in 2010, and work has started on the sampling methodology. The total sample size is to be 120. The researcher responsible has heard the terms proportional allocation and optimal allocation. Explain what these terms mean, and find the stratum sample sizes each method of allocation would give using the 2009 data.

(6 Marks)

Question 3

(a) The variance of a stratified random sample mean can be written as

$$\mathrm{Var}(\bar{y}_{st}) = \sum_{i=1}^{k} \frac{N_i^2}{N^2}\left(\frac{1}{n_i} - \frac{1}{N_i}\right) S_i^2 .$$

Explain the (conventional) notations used here.

Suppose the total cost of sampling is

$$C = c_0 + \sum_{i=1}^{k} c_i n_i ,$$

where $c_0, c_1, \ldots, c_k$ are positive constants.

If $\mathrm{Var}(\bar{y}_{st})$ is fixed and the stratum sample sizes are chosen to minimise the total cost of sampling, show that the ith stratum sample size n_i is proportional to

$$\frac{N_i S_i / \sqrt{c_i}}{\sum_{i=1}^{k} \left(N_i S_i / \sqrt{c_i}\right)}.$$

(8 Marks)

(b) In one region there are 227 orchards located in three different districts of the region. The number of orchards, an estimate of the standard deviation of the areas of the orchards (in hectares), and the sampling cost per orchard are given below.

District	*Total number of orchards*	*Estimated standard deviation (ha)*	*Sampling cost per orchard*
1	68	34	10
2	143	20	14
3	16	59	18

Find the stratum sample sizes required to estimate the total area of orchards in the region to within 300 ha, given that the total cost of sampling is to be minimised and also that we are prepared to accept a one in twenty chance that the estimate will be more than 300 ha from the true total area.

Find the total cost of sampling in this case, if the overhead cost is 200 units.

(12 Marks)

Question 4

A political scientist has developed a test designed to measure the degree of awareness of current events. She wants to estimate the average score per person that would be achieved on this test by all students in a certain school. She selects 10 classes at random from a total of 108 classes and gives the test to each member of the sampled classes, with the following results.

Class	*Number of students*	*Total score*
1	29	1510
2	38	1990
3	22	1080
4	30	1620
5	18	710
6	40	1980
7	22	1310
8	19	860
9	31	1590
10	21	1140
Total	270	13790

(a) Explain why the above sample is a cluster sample and what its advantages are in this context.

(3 Marks)

(b) Explain why the average score per student based on the above sample is a ratio estimator.

(2 Marks)

(c) Variates y_i and m_i are measured on each unit of a simple random sample of size n, assumed large.

Show that the variance of $r = \frac{\bar{y}}{\bar{m}}$ is approximately

$$\frac{1-f}{n\bar{M}^2}\sum_{i=1}^{N}\frac{(y_i - Rm_i)^2}{N-1}$$

where $R = \bar{Y}/\bar{M}$ is the ratio of the population means and $f = n/N$.

(8 Marks)

(d) Estimate the average score per student that would be achieved in this test by all students in the school, and give a 95% confidence interval for this average.

(7 Marks)

FORMULA SHEET

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i}.$$

$$\text{var}(\hat{\tau}_{hh}) = \frac{1}{n}\sum_{i=1}^{N}p_i\left(\frac{y_i}{p_i}-\tau\right)^2$$

$$\widehat{\text{var}}(\hat{\tau}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}p_i\left(\frac{y_i}{p_i}-\hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{1}{Nn}\sum_{i=1}^{n}\frac{y_i}{p_i}$$

$$\widehat{\text{var}}(\hat{\mu}_{hh}) = \frac{1}{N^2 n(n-1)}\sum_{i=1}^{n}p_i\left(\frac{y_i}{p_i}-\hat{\tau}_{hh}\right)^2$$

$$\hat{\tau}_{hh} \pm z^*\sqrt{\text{var}(\hat{\tau}_{hh})}$$

$$\hat{\mu}_{hh} \pm z^*\sqrt{\text{var}(\hat{\mu}_{hh})}$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\hat{\mu}_{ht} = \frac{1}{N}\sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\text{var}(\hat{\tau}_{ht}) = \sum_{i=1}^{N}\left(\frac{1}{\pi_i}-1\right)y_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\pi_{ij}}{\pi_i\pi_j}-1\right)y_iy_j$$

$$\widehat{\text{var}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2}-\frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j}-\frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\widehat{\text{var}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\widehat{\text{var}}(\hat{\tau}_{ht})$$

$$\widehat{\text{var}}_{syg}(\hat{\tau}_{ht}) = \sum\sum_{j>i}^{\nu}\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\left(\frac{\pi_i\pi_j}{\pi_{ij}}-1\right)$$

$$\widehat{\text{var}}(\hat{\tau}_{ht}) = \frac{N-\nu}{N}\frac{s_t^2}{\nu}$$

$$s_t^2 = \frac{1}{(\nu-1)}\sum_{i=1}^{\nu}(t_i-\hat{\tau}_{ht})^2$$

$$t_i = \frac{\nu y_i}{\pi_i}$$

$$\hat{\mu}_{ht} \pm z^*\sqrt{\widehat{var}(\hat{\mu}_{ht})}$$

$$\hat{\tau}_{ht} \pm z^*\sqrt{\widehat{var}(\hat{\tau}_{ht})}$$

$$\hat{\mu}_{ht} \pm t^*\sqrt{\widehat{var}(\hat{\mu}_{ht})}$$

$$\hat{\tau}_{ht} \pm t^*\sqrt{\widehat{var}(\hat{\tau}_{ht})}$$

$$\tau = \sum_{i=1}^{N}y_i$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N}y_i = \tau/N$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\mu)^2$$

$$\sigma^2 = \left(\frac{1}{N-1}\right)\left(\sum_{i=1}^{N}y_i^2-\frac{\tau^2}{N}\right) = \left(\frac{1}{N-1}\right)\left(\sum_{i=1}^{N}y_i^2-N\mu^2\right)$$

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\overline{y})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}y_i^2-\frac{(\sum_{i=1}^{n}y_i)^2}{n}\right)$$

$$\hat{\mu} = \overline{y} = \frac{1}{n}\sum_{i=1}^{n}y_i$$

$$\hat{\tau} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\overline{y}.$$

$$\text{var}(\hat{\mu}) = \text{var}(\overline{y}) = \left(\frac{N-n}{N}\right)\frac{\sigma^2}{n}$$

$$\text{var}(\hat{\tau}) = N(N-n)\frac{\sigma^2}{n}$$

$$\widehat{\text{var}}(\hat{\mu}) = \widehat{\text{var}}(\overline{y}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\widehat{\text{var}}(\hat{\tau}) = N(N-n)\frac{s^2}{n}$$

$$\overline{y} \pm z^*\frac{s}{\sqrt{n}}$$

$$\overline{y} \pm t^*\frac{s}{\sqrt{n}}$$

$$\overline{y} \pm z^*\sqrt{\left(\frac{N-n}{N}\right)\frac{s^2}{n}}$$

$$N\overline{y} \pm z^*\sqrt{N(N-n)\frac{s^2}{n}}$$

$$\overline{y} \pm t^*\sqrt{\left(\frac{N-n}{N}\right)\frac{s^2}{n}}$$

$$N\overline{y} \pm t^*\sqrt{N(N-n)\frac{s^2}{n}}$$

$$n = \frac{1}{\frac{d^2}{z^2\sigma^2} + \frac{1}{N}} = \frac{1}{\frac{1}{n_0} + \frac{1}{N}}$$

$$n_0 = \frac{z^2\sigma^2}{d^2}$$

$$n = \frac{1}{\frac{d^2}{N^2z^2\sigma^2} + \frac{1}{N}} = \frac{1}{\frac{1}{n_0} + \frac{1}{N}}$$

$$n_0 = \frac{N^2z^2\sigma^2}{d^2}$$

$$\hat{p} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - p)^2 = \left(\frac{N}{N-1}\right)p(1-p)$$

$$\text{var}(\hat{p}) = \left(\frac{N-n}{N}\right)\frac{\sigma^2}{n}$$

$$\widehat{\text{var}}(\hat{p}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n} = \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}$$

$$\hat{p} \pm z^*\sqrt{\widehat{\text{var}}(\hat{p})}$$

$$n = \frac{Np(1-p}{(N-1)\frac{d^2}{z^2} + p(1-p)} = \frac{1}{\frac{N-1}{Nn_0} + \frac{1}{N}} \approx \frac{1}{\frac{1}{n_0} + \frac{1}{N}}$$

$$n_0 = \frac{z^2p(1-p)}{d^2}$$

$$R = \frac{\mu_y}{\mu_x} = \frac{\tau_y}{\tau_x}$$

$$\mu_x = \left(\sum_{i=1}^{N} x_i\right)/N \qquad \tau_x = \sum_{i=1}^{N} x_i \qquad \mu_y = \left(\sum_{i=1}^{N} y_i\right)/N \qquad \tau_y = \sum_{i=1}^{N} y_i$$

$$\overline{x} = \sum_{i=1}^{n} x_i/n$$

$$\overline{y} = \sum_{i=1}^{n} y_i/n$$

$$r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} = \frac{\overline{y}}{\overline{x}}$$

$$\widehat{\text{Var}}(r) = \left(\frac{N-n}{N\mu_x^2}\right)\frac{s_r^2}{n} = \left(\frac{N(N-n)}{\tau_x^2}\right)\frac{s_r^2}{n}$$

$$s_r^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - rx_i)^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 + r^2\sum_{i=1}^{n} x_i^2 - 2r\sum_{i=1}^{n} x_i y_i\right)$$

$$\hat{\mu}_r = r\mu_x$$

$$\widehat{\text{Var}}(\hat{\mu}_r) = \left(\frac{N-n}{N}\right)\frac{s_r^2}{n}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x$$

$$\widehat{\text{Var}}(\hat{\tau}_r) = N(N-n)\frac{s_r^2}{n}$$

$$\widehat{\text{Var}}(\hat{\mu}_r) = \left(\frac{\mu_x^2}{\overline{x}^2}\right)\widehat{\text{Var}}(\hat{\mu}_r)$$

$$\widehat{\text{Var}}(\hat{\tau}_r) = \left(\frac{\mu_x^2}{\overline{x}^2}\right)\widehat{\text{Var}}(\hat{\tau}_r)$$

$r \pm z^* \sqrt{\widehat{\text{Var}}(r)}$ $\qquad$ $\hat{\mu}_r \pm z^* \sqrt{\widehat{\text{Var}}(\hat{\mu}_r)}$ $\qquad$ $\hat{\tau}_r \pm z^* \sqrt{\widehat{\text{Var}}(\hat{\tau}_r)}$

$r \pm t^* \sqrt{\widehat{\text{Var}}(r)}$ $\qquad$ $\hat{\mu}_r \pm t^* \sqrt{\widehat{\text{Var}}(\hat{\mu}_r)}$ $\qquad$ $\hat{\tau}_r \pm t^* \sqrt{\widehat{\text{Var}}(\hat{\tau}_r)}$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_y \sigma_x}$$

$$\sigma_{xy} = \frac{\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)}{N-1}$$

$$\text{Var}(\hat{\tau}_r) = \frac{N(N-n)}{n}\left(\sigma_y^2 - 2R\rho_{xy}\sigma_y\sigma_x + R^2\sigma_x^2\right)$$

$$\text{Var}(\hat{\mu}_r) = \frac{N-n}{Nn}\left(\sigma_y^2 - 2R\rho_{xy}\sigma_y\sigma_x + R^2\sigma_x^2\right)$$

$$\hat{C}_x = \frac{s_x}{\overline{x}} \qquad \hat{C}_y = \frac{s_y}{\overline{y}}$$

$$\hat{\rho} = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \overline{x}}{s_x}\right)\left(\frac{y_i - \overline{y}}{s_y}\right) = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{(n-1)s_x s_y}$$

$$a = \overline{y} - b\overline{x} \qquad b = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} = \frac{n\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}$$

$$\hat{\mu}_L = a + b\mu_x = (\overline{y} - b\overline{x}) + b\mu_x = \overline{y} + b(\mu_x - \overline{x})$$

$$\widehat{\text{var}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\widehat{\text{var}}(\hat{\mu}_L) \approx \frac{1}{n(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\widehat{\text{var}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}\left[\sum_{i=1}^{n} y_i^2 - n\overline{y}^2 - b^2\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right)\right]$$

$$\hat{\tau}_L = N\hat{\mu}_L = N(a + b\mu_x) = N\overline{y} + b(\tau_x - N\overline{x})$$

$$\widehat{\text{var}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\widehat{\text{var}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}SSE = \frac{N-n}{Nn}MSE$$

$$\widehat{\text{var}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-2)}SSE = \frac{N(N-n)}{n}MSE$$

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i$$

$$\hat{\mu}_L = b_0 + \sum_{i=1}^{k} b_i \mu_i$$

$$\hat{y} = b_0 + b_1 x + b_2 x^2 + \cdots + b_k x^k$$

$$\hat{\mu}_L = b_0 + b_1\mu_x + b_2\mu_x^2 + \cdots + b_k\mu_x^k$$

$$\widehat{\text{var}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-k-1)}SSE = \frac{N-n}{Nn}MSE$$

$$z_{\alpha/2}\sqrt{\widehat{\text{Var}}(r)} = z_{\alpha/2}\sqrt{\left(\frac{N-n}{N\mu_x^2}\right)\frac{\sigma_r^2}{n}}$$

$$z_{\alpha/2}\sqrt{\widehat{\text{Var}}(\hat{\mu}_r)} = z_{\alpha/2}\sqrt{\left(\frac{N-n}{N}\right)\frac{\sigma_r^2}{n}}$$

$$z_{\alpha/2}\sqrt{\widehat{\text{Var}}(\hat{\tau}_r)} = z_{\alpha/2}\sqrt{N(N-n)\frac{\sigma_r^2}{n}}$$

For R : $n = \dfrac{1}{\frac{1}{n_0} + \frac{1}{N}}$ where $n_0 = \dfrac{z_{\alpha/2}^2\hat{\sigma}_r^2}{d^2\mu_x^2}$

For μ_y : $n = \dfrac{1}{\frac{1}{n_0} + \frac{1}{N}}$ where $n_0 = \dfrac{z^2_{\alpha/2}\,\widehat{\sigma}^2_r}{d^2}$

For τ_y : $n = \dfrac{1}{\frac{1}{n_0} + \frac{1}{N}}$ where $n_0 = \dfrac{N^2 z^2_{\alpha/2}\,\widehat{\sigma}^2_r}{d^2}$

For μ_y : $n = \dfrac{1}{\frac{1}{n_0} + \frac{1}{N}}$ where $n_0 = \dfrac{z^2_{\alpha/2}\,\widehat{\sigma}^2_{regr}}{d^2}$

For τ_y : $n = \dfrac{1}{\frac{1}{n_0} + \frac{1}{N}}$ where $n_0 = \dfrac{N^2 z^2_{\alpha/2}\,\widehat{\sigma}^2_{regr}}{d^2}$

$\tau_{y(d)} = \sum_{i \in U_d} y_i \qquad \mu_{y(d)} = \tau_{y(d)}/N_d = \left(\sum_{i \in U_d} y_i\right)/N_d$

$\widehat{\mu}_{y(d)} = \left(\sum_{i \in S_d} y_i\right)/n_d = \overline{y}_d \qquad \widehat{\tau}_{y(d)} = \dfrac{N_d}{n_d}\left(\sum_{i \in S_d} y_i\right) = N_d\overline{y}_d$

$$\widehat{\text{var}}(\overline{y}_d) = \widehat{\text{var}}(r_d) = \left(\frac{N-n}{N p^2_{s(d)}}\right)\frac{s_r^2}{n} = \left(\frac{N-n}{N(N_d/N)^2}\right)\frac{s_r^2}{n} = \left(\frac{N-n}{N}\right)\left(\frac{N}{N_d}\right)^2\frac{s_r^2}{n}$$

$$s_r^2 = \frac{1}{n-1}\left(\sum_{i \in S_d} y_i^2 + n_d r_d^2 - 2r_d \sum_{i \in S_d} y_i\right)$$

$\tau_h = \sum_{i=1}^{N_h} y_{hi} \qquad \tau = \sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \sum_{h=1}^{L} \tau_h$

$\mu_h = \dfrac{\tau_h}{N_h} \qquad \mu = \dfrac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$

$\widehat{\mu}_h = \overline{y}_h = \dfrac{1}{n_h}\sum_{i=1}^{n_h} y_{hi} \qquad \widehat{\tau}_h = N_h\overline{y}_h = \dfrac{N_h}{n_h}\sum_{i=1}^{n_h} y_{hi}$

$\widehat{\mu}_{st} = \dfrac{\widehat{\tau}_{st}}{N} = \dfrac{1}{N}\sum_{h=1}^{L} N_h\overline{y}_h \qquad \widehat{\mu}_{st} = \sum_{h=1}^{L} W_h\overline{y}_h$

$\widehat{\tau}_{st} = \sum_{h=1}^{L}\widehat{\tau}_h = \sum_{h=1}^{L} N_h\overline{y}_h$

$\text{var}(\widehat{\mu}_h) = \left(\dfrac{N_h - n_h}{N_h}\right)\dfrac{\sigma_h^2}{n_h} \qquad \text{var}(\widehat{\tau}_h) = N_h(N_h - n_h)\dfrac{\sigma_h^2}{n_h}$

$$\sigma_h^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \mu_h)^2$$

$$\text{var}(\widehat{\tau}_{st}) = \sum_{i=1}^{L}\text{var}(\widehat{\tau}_h) = \sum_{i=1}^{L} N_h(N_h - n_h)\frac{\sigma_h^2}{n_h}$$

$$\text{var}(\widehat{\mu}_{st}) = \left(\frac{1}{N^2}\right)\text{var}(\widehat{\tau}_{st}) = \left(\frac{1}{N^2}\right)\sum_{i=1}^{L} N_h(N_h - n_h)\frac{\sigma_h^2}{n_h}$$

$\widehat{\text{var}}(\widehat{\tau}_{st}) = \sum_{h=1}^{L} N_h(N_h - n_h)\dfrac{s_h^2}{n_h} \qquad \widehat{\text{var}}(\widehat{\mu}_{st}) = \left(\dfrac{1}{N^2}\right)\sum_{h=1}^{L} N_h(N_h - n_h)\dfrac{s_h^2}{n_h}$

$\widehat{\mu}_{st} \pm z^*\sqrt{\widehat{\text{var}}(\widehat{\mu}_{st})} \qquad \widehat{\tau}_{st} \pm z^*\sqrt{\widehat{\text{var}}(\widehat{\tau}_{st})}$

$\widehat{\mu}_{st} \pm t^*\sqrt{\widehat{\text{var}}(\widehat{\mu}_{st})} \qquad \widehat{\tau}_{st} \pm t^*\sqrt{\widehat{\text{var}}(\widehat{\tau}_{st})}$

$$d = \frac{\left(\sum_{h=1}^{L} a_h s_h^2\right)^2}{\sum_{h=1}^{L}(a_h s_h^2)^2/(n_h - 1)} = \frac{(\widehat{\text{var}}(\widehat{\tau}_{st}))^2}{\sum_{h=1}^{L}(a_h s_h^2)^2/(n_h - 1)}$$

$$n_h = \frac{(C - c_0)N_h\sigma_h/\sqrt{c_h}}{\sum_{h=1}^{L} N_h\sigma_h\sqrt{c_h}}$$

$p_h = \dfrac{1}{N_h}\sum_{i=1}^{N_h} y_{hi} \qquad \tau = \sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$

$\widehat{p}_h = \dfrac{1}{n_h}\sum_{i=1}^{n_h} y_{hi} \qquad \widehat{p}_{st} = \sum_{h=1}^{L}\dfrac{N_h}{N}\widehat{p}_h = \sum_{h=1}^{L} W_h\widehat{p}_h$

$$\text{var}(\widehat{p}_{st}) = \sum_{h=1}^{L} W_h^2\,\text{var}(\widehat{p}_h) = \sum_{h=1}^{L} W_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{\sigma_h^2}{n_h}$$

$$\widehat{\text{var}}(\widehat{p}_{st}) = \sum_{h=1}^{L} W_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h} = \sum_{h=1}^{L} W_h^2\,\frac{N_h - n_h}{N_h}\,\frac{\widehat{p}_h(1 - \widehat{p}_h)}{n_h - 1}$$

$$\hat{p}_{st} \pm t^* \sqrt{\widehat{\mathrm{var}}(\hat{p}_{st})}$$

$$\hat{\mu}_h = \frac{\overline{y}_h}{\overline{x}_h} \mu_{h(x)} = r_h \mu_{h(x)}$$

$$\hat{\mu}_{rst} = \sum_{h=1}^{L} W_h r_h \mu_{h(x)} = \sum_{h=1}^{L} \frac{N_h}{N} \frac{\overline{y}_h}{\overline{x}_h} \mu_{h(x)}$$

$$\begin{aligned} \widehat{\mathrm{var}}(\hat{\mu}_{rst}) &= \sum_{h=1}^{L} \left[\left(\frac{N_h}{N}\right)^2 \frac{N_h - n_h}{N_h n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_h^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2r_h \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \\ &= \sum_{h=1}^{L} \left[W_h^2 \frac{1 - f_h}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_h^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2r_h \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \end{aligned}$$

$$\hat{\mu}_{rst} \pm z^* \sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{rst})} \qquad \hat{\mu}_{rst} \pm t^* \sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{rst})}$$

$$\overline{x}_{st} = \sum_{h=1}^{L} \frac{N_h}{N} \overline{x}_h \qquad \overline{y}_{st} = \sum_{h=1}^{L} \frac{N_h}{N} \overline{y}_h \qquad r_c = \frac{\overline{y}_{st}}{\overline{x}_{st}}$$

$$\begin{aligned} \widehat{\mathrm{var}}(\hat{\mu}_{rcst}) &= \sum_{h=1}^{L} \left[W_h^2 \frac{(1 - f_h)}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_c^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2r_c \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \\ &= \sum_{h=1}^{L} \left[\left(\frac{N_h}{N}\right)^2 \frac{(1 - f_h)}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_c^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2r_c \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \end{aligned}$$

$$\hat{y} = a_h + b_h x$$

$$b_h = \frac{n_h \sum_{i=1}^{n_h} x_{hi} y_{hi} - (\sum_{i=1}^{n_h} x_{hi})(\sum_{i=1}^{n_h} y_{hi})}{n_h \sum_{i=1}^{n_h} x_{hi}^2 - (\sum_{i=1}^{n_h} x_{hi})^2}$$

$$\begin{aligned} \hat{\mu}_{Lh} &= a_h + b_h \mu_{h(x)} \\ &= \overline{y}_h + b_h (\mu_{h(x)} - \overline{x}_h) \end{aligned}$$

$$\begin{aligned} \hat{\mu}_{Lsst} &= \sum_{h=1}^{L} \frac{N_h}{N} \left(a_h + b_h \mu_{h(x)} \right) \\ &= \sum_{h=1}^{L} \frac{N_h}{N} \left(\overline{y}_h + b_h (\mu_{h(x)} - \overline{x}_h) \right) \end{aligned}$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{Lsst}) = \sum_{h=1}^{L} \left(\frac{N_h}{N}\right)^2 \frac{(1 - f_h)}{n_h (n_h - 2)} SSE_h = \sum_{h=1}^{L} \left(\frac{N_h}{N}\right)^2 \frac{(1 - f_h)}{n_h} MSE_h$$

$$\hat{\tau}_{Lsst} = N \hat{\mu}_{Lsst} \qquad \widehat{\mathrm{var}}(\hat{\tau}_{Lsst}) = N^2 \widehat{\mathrm{var}}(\hat{\mu}_{Lsst})$$

$$\mu_{Lcst} = \hat{\mu}_{st} + b_c (\mu_x - \overline{x}_{st}) \qquad b_c = \frac{\sum_{h=1}^{L} c_h b_h}{\sum_{h=1}^{L} c_h}$$

$$c_h = \left(\frac{N_h}{N}\right)^2 \frac{(1 - f_h)}{n_h} s_{xh}^2$$

$$\mu_{Lcst} = \hat{\mu}_{st} + b_c (\mu_x - \overline{x}_{st})$$

$$\widehat{\mathrm{var}}(\mu_{Lcst}) = \sum_{h=1}^{L} \left[\left(\frac{N_h}{N}\right)^2 \frac{(1 - f_h)}{n_h (n_h - 2)} \sum_{i=1}^{n_h} [(y_{hi} - \overline{y}_h) - b_c (x_{hi} - \overline{x}_h)]^2 \right]$$

$$\hat{\mu}_{pst} = \frac{1}{N} \sum_{h=1}^{L} N_h \overline{y}_h \qquad \hat{\tau}_{pst} = \sum_{h=1}^{L} N_h \overline{y}_h$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{pst}) = \frac{N - n}{nN} \sum_{i=1}^{L} \left(\frac{N_h}{N}\right) s_h^2 + \frac{1}{n^2} \left(\frac{N - n}{N - 1}\right) \sum_{i=1}^{L} \frac{N - N_h}{N} s_h^2$$

$$\widehat{\mathrm{var}}(\hat{\tau}_{pst}) = \frac{N - n}{n} \sum_{i=1}^{L} N_h s_h^2 + \left(\frac{N^2}{n^2}\right) \left(\frac{N - n}{N - 1}\right) \sum_{i=1}^{L} \frac{N - N_h}{N} s_h^2$$

$$\overline{y}_i = \frac{y_i}{M_i} \qquad y_i = \sum_{j=1}^{M_i} y_{ij} \qquad s_u^2 = \frac{\sum_{i=1}^{n} (y_i - \overline{y})^2}{n - 1}$$

$$\tau = \sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} = \sum_{i=1}^{N} y_i \qquad \mu = \frac{1}{M}\sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} = \frac{\tau}{M}$$

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{N} y_i \qquad \sigma_u^2 = \frac{\sum_{i=1}^{N}(y_i - \mu_1)^2}{N-1}$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\overline{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\overline{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

$$\mathrm{var}(\hat{\tau}_{cl}) = N(N-n)\frac{\sigma_u^2}{n} \qquad \mathrm{var}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{\sigma_u^2}{n}$$

$$\widehat{\mathrm{var}}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \widehat{\mathrm{var}}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

$$\hat{\mu}_{cl} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{cl})} \qquad \hat{\tau}_{cl} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau}_{cl})}$$

$$\hat{\tau}_{sys} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\overline{y} \quad \hat{\mu}_{sys} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\overline{y}}{L} = \frac{\hat{\tau}_{sys}}{M}$$

$$\widehat{\mathrm{var}}(\hat{\tau}_{sys}) = N(N-n)\frac{s_u^2}{n} \qquad \widehat{\mathrm{var}}(\hat{\mu}_{sys}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

$$\hat{\mu}_{sys} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{sys})} \qquad \hat{\tau}_{sys} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau}_{sys})}$$

$$\mu = \frac{\sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} M_i} = \frac{\sum_{i=1}^{N} y_i}{M} \qquad \mu = \left(\frac{N}{M}\right)\sum_{i=1}^{N}\frac{y_i}{N}$$

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m}$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\overline{y}_i - \hat{\mu}_{c(a)})^2$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{c(a)}) = \frac{(N-n)n}{N(n-1)}\sum_{i=1}^{n}\left(\frac{M_i}{m}\right)^2(\overline{y}_i - \hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i - \overline{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{c(b)}) = \frac{(N-n)n}{(n-1)Nm^2}\sum_{i=1}^{n}(y_i - \overline{y})^2 = \frac{(N-n)n}{Nm^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n}$$

$$\mathrm{var}(\hat{p}_c) = \left(\frac{N-n}{nN}\right)\sum_{i=1}^{N}\frac{(p_i - p)^2}{N-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{N}\frac{(p_i - p)^2}{N-1}$$

$$\widehat{\mathrm{var}}(\hat{p}_c) = \left(\frac{N-n}{nN}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i}$$

$$\mathrm{var}(\hat{p}_c) \approx = \left(\frac{1-f}{n\overline{M}^2}\right)\frac{\sum_{i=1}^{N}(y_i - pM_i)^2}{N-1}$$

$$\widehat{\mathrm{var}}(\hat{p}_c) \approx \left(\frac{1-f}{n\overline{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i - p_cM_i)^2}{n-1}$$

$$= \left(\frac{1-f}{n\overline{m}^2}\right)\frac{\sum_{i=1}^{n} y_i^2 - 2p_c\sum_{i=1}^{n} y_iM_i + p_c^2\sum_{i=1}^{n} M_i^2}{n-1}$$

$$\hat{y}_i = \frac{M_i}{m_i}\sum_{j=1}^{m_i} y_{ij} = M_i\overline{y}_i \quad \hat{\tau} = \frac{N}{n}\sum_{i=1}^{n}\hat{y}_i \quad \hat{\mu}_1 = \frac{\sum_{i=1}^{n}\hat{y}_i}{n}$$

$$\mathrm{var}(\hat{\tau}) = N(N-n)\frac{\sigma_u^2}{n} + \frac{N}{n}\sum_{i=1}^{N} M_i(M_i - m_i)\frac{\sigma_i^2}{m_i}$$

$$\sigma_u^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \mu_1)^2 \quad \sigma_i^2 = \frac{1}{M_i - 1}\sum_{j=1}^{M_i}(y_{ij} - \mu_{i\cdot})^2$$

$$\widehat{\mathrm{var}}(\hat{\tau}) = N(N-n)\frac{s_u^2}{n} + \frac{N}{n}\sum_{i=1}^{n} M_i(M_i - m_i)\frac{s_i^2}{m_i}$$

$$s_u^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\hat{y}_i - \hat{\mu}_1)^2 \quad s_i^2 = \frac{1}{m_i - 1}\sum_{j=1}^{m_i}(y_{ij} - \overline{y}_i)^2$$

$$\hat{\tau} = \frac{M}{nl}\sum_{i=1}^{n}\sum_{j=1}^{l} y_{ij} = \frac{M}{n}\sum_{i=1}^{n}\overline{y}_i$$

$$\hat{\mu} = \frac{1}{nl}\sum_{i=1}^{n}\sum_{j=1}^{l} y_{ij} = \frac{1}{n}\sum_{i=1}^{n}\overline{y}_i$$

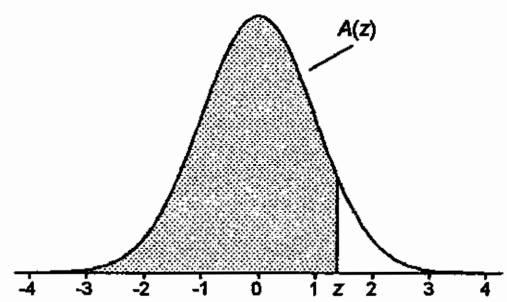
$$\widehat{\mathrm{var}}(\hat{\tau}) = N(N-n)\frac{s_u^2}{n} + \frac{M(L-l)}{nl}\sum_{i=1}^{n} s_i^2$$

$$\widehat{\mathrm{var}}(\hat{\mu}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n} + \frac{L-l}{Mnl}\sum_{i=1}^{n} s_i^2$$

$$\hat{\tau} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau})} \qquad \hat{\mu} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu})}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of t

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

TABLE A.3 (continued)

F Distribution: Critical Values of F (5% significance level)

v_2 \ v_1	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19

TABLE A.3

F Distribution: Critical Values of F (5% significance level)

ν_1 / ν_2	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

TABLE A.3 (continued)

F Distribution: Critical Values of F (1% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6142.67	6170.10	6191.53	6208.73
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.44	99.45
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.92	26.83	26.75	26.69
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.25	14.15	14.08	14.02
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.77	9.68	9.61	9.55
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.60	7.52	7.45	7.40
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.36	6.28	6.21	6.16
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.56	5.48	5.41	5.36
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	5.01	4.92	4.86	4.81
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.60	4.52	4.46	4.41
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.29	4.21	4.15	4.10
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.05	3.97	3.91	3.86
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.86	3.78	3.72	3.66
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.70	3.62	3.56	3.51
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.56	3.49	3.42	3.37
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.45	3.37	3.31	3.26
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.35	3.27	3.21	3.16
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.27	3.19	3.13	3.08
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.19	3.12	3.05	3.00
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.13	3.05	2.99	2.94
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.07	2.99	2.93	2.88
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.02	2.94	2.88	2.83
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.97	2.89	2.83	2.78
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.93	2.85	2.79	2.74
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.89	2.81	2.75	2.70
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.86	2.78	2.72	2.66
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.82	2.75	2.68	2.63
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.79	2.72	2.65	2.60
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.77	2.69	2.63	2.57
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.74	2.66	2.60	2.55
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.64	2.56	2.50	2.44
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.56	2.48	2.42	2.37
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.46	2.38	2.32	2.27
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.39	2.31	2.25	2.20
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.35	2.27	2.20	2.15
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.31	2.23	2.17	2.12
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.29	2.21	2.14	2.09
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.27	2.19	2.12	2.07
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.23	2.15	2.09	2.03
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.20	2.12	2.06	2.00
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.27	2.17	2.09	2.03	1.97
250	6.74	4.69	3.86	3.40	3.09	2.87	2.71	2.58	2.48	2.39	2.26	2.15	2.07	2.01	1.95
300	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38	2.24	2.14	2.06	1.99	1.94
400	6.70	4.66	3.83	3.37	3.06	2.85	2.68	2.56	2.45	2.37	2.23	2.13	2.05	1.98	1.92
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.22	2.12	2.04	1.97	1.92
600	6.68	4.64	3.81	3.35	3.05	2.83	2.67	2.54	2.44	2.35	2.21	2.11	2.03	1.96	1.91
750	6.67	4.63	3.81	3.34	3.04	2.83	2.66	2.53	2.43	2.34	2.21	2.11	2.02	1.96	1.90
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.20	2.10	2.02	1.95	1.90

Table A.3 (continued)

***F* Distribution: Critical Values of *F* (1% significance level)**

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	6239.83	6260.65	6275.57	6286.78	6302.52	6313.03	6323.56	6334.11	6344.68	6349.97
2	99.46	99.47	99.47	99.47	99.48	99.48	99.49	99.49	99.49	99.49
3	26.58	26.50	26.45	26.41	26.35	26.32	26.28	26.24	26.20	26.18
4	13.91	13.84	13.79	13.75	13.69	13.65	13.61	13.58	13.54	13.52
5	9.45	9.38	9.33	9.29	9.24	9.20	9.17	9.13	9.09	9.08
6	7.30	7.23	7.18	7.14	7.09	7.06	7.02	6.99	6.95	6.93
7	6.06	5.99	5.94	5.91	5.86	5.82	5.79	5.75	5.72	5.70
8	5.26	5.20	5.15	5.12	5.07	5.03	5.00	4.96	4.93	4.91
9	4.71	4.65	4.60	4.57	4.52	4.48	4.45	4.41	4.38	4.36
10	4.31	4.25	4.20	4.17	4.12	4.08	4.05	4.01	3.98	3.96
11	4.01	3.94	3.89	3.86	3.81	3.78	3.74	3.71	3.67	3.66
12	3.76	3.70	3.65	3.62	3.57	3.54	3.50	3.47	3.43	3.41
13	3.57	3.51	3.46	3.43	3.38	3.34	3.31	3.27	3.24	3.22
14	3.41	3.35	3.30	3.27	3.22	3.18	3.15	3.11	3.08	3.06
15	3.28	3.21	3.17	3.13	3.08	3.05	3.01	2.98	2.94	2.92
16	3.16	3.10	3.05	3.02	2.97	2.93	2.90	2.86	2.83	2.81
17	3.07	3.00	2.96	2.92	2.87	2.83	2.80	2.76	2.73	2.71
18	2.98	2.92	2.87	2.84	2.78	2.75	2.71	2.68	2.64	2.62
19	2.91	2.84	2.80	2.76	2.71	2.67	2.64	2.60	2.57	2.55
20	2.84	2.78	2.73	2.69	2.64	2.61	2.57	2.54	2.50	2.48
21	2.79	2.72	2.67	2.64	2.58	2.55	2.51	2.48	2.44	2.42
22	2.73	2.67	2.62	2.58	2.53	2.50	2.46	2.42	2.38	2.36
23	2.69	2.62	2.57	2.54	2.48	2.45	2.41	2.37	2.34	2.32
24	2.64	2.58	2.53	2.49	2.44	2.40	2.37	2.33	2.29	2.27
25	2.60	2.54	2.49	2.45	2.40	2.36	2.33	2.29	2.25	2.23
26	2.57	2.50	2.45	2.42	2.36	2.33	2.29	2.25	2.21	2.19
27	2.54	2.47	2.42	2.38	2.33	2.29	2.26	2.22	2.18	2.16
28	2.51	2.44	2.39	2.35	2.30	2.26	2.23	2.19	2.15	2.13
29	2.48	2.41	2.36	2.33	2.27	2.23	2.20	2.16	2.12	2.10
30	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.13	2.09	2.07
35	2.35	2.28	2.23	2.19	2.14	2.10	2.06	2.02	1.98	1.96
40	2.27	2.20	2.15	2.11	2.06	2.02	1.98	1.94	1.90	1.87
50	2.17	2.10	2.05	2.01	1.95	1.91	1.87	1.82	1.78	1.76
60	2.10	2.03	1.98	1.94	1.88	1.84	1.79	1.75	1.70	1.68
70	2.05	1.98	1.93	1.89	1.83	1.78	1.74	1.70	1.65	1.62
80	2.01	1.94	1.89	1.85	1.79	1.75	1.70	1.65	1.61	1.58
90	1.99	1.92	1.86	1.82	1.76	1.72	1.67	1.62	1.57	1.55
100	1.97	1.89	1.84	1.80	1.74	1.69	1.65	1.60	1.55	1.52
120	1.93	1.86	1.81	1.76	1.70	1.66	1.61	1.56	1.51	1.48
150	1.90	1.83	1.77	1.73	1.66	1.62	1.57	1.52	1.46	1.43
200	1.87	1.79	1.74	1.69	1.63	1.58	1.53	1.48	1.42	1.39
250	1.85	1.77	1.72	1.67	1.61	1.56	1.51	1.46	1.40	1.36
300	1.84	1.76	1.70	1.66	1.59	1.55	1.50	1.44	1.38	1.35
400	1.82	1.75	1.69	1.64	1.58	1.53	1.48	1.42	1.36	1.32
500	1.81	1.74	1.68	1.63	1.57	1.52	1.47	1.41	1.34	1.31
600	1.80	1.73	1.67	1.63	1.56	1.51	1.46	1.40	1.34	1.30
750	1.80	1.72	1.66	1.62	1.55	1.50	1.45	1.39	1.33	1.29
1000	1.79	1.72	1.66	1.61	1.54	1.50	1.44	1.38	1.32	1.28

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4.05e05	5.00e05	5.40e05	5.62e05	5.76e05	5.86e05	5.93e05	5.98e05	6.02e05	6.06e05	6.11e05	6.14e05	6.17e05	6.19e05	6.21e05
2	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39	999.40	999.42	999.43	999.44	999.44	999.45
3	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	129.25	128.32	127.64	127.14	126.74	126.42
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.95	46.60	46.32	46.10
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92	26.42	26.06	25.78	25.57	25.39
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	17.99	17.68	17.45	17.27	17.12
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.43	13.23	13.06	12.93
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54	11.19	10.94	10.75	10.60	10.48
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.33	9.15	9.01	8.90
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75	8.45	8.22	8.05	7.91	7.80
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.41	7.24	7.11	7.01
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.79	6.63	6.51	6.40
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.31	6.16	6.03	5.93
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.13	5.93	5.78	5.66	5.56
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.62	5.46	5.35	5.25
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81	5.55	5.35	5.20	5.09	4.99
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.13	4.99	4.87	4.78
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.94	4.80	4.68	4.59
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.78	4.64	4.52	4.43
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.64	4.49	4.38	4.29
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.51	4.37	4.26	4.17
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.40	4.26	4.15	4.06
23	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.30	4.16	4.05	3.96
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.21	4.07	3.96	3.87
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56	4.31	4.13	3.99	3.88	3.79
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	4.06	3.92	3.81	3.72
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.99	3.86	3.75	3.66
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.93	3.80	3.69	3.60
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.88	3.74	3.63	3.54
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.82	3.69	3.58	3.49
35	12.90	8.47	6.79	5.88	5.30	4.89	4.59	4.36	4.18	4.03	3.79	3.62	3.48	3.38	3.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.47	3.34	3.23	3.14
50	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	3.67	3.44	3.27	3.41	3.04	2.95
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.32	3.15	3.02	2.91	2.83
70	11.80	7.64	6.06	5.20	4.66	4.28	3.99	3.77	3.60	3.45	3.23	3.06	2.93	2.83	2.74
80	11.67	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53	3.39	3.16	3.00	2.87	2.76	2.68
90	11.57	7.47	5.91	5.06	4.53	4.15	3.87	3.65	3.48	3.34	3.11	2.95	2.82	2.71	2.63
100	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	3.30	3.07	2.91	2.78	2.68	2.59
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.85	2.72	2.62	2.53
150	11.27	7.24	5.71	4.88	4.35	3.98	3.71	3.49	3.32	3.18	2.96	2.80	2.67	2.56	2.48
200	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26	3.12	2.90	2.74	2.61	2.51	2.42
250	11.09	7.10	5.59	4.77	4.25	3.88	3.61	3.40	3.23	3.09	2.87	2.71	2.58	2.48	2.39
300	11.04	7.07	5.56	4.75	4.22	3.86	3.59	3.38	3.21	3.07	2.85	2.69	2.56	2.46	2.37
400	10.99	7.03	5.53	4.71	4.19	3.83	3.56	3.35	3.18	3.04	2.82	2.66	2.53	2.43	2.34
500	10.96	7.00	5.51	4.69	4.18	3.81	3.54	3.33	3.16	3.02	2.81	2.64	2.52	2.41	2.33
600	10.94	6.99	5.49	4.68	4.16	3.80	3.53	3.32	3.15	3.01	2.80	2.63	2.51	2.40	2.32
750	10.91	6.97	5.48	4.67	4.15	3.79	3.52	3.31	3.14	3.00	2.78	2.62	2.49	2.39	2.31
1000	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	2.99	2.77	2.61	2.48	2.38	2.30

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	6.24e05	6.26e05	6.28e05	6.29e05	6.30e05	6.31e05	6.32e05	6.33e05	6.35e05	6.35e05
2	999.46	999.47	999.47	999.47	999.48	999.48	999.49	999.49	999.49	999.49
3	125.84	125.45	125.17	124.96	124.66	124.47	124.27	124.07	123.87	123.77
4	45.70	45.43	45.23	45.09	44.88	44.75	44.61	44.47	44.33	44.26
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.80	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.96	7.93
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.37	5.30	5.24	5.17	5.10	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.70	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	4.00	3.93	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.86	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.43	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.18	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.21	2.11	2.05	1.98	1.91	1.83	1.79
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.03	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.80	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.45	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

	Significance level		
Degrees of freedom	5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588