

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2010

TITLE OF PAPER : INTRODUCTION TO REGRESSION ANALYSIS
COURSE CODE : ST304
TIME ALLOWED : 2 (TWO) HOURS
**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**
**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[10 + 5 + 5 + 5 marks]

- 1.1 What is Multicollinearity? Discuss the effects of multicollinearity, when it is present in the model, (i) on the Regression Coefficients, (ii) on the Standard Errors of the estimated regression coefficients, $s\{b_k\}$, and (iii) on F -test.
- 1.2 State some informal indications of the presence of serious multicollinearity.
- 1.3 Explain how variance inflation factors (VIF) can be used to detect the presence of multicollinearity.
- 1.4 Discuss the different remedial measures in case of the presence for serious multicollinearity.

QUESTION TWO.

[4 + 4 + 5 + 8 + 4 marks]

- 2.1 State the first-order autoregressive multiple linear regression model.
- 2.2 We have four important consequences to use ordinary least square procedure when autocorrelation is present. State those.
- 2.3 The following residuals were found running a multiple regression model with 3 predictor variables:

0.156	0.485	-0.442	-0.396
0.039	0.354	0.048	-0.550
0.803	-0.124	-0.224	-0.524
0.266	-0.087	-0.388	0.538

Use Durbin-Watson test statistic to test the existence of autocorrelation. Choose your α suitable according to attached table.

- 2.4 Given a sample of 50 observations and 4 predictor variables with 5% level of significance, what can you say about autocorrelation (positive or negative) if
 - (i) $D = 1.05$?
 - (ii) $D = 1.40$?
 - (iii) $D = 2.50$?
 - (iv) $D = 3.97$?

[You must show all your works regarding your conclusions]

- 2.5 State few remedial measures for autocorrelation.

QUESTION THREE.

[8 + 8 + 9 marks]

- 3.1 What is Heteroscedasticity? What happens to OLS estimator in the presence of heteroscedasticity?
- 3.2 What is Generalized Least Squares (GLS) method? Explain how the GLS method works to obtain estimates of regression coefficients when heteroscedasticity is present.
- 3.3 Discuss at least three methods of detecting heteroscedasticity.

QUESTION FOUR.

[7 + 4 + 4 + 5 + 5 marks]

- 4.1 State the General Linear Regression Model in matrix form including all assumptions with clearly stated vectors and matrices.
- 4.2 The following results were computed from a multiple regression model:

$$(X'X)^{-1} = \begin{bmatrix} 29.7289 & .0722 & -1.9926 \\ .0722 & .00037 & -.0056 \\ -1.9926 & -.0056 & .1363 \end{bmatrix}, X'Y = \begin{bmatrix} 3820 \\ 249643 \\ 66073 \end{bmatrix}, n = 21 \text{ and } MSE = 121.1626$$

- Find the estimated regression function.
- Estimate the variance-covariance matrix.
- Construct a 99% confidence interval for β_0 .
- Test the hypothesis, $\beta_1 = 1$ against $\beta_1 \neq 1$.

QUESTION FIVE.

[4 + 5 + 1 + 4 + 6 + 5 marks]

The following tables are the partial SPSS output of a multiple regression problem:

ANOVA						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	23.446	5	4.689	9.076	.000
	Residual	22.734	44	.517		
	Total	46.180	49			

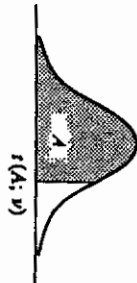
Coefficients						
		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.518	.564		4.467	.000
	x1	.297	.229	.170	1.299	.201
	x2	.179	.135	.158	1.326	.192
	x3	-.379	.112	-.522	-3.399	.001
	x4	-.086	.128	-.087	-.670	.506
	x5	-.016	.116	-.024	-.135	.893

Answer the following questions based on the above two tables:

- State the fitted regression line.
- Test the goodness of fitted model. You must state the null and alternative hypotheses and clearly state the conclusion.
- What is the estimated value of σ^2 ?
- Calculate the coefficient of multiple determination R^2 . How is it interpreted here?
- Construct a 95% joint confidence interval for β_1 and β_2 .
- Do the data present sufficient evidence to indicate that x_3 contributes information for the prediction of y .

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



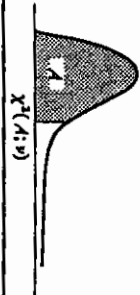
ν	A					
	.50	.70	.80	.85	.90	.95
1	0.325	0.727	1.376	1.963	3.078	6.314
2	0.289	0.617	1.061	1.386	2.920	4.303
3	0.277	0.584	0.978	1.250	2.353	3.182
4	0.271	0.569	0.941	1.190	2.132	2.776
5	0.267	0.559	0.920	1.156	2.015	2.571
6	0.265	0.553	0.906	1.134	1.943	2.447
7	0.263	0.549	0.896	1.119	1.893	2.365
8	0.262	0.546	0.889	1.108	1.860	2.306
9	0.261	0.543	0.883	1.100	1.833	2.262
10	0.260	0.542	0.879	1.093	1.812	2.228
11	0.260	0.540	0.876	1.088	1.796	2.201
12	0.259	0.539	0.873	1.083	1.782	2.179
13	0.259	0.537	0.870	1.079	1.771	2.160
14	0.258	0.537	0.868	1.076	1.761	2.145
15	0.258	0.536	0.866	1.074	1.753	2.131
16	0.258	0.535	0.865	1.071	1.746	2.120
17	0.257	0.534	0.863	1.069	1.740	2.110
18	0.257	0.534	0.862	1.067	1.734	2.101
19	0.257	0.533	0.861	1.066	1.729	2.093
20	0.257	0.533	0.860	1.064	1.725	2.086
21	0.257	0.532	0.859	1.063	1.721	2.080
22	0.256	0.532	0.858	1.061	1.717	2.074
23	0.256	0.532	0.858	1.060	1.714	2.069
24	0.256	0.531	0.857	1.059	1.711	2.064
25	0.256	0.531	0.856	1.058	1.708	2.060
26	0.256	0.531	0.856	1.058	1.706	2.056
27	0.256	0.531	0.855	1.057	1.703	2.052
28	0.256	0.530	0.855	1.056	1.701	2.048
29	0.256	0.530	0.854	1.055	1.699	2.045
30	0.256	0.530	0.854	1.055	1.697	2.042
40	0.255	0.529	0.851	1.050	1.684	2.021
60	0.254	0.527	0.848	1.045	1.671	2.000
120	0.254	0.526	0.845	1.041	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.645	1.960

TABLE A.2 (continued) Percentiles of the *t* Distribution

ν	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.00757	0.0157	0.0308	0.0539	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0461	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-99.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

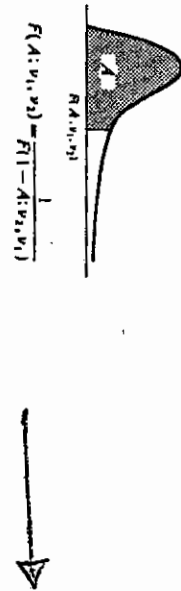


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
.95	161	200	216	225	230	234	237	239	241
.975	648	800	864	900	922	937	948	957	963
.99	4,052	5,000	5,403	5,625	5,764	5,839	5,928	5,981	6,022
.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32
.90	8.33	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20	
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	63.5	
.95	242	244	246	248	249	250	252	253	254	255	
.975	969	977	985	993	997	1,001	1,010	1,014	1,018	1,021	
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366	6,393	
.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	25,569	
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620	639,280	
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44	
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	9.50	
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5	
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	
.995	199	199	199	199	199	199	199	199	200	200	
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	999.5	
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27	
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.17	5.15	5.14	5.13	
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53	8.51	
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9	13.9	
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1	26.1	
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8	41.8	
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	123.5	
4	.50	1.11	1.13	1.14	1.15	1.16	1.18	1.18	1.19	1.19	
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76	3.75	
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63	5.61	
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	8.23	
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	13.5	
.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3	19.3	
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	44.1	
5	.50	1.07	1.09	1.10	1.11	1.12	1.14	1.14	1.15	1.15	
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11	3.11	
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37	4.37	
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	6.00	
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	9.00	
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	12.1	
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	23.8	
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12	
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72	2.72	
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67	3.67	
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	4.85	
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88	6.88	
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	8.88	
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	15.7	
7	.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10	
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47	2.47	
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23	3.23	
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14	4.14	
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	5.65	
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	7.08	
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	11.7	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
8	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34
.999	23.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8
9	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
.95	5.12	4.26	3.86	3.53	3.48	3.37	3.29	3.23	3.18
.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
.995	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54
.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1
10	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
.99	10.0	7.36	6.35	5.99	5.64	5.39	5.20	5.06	4.94
.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96
12	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48
15	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
.995	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26
20	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
24	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
.999	14.0	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df									
	10	12	15	20	24	30	60	120	∞	
8	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09	
.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29	
.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.92	
.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67	
.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	
.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95	
.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33	
9	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08	
.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	
.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	
.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	
.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	
.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19	
.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81	
10	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07	
.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06	
.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54	
.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	
.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	
.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64	
.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94	6.76	
12	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06	
.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	
.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	
.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72	
.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	
.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90	
.999	7.29	7.00	6.71	6.40	6.25	6.09	5.76	5.59	5.42	
15	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05	
.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	
.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	
.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	
.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87	
.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37	3.26	
.999	6.08	5.81	5.54	5.25	5.10	4.95	4.64	4.48	4.31	
20	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03	
.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	
.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	
.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09	
.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	
.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69	
.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54	3.38	
24	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03	
.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	
.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	
.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	
.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	
.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43	
.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97	

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.838	0.860	0.912	0.927	0.939	0.948
.50	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.90	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.95	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.975	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.99	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.995	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.50	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.90	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.95	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.975	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.99	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.995	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.50	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.90	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.95	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.975	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.99	8.18	5.54	4.50	3.92	3.53	3.28	3.09	2.93	2.81
.995	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10
∞	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.50	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.90	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.95	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.975	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.99	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.995	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	Numerator df								
	10	12	15	20	24	30	60	120	∞
30	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02
.50	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46
.90	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62
.95	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79
.975	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01
.99	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18
.995	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59
.999									
60	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01
.50	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29
.90	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39
.95	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48
.975	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60
.99	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69
.995	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89
.999									
120	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01
.50	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19
.90	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25
.95	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31
.975	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38
.99	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43
.995	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54
.999									
∞	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00
.50	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00
.90	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00
.95	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00
.975	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00
.99	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00
.995	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00
.999									

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TABLE B.7
Durbin-Watson
Test Bounds.

n	Level of Significance $\alpha = .05$									
	$p - 1 = 1$		$p - 1 = 2$		$p - 1 = 3$		$p - 1 = 4$		$p - 1 = 5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.59	1.73	1.57	1.75	1.54	1.78
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

TABLE B.7
(concluded)
Durbin-Watson
Test Bounds.

n	Level of Significance $\alpha = .01$									
	$p - 1 = 1$		$p - 1 = 2$		$p - 1 = 3$		$p - 1 = 4$		$p - 1 = 5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	0.81	1.07	0.70	1.25	0.59	1.46	0.49	1.70	0.39	1.96
16	0.84	1.09	0.74	1.25	0.63	1.44	0.53	1.66	0.44	1.90
17	0.87	1.10	0.77	1.25	0.67	1.43	0.57	1.63	0.48	1.85
18	0.90	1.12	0.80	1.26	0.71	1.42	0.61	1.60	0.52	1.80
19	0.93	1.13	0.83	1.26	0.74	1.41	0.65	1.58	0.56	1.77
20	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
21	0.97	1.16	0.89	1.27	0.80	1.41	0.72	1.55	0.63	1.71
22	1.00	1.17	0.91	1.28	0.83	1.40	0.75	1.54	0.66	1.69
23	1.02	1.19	0.94	1.29	0.86	1.40	0.77	1.53	0.70	1.67
24	1.04	1.20	0.96	1.30	0.88	1.41	0.80	1.53	0.72	1.66
25	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.64
26	1.07	1.22	1.00	1.31	0.93	1.41	0.85	1.52	0.78	1.64
27	1.09	1.23	1.02	1.32	0.95	1.41	0.88	1.51	0.81	1.63
28	1.10	1.24	1.04	1.32	0.97	1.41	0.90	1.51	0.83	1.62
29	1.12	1.25	1.05	1.33	0.99	1.42	0.92	1.51	0.85	1.61
30	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
31	1.15	1.27	1.08	1.34	1.02	1.42	0.96	1.51	0.90	1.60
32	1.16	1.28	1.10	1.35	1.04	1.43	0.98	1.51	0.92	1.60
33	1.17	1.29	1.11	1.36	1.05	1.43	1.00	1.51	0.94	1.59
34	1.18	1.30	1.13	1.36	1.07	1.43	1.01	1.51	0.95	1.59
35	1.19	1.31	1.14	1.37	1.08	1.44	1.03	1.51	0.97	1.59
36	1.21	1.32	1.15	1.38	1.10	1.44	1.04	1.51	0.99	1.59
37	1.22	1.32	1.16	1.38	1.11	1.45	1.06	1.51	1.00	1.59
38	1.23	1.33	1.18	1.39	1.12	1.45	1.07	1.52	1.02	1.58
39	1.24	1.34	1.19	1.39	1.14	1.45	1.09	1.52	1.03	1.58
40	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
45	1.29	1.38	1.24	1.42	1.20	1.48	1.16	1.53	1.11	1.58
50	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
55	1.36	1.43	1.32	1.47	1.28	1.51	1.25	1.55	1.21	1.59
60	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
65	1.41	1.47	1.38	1.50	1.35	1.53	1.31	1.57	1.28	1.61
70	1.43	1.49	1.40	1.52	1.37	1.55	1.34	1.58	1.31	1.61
75	1.45	1.50	1.42	1.53	1.39	1.56	1.37	1.59	1.34	1.62
80	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
85	1.48	1.53	1.46	1.55	1.43	1.58	1.41	1.60	1.39	1.63
90	1.50	1.54	1.47	1.56	1.45	1.59	1.43	1.61	1.41	1.64
95	1.51	1.55	1.49	1.57	1.47	1.60	1.45	1.62	1.42	1.64
100	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

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