UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2009

TITLE OF PAPER	**: SAMPLE SURVEY THEORY**
COURSE CODE	**: ST 306**
TIME ALLOWED	**: TWO (2) HOURS**
REQUIREMENTS	**: CALCULATOR AND STATISTICAL TABLES FORMULA SHEET ATTACHED**
INSTRUCTIONS	**: ANSWER ANY THREE QUESTIONS**

Question 2

a) The formula for the variance of the estimator of a population mean based on a stratified (random) sample is

$$V = \sum_{h=1}^{L} W_h^2 \left(1 - \frac{n_h}{N_h}\right) \frac{S_h^2}{n_h}.$$

Define the symbols in the above formula. Explain the conditions under which stratified sampling may be superior to simple random sampling.

(5 Marks)

b) A region aims to estimate the total number of children who have been absent from lessons without good reason (truants) in the past week. The region is divided into four education authorities (strata) and a sample of ten schools is taken from each education authority. The results are as follows.

Education authority, h	*Total number of schools*	*Number of truants (y) in schools selected*										$\bar{y}_h$	s_h
		1	2	3	4	5	6	7	8	9	10		
1	141	43	84	98	0	10	44	0	124	13	0	41.6	45.69
2	471	50	147	62	87	84	150	170	104	56	160	107.0	46.00
3	256	228	262	110	232	139	170	334	0	63	220	175.8	99.80
4	1499	17	34	25	34	36	0	25	7	15	31	22.4	12.31

i. Obtain a point estimate and a 95% confidence interval for the total number of children in the region who have played truant from school in the past week.

(7 Marks)

ii. Suppose that the cost of sampling in different strata varies and let c_h be the cost of sampling an element from stratum h. If c_1 = E25, c_2 = E10, c_3 = E10, c_4 = E5, and the total sample size is 40, use the results of this survey to compute the sample sizes in each stratum under

(*A*) optimal allocation,
(*B*) proportional allocation.

For each method, state the total cost of sampling, and obtain an estimate of the variance of the estimated total number of children who have played truant in this region.

(8 Marks)

Question 3

A simple random sample of 10 hospitals was selected from a population of 33 hospitals that had received state funding to upgrade their emergency medical services. Within each of the selected hospitals, the records of all patients hospitalised in the past 12 months for traumatic injuries (i.e. accidents, poisonings, violence, burns, etc) were examined. The numbers of patients hospitalised for trauma conditions and the numbers discharged dead for the selected hospitals are given below.

Hospital	*Number of patients hospitalised for trauma conditions*	*Number with trauma conditions discharged dead*
1	560	4
2	190	4
3	260	2
4	370	4
5	190	4
6	130	0
7	170	9
8	170	2
9	60	0
10	110	1

a) Explain why this design may be considered as a cluster sample. What are the first-stage and second-stage units?

(2 Marks)

b) Obtain a point estimate and an approximate 95% confidence interval for the total number of persons hospitalised for trauma conditions for the 33 hospitals. State the properties of your estimator.

(6 Marks)

c) Obtain a point estimate of the proportion of persons discharged dead among those hospitalised for trauma conditions for the 33 hospitals, using the cluster totals. Hence calculate an approximate 95% confidence interval for this proportion, and comment on the validity of the assumptions necessary for this calculation.

(6 Marks)

d) Give reasons why, for this survey, cluster sampling might be preferred to stratified random sampling. What might be the drawbacks of cluster sampling?
Discuss, with reasons, any improvements you might make if another survey was being planned on the same topic.

(6 Marks)

Question 4

At an experimental station 100 fields (each of area one hectare) were sown with sorghum. Each field was divided into sixteen equal plots. A simple random sample of 10 fields was selected, and from each of these fields a simple random sample of 4 plots was selected. The yields in kg/plot are given below:

	Plot within field			
Field	*1*	*2*	*3*	*4*
1	4.28	4.36	3.00	3.52
2	4.20	4.66	3.64	5.00
3	4.40	4.72	4.04	3.98
4	5.16	4.24	4.96	3.84
5	4.08	3.96	3.42	3.08
6	4.12	4.68	3.46	4.02
7	4.00	4.84	4.32	3.72
8	3.06	4.24	4.76	3.12
9	4.16	4.36	3.50	5.00
10	4.32	4.84	3.96	4.04

(a) Estimate the sorghum yield per hectare for the experimental station and its standard error. (12 Marks)

(b) Appraise the design effect (deff) of this estimator. (8 Marks)

STATISTICAL TABLES

Cumulative normal distribution

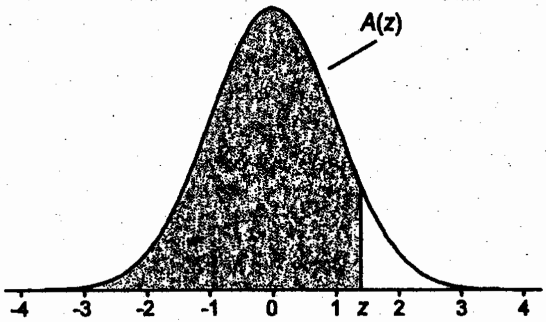
Critical values of the *t* distribution

Critical values of the *F* distribution

Critical values of the chi-squared distribution

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Table A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4.05e05	5.00e05	5.40e05	5.62e05	5.76e05	5.86e05	5.93e05	5.98e05	6.02e05	6.06e05	6.11e05	6.14e05	6.17e05	6.19e05	6.21e05
2	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39	999.40	999.42	999.43	999.44	999.44	999.45
3	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86	129.25	128.32	127.64	127.14	126.74	126.42
4	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47	48.05	47.41	46.95	46.60	46.32	46.10
5	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24	26.92	26.42	26.06	25.78	25.57	25.39
6	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	18.41	17.99	17.68	17.45	17.27	17.12
7	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	14.08	13.71	13.43	13.23	13.06	12.93
8	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77	11.54	11.19	10.94	10.75	10.60	10.48
9	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	9.89	9.57	9.33	9.15	9.01	8.90
10	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	8.75	8.45	8.22	8.05	7.91	7.80
11	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12	7.92	7.63	7.41	7.24	7.11	7.01
12	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48	7.29	7.00	6.79	6.63	6.51	6.40
13	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98	6.80	6.52	6.31	6.16	6.03	5.93
14	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	6.40	6.13	5.93	5.78	5.66	5.56
15	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26	6.08	5.81	5.62	5.46	5.35	5.25
16	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	5.81	5.55	5.35	5.20	5.09	4.99
17	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75	5.58	5.32	5.13	4.99	4.87	4.78
18	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	5.39	5.13	4.94	4.80	4.68	4.59
19	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	5.22	4.97	4.78	4.64	4.52	4.43
20	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	5.08	4.82	4.64	4.49	4.38	4.29
21	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	4.95	4.70	4.51	4.37	4.26	4.17
22	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	4.83	4.58	4.40	4.26	4.15	4.06
23	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	4.73	4.48	4.30	4.16	4.05	3.96
24	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	4.64	4.39	4.21	4.07	3.96	3.87
25	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71	4.56	4.31	4.13	3.99	3.88	3.79
26	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.24	4.06	3.92	3.81	3.72
27	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.17	3.99	3.86	3.75	3.66
28	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.11	3.93	3.80	3.69	3.60
29	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.05	3.88	3.74	3.63	3.54
30	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.00	3.82	3.69	3.58	3.49
35	12.90	8.47	6.79	5.88	5.30	4.89	4.59	4.36	4.18	4.03	3.79	3.62	3.48	3.38	3.29
40	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.64	3.47	3.34	3.23	3.14
50	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	3.67	3.44	3.27	3.41	3.04	2.95
60	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	3.54	3.32	3.15	3.02	2.91	2.83
70	11.80	7.64	6.06	5.20	4.66	4.28	3.99	3.77	3.60	3.45	3.23	3.06	2.93	2.83	2.74
80	11.67	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.53	3.39	3.16	3.00	2.87	2.76	2.68
90	11.57	7.47	5.91	5.06	4.53	4.15	3.87	3.65	3.48	3.34	3.11	2.95	2.82	2.71	2.63
100	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	3.30	3.07	2.91	2.78	2.68	2.59
120	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.02	2.85	2.72	2.62	2.53
150	11.27	7.24	5.71	4.88	4.35	3.98	3.71	3.49	3.32	3.18	2.96	2.80	2.67	2.56	2.48
200	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26	3.12	2.90	2.74	2.61	2.51	2.42
250	11.09	7.10	5.59	4.77	4.25	3.88	3.61	3.40	3.23	3.09	2.87	2.71	2.58	2.48	2.39
300	11.04	7.07	5.56	4.75	4.22	3.86	3.59	3.38	3.21	3.07	2.85	2.69	2.56	2.46	2.37
400	10.99	7.03	5.53	4.71	4.19	3.83	3.56	3.35	3.18	3.04	2.82	2.66	2.53	2.43	2.34
500	10.96	7.00	5.51	4.69	4.18	3.81	3.54	3.33	3.16	3.02	2.81	2.64	2.52	2.41	2.33
600	10.94	6.99	5.49	4.68	4.16	3.80	3.53	3.32	3.15	3.01	2.80	2.63	2.51	2.40	2.32
750	10.91	6.97	5.48	4.67	4.15	3.79	3.52	3.31	3.14	3.00	2.78	2.62	2.49	2.39	2.31
1000	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	2.99	2.77	2.61	2.48	2.38	2.30

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (0.1% significance level)

ν_2 \ ν_1	25	30	35	40	50	60	75	100	150	200
1	6.24e05	6.26e05	6.28e05	6.29e05	6.30e05	6.31e05	6.32e05	6.33e05	6.35e05	6.35e05
2	999.46	999.47	999.47	999.47	999.48	999.48	999.49	999.49	999.49	999.49
3	125.84	125.45	125.17	124.96	124.66	124.47	124.27	124.07	123.87	123.77
4	45.70	45.43	45.23	45.09	44.88	44.75	44.61	44.47	44.33	44.26
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.80	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.96	7.93
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.37	5.30	5.24	5.17	5.10	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.70	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	4.00	3.93	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.86	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.43	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.18	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.21	2.11	2.05	1.98	1.91	1.83	1.79
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.03	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.80	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.45	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

Degrees of freedom	*Significance level* 5%	1%	0.1%
1	3.841	6.635	10.828
2	5.991	9.210	13.816
3	7.815	11.345	16.266
4	9.488	13.277	18.467
5	11.070	15.086	20.515
6	12.592	16.812	22.458
7	14.067	18.475	24.322
8	15.507	20.090	26.124
9	16.919	21.666	27.877
10	18.307	23.209	29.588

FORMULA SHEET

$$\widehat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i}.$$

$$\text{var}(\widehat{\tau}_{hh}) = \frac{1}{n}\sum_{i=1}^{N}p_i\left(\frac{y_i}{p_i}-\tau\right)^2$$

$$\widehat{\text{var}}(\widehat{\tau}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}p_i\left(\frac{y_i}{p_i}-\widehat{\tau}_{hh}\right)^2$$

$$\widehat{\mu}_{hh} = \frac{1}{Nn}\sum_{i=1}^{n}\frac{y_i}{p_i}$$

$$\widehat{\text{var}}(\widehat{\mu}_{hh}) = \frac{1}{N^2n(n-1)}\sum_{i=1}^{n}p_i\left(\frac{y_i}{p_i}-\widehat{\tau}_{hh}\right)^2$$

$$\widehat{\tau}_{hh} \pm z^*\sqrt{\text{var}(\widehat{\tau}_{hh})}$$

$$\widehat{\mu}_{hh} \pm z^*\sqrt{\text{var}(\widehat{\mu}_{hh})}$$

$$\widehat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\widehat{\mu}_{ht} = \frac{1}{N}\sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\text{var}(\widehat{\tau}_{ht}) = \sum_{i=1}^{N}\left(\frac{1}{\pi_i}-1\right)y_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\pi_{ij}}{\pi_i\pi_j}-1\right)y_iy_j$$

$$\widehat{\text{var}}(\widehat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2}-\frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j}-\frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\widehat{\text{var}}(\widehat{\mu}_{ht}) = \frac{1}{N^2}\widehat{\text{var}}(\widehat{\tau}_{ht})$$

$$\widehat{\text{var}}_{syg}(\widehat{\tau}_{ht}) = \sum^{\nu}\sum_{j>i}^{\nu}\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\left(\frac{\pi_i\pi_j}{\pi_{ij}}-1\right)$$

$$\widehat{\text{var}}(\hat{\tau}_{bh}) = \frac{N-\nu}{N}\frac{s_t^2}{\nu}$$

$$s_t^2 = \frac{1}{(\nu-1)}\sum_{i=1}^{\nu}(t_i - \hat{\tau}_{ht})^2$$

$$t_i = \frac{\nu y_i}{\pi_i}$$

$$\hat{\mu}_{ht} \pm z^*\sqrt{\widehat{var}(\hat{\mu}_{ht})}$$

$$\hat{\tau}_{ht} \pm z^*\sqrt{\widehat{var}(\hat{\tau}_{ht})}$$

$$\hat{\mu}_{ht} \pm t^*\sqrt{\widehat{var}(\hat{\mu}_{ht})}$$

$$\hat{\tau}_{ht} \pm t^*\sqrt{\widehat{var}(\hat{\tau}_{ht})}$$

$$\tau = \sum_{i=1}^{N} y_i$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} y_i = \tau/N$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \mu)^2$$

$$\sigma^2 = \left(\frac{1}{N-1}\right)\left(\sum_{i=1}^{N} y_i^2 - \frac{\tau^2}{N}\right) = \left(\frac{1}{N-1}\right)\left(\sum_{i=1}^{N} y_i^2 - N\mu^2\right)$$

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}\right)$$

$$\hat{\mu} = \overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\widehat{\tau}_{st} = \sum_{h=1}^{L} \widehat{\tau}_h = \sum_{h=1}^{L} N_h \overline{y}_h$$

$$\text{var}(\widehat{\mu}_h) = \left(\frac{N_h - n_h}{N_h}\right) \frac{\sigma_h^2}{n_h} \qquad \text{var}(\widehat{\tau}_h) = N_h(N_h - n_h)\frac{\sigma_h^2}{n_h}$$

$$\sigma_h^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \mu_h)^2$$

$$\text{var}(\widehat{\tau}_{st}) = \sum_{i=1}^{L} \text{var}(\widehat{\tau}_h) = \sum_{i=1}^{L} N_h(N_h - n_h)\frac{\sigma_h^2}{n_h}$$

$$\text{var}(\widehat{\mu}_{st}) = \left(\frac{1}{N^2}\right) \text{var}(\widehat{\tau}_{st}) = \left(\frac{1}{N^2}\right) \sum_{i=1}^{L} N_h(N_h - n_h)\frac{\sigma_h^2}{n_h}$$

$$\widehat{\text{var}}(\widehat{\tau}_{st}) = \sum_{h=1}^{L} N_h(N_h - n_h)\frac{s_h^2}{n_h} \qquad \widehat{\text{var}}(\widehat{\mu}_{st}) = \left(\frac{1}{N^2}\right) \sum_{h=1}^{L} N_h(N_h - n_h)\frac{s_h^2}{n_h}$$

$$\widehat{\mu}_{st} \pm z^* \sqrt{\widehat{\text{var}}(\widehat{\mu}_{st})} \qquad \widehat{\tau}_{st} \pm z^* \sqrt{\widehat{\text{var}}(\widehat{\tau}_{st})}$$

$$\widehat{\mu}_{st} \pm t^* \sqrt{\widehat{\text{var}}(\widehat{\mu}_{st})} \qquad \widehat{\tau}_{st} \pm t^* \sqrt{\widehat{\text{var}}(\widehat{\tau}_{st})}$$

$$d = \frac{\left(\sum_{h=1}^{L} a_h s_h^2\right)^2}{\sum_{h=1}^{L} (a_h s_h^2)^2/(n_h - 1)} = \frac{(\widehat{\text{var}}(\widehat{\tau}_{st}))^2}{\sum_{h=1}^{L} (a_h s_h^2)^2/(n_h - 1)}$$

$$n_h = \frac{(C - c_0) N_h \sigma_h / \sqrt{c_h}}{\sum_{h=1}^{L} N_h \sigma_h \sqrt{c_h}}$$

$$p_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi} \qquad \tau = \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{hi}$$

$$\widehat{p}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi} \qquad \widehat{p}_{st} = \sum_{h=1}^{L} \frac{N_h}{N} \widehat{p}_h = \sum_{h=1}^{L} W_h \widehat{p}_h$$

$$\text{var}(\widehat{p}_{st}) = \sum_{h=1}^{L} W_h^2 \,\text{var}(\widehat{p}_h) = \sum_{h=1}^{L} W_h^2 \left(\frac{N_h - n_h}{N_h}\right) \frac{\sigma_h^2}{n_h}$$

$$\widehat{\text{var}}(\widehat{p}_{st}) = \sum_{h=1}^{L} W_h^2 \left(\frac{N_n - n_h}{N_h}\right) \frac{s_h^2}{n_h} = \sum_{h=1}^{L} W_h^2 \frac{N_h - n_h}{N_h} \frac{\widehat{p}_h(1 - \widehat{p}_h)}{n_h - 1}$$

$$\widehat{p}_{st} \pm t^* \sqrt{\widehat{\text{var}}(\widehat{p}_{st})}$$

$$\widehat{\mu}_h = \frac{\overline{y}_h}{\overline{x}_h} \mu_{h(x)} = r_h \mu_{h(x)}$$

$$\widehat{\mu}_{sst} = \sum_{h=1}^{L} W_h r_h \mu_{h(x)} = \sum_{h=1}^{L} \frac{N_h}{N} \frac{\overline{y}_h}{\overline{x}_h} \mu_{h(x)}$$

$$\begin{aligned}
\widehat{\text{var}}(\widehat{\mu}_{sst}) &= \sum_{h=1}^{L} \left[\left(\frac{N_h}{N} \right)^2 \frac{N_h - n_h}{N_h n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_h^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2 r_h \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \\
&= \sum_{h=1}^{L} \left[W_h^2 \frac{1 - f_h}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_h^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2 r_h \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right]
\end{aligned}$$

$$\widehat{\mu}_{sst} \pm z^* \sqrt{\widehat{\text{var}}(\widehat{\mu}_{sst})} \qquad \widehat{\mu}_{sst} \pm t^* \sqrt{\widehat{\text{var}}(\widehat{\mu}_{sst})}$$

$$\overline{x}_{st} = \sum_{h=1}^{L} \frac{N_h}{N} \overline{x}_h \qquad \overline{y}_{st} = \sum_{h=1}^{L} \frac{N_h}{N} \overline{y}_h \qquad r_c = \frac{\overline{y}_{st}}{\overline{x}_{st}}$$

$$\begin{aligned}
\widehat{\text{var}}(\widehat{\mu}_{cst}) &= \sum_{h=1}^{L} \left[W_h^2 \frac{(1 - f_h)}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_c^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2 r_c \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right] \\
&= \sum_{h=1}^{L} \left[\left(\frac{N_h}{N} \right)^2 \frac{(1 - f_h)}{n_h (n_h - 1)} \left(\sum_{i=1}^{n_h} y_{hi}^2 + r_c^2 \sum_{i=1}^{n_h} x_{hi}^2 - 2 r_c \sum_{i=1}^{n_h} x_{hi} y_{hi} \right) \right]
\end{aligned}$$

$$\widehat{y} = a_h + b_h x$$

$$b_h = \frac{n_h \sum_{i=1}^{n_h} x_{hi} y_{hi} - (\sum_{i=1}^{n_h} x_{hi})(\sum_{i=1}^{n_h} y_{hi})}{n_h \sum_{i=1}^{n_h} x_{hi}^2 - (\sum_{i=1}^{n_h} x_{hi})^2}$$

$$\begin{aligned}
\widehat{\mu}_{Lh} &= a_h + b_h \mu_{h(x)} \\
&= \overline{y}_h + b_h (\mu_{h(x)} - \overline{x}_h)
\end{aligned}$$

$$\widehat{\mu}_{Lsst} = \sum_{h=1}^{L} \frac{N_h}{N}\left(a_h + b_h \mu_{h(x)}\right)$$

$$= \sum_{h=1}^{L} \frac{N_h}{N}\left(\overline{y}_h + b_h(\mu_{h(x)} - \overline{x}_h)\right)$$

$$\widehat{\text{var}}(\widehat{\mu}_{Lsst}) = \sum_{h=1}^{L}\left(\frac{N_h}{N}\right)^2 \frac{(1-f_h)}{n_h(n_h-2)} SSE_h = \sum_{h=1}^{L}\left(\frac{N_h}{N}\right)^2 \frac{(1-f_h)}{n_h} MSE_h$$

$$\widehat{\tau}_{Lsst} = N\widehat{\mu}_{Lsst} \qquad \widehat{\text{var}}(\widehat{\tau}_{Lsst}) = N^2 \widehat{\text{var}}(\widehat{\mu}_{Lsst})$$

$$\mu_{Lcst} = \widehat{\mu}_{st} + b_c(\mu_x - \overline{x}_{st}) \qquad b_c = \frac{\sum_{h=1}^{L} c_h b_h}{\sum_{h=1}^{L} c_h}$$

$$c_h = \left(\frac{N_h}{N}\right)^2 \frac{(1-f_h)}{n_h} s_{xh}^2$$

$$\mu_{Lcst} = \widehat{\mu}_{st} + b_c(\mu_x - \overline{x}_{st})$$

$$\widehat{\text{var}}(\mu_{Lcst}) = \sum_{h=1}^{L}\left[\left(\frac{N_h}{N}\right)^2 \frac{(1-f_h)}{n_h(n_h-2)} \sum_{i=1}^{n_h}[(y_{hi} - \overline{y}_h) - b_c(x_{hi} - \overline{x}_h)]^2\right]$$

$$\widehat{\mu}_{pst} = \frac{1}{N}\sum_{h=1}^{L} N_h \overline{y}_h \qquad \widehat{\tau}_{pst} = \sum_{h=1}^{L} N_h \overline{y}_h$$

$$\widehat{\text{var}}(\widehat{\mu}_{pst}) = \frac{N-n}{nN}\sum_{i=1}^{L}\left(\frac{N_h}{N}\right) s_h^2 + \frac{1}{n^2}\left(\frac{N-n}{N-1}\right)\sum_{i=1}^{L}\frac{N-N_h}{N} s_h^2$$

$$\widehat{\text{var}}(\widehat{\tau}_{pst}) = \frac{N-n}{n}\sum_{i=1}^{L} N_h s_h^2 + \left(\frac{N^2}{n^2}\right)\left(\frac{N-n}{N-1}\right)\sum_{i=1}^{L}\frac{N-N_h}{N} s_h^2$$

$$\overline{y}_i = \frac{y_i}{M_i} \qquad y_i = \sum_{j=1}^{M_i} y_{ij} \qquad s_u^2 = \frac{\sum_{i=1}^{n}(y_i - \overline{y})^2}{n-1}$$

$$\tau = \sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} = \sum_{i=1}^{N} y_i \qquad \mu = \frac{1}{M}\sum_{i=1}^{N}\sum_{j=1}^{M_i} y_{ij} = \frac{\tau}{M}$$

$$\mu_1 = \frac{1}{N}\sum_{i=1}^{N} y_i \qquad \sigma_u^2 = \frac{\sum_{i=1}^{N}(y_i - \mu_1)^2}{N-1}$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\overline{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\overline{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

$$\mathrm{var}(\hat{\tau}_{cl}) = N(N-n)\,\frac{\sigma_u^2}{n} \qquad \mathrm{var}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\,\frac{\sigma_u^2}{n}$$

$$\widehat{\mathrm{var}}(\hat{\tau}_{cl}) = N(N-n)\,\frac{s_u^2}{n} \qquad \widehat{\mathrm{var}}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\,\frac{s_u^2}{n}$$

$$\hat{\mu}_{cl} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{cl})} \qquad \hat{\tau}_{cl} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau}_{cl})}$$

$$\hat{\tau}_{sys} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\overline{y} \quad \hat{\mu}_{sys} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\overline{y}}{L} = \frac{\hat{\tau}_{sys}}{M}$$

$$\widehat{\mathrm{var}}(\hat{\tau}_{sys}) = N(N-n)\frac{s_u^2}{n} \qquad \widehat{\mathrm{var}}(\hat{\mu}_{sys}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

$$\hat{\mu}_{sys} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu}_{sys})} \qquad \hat{\tau}_{sys} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau}_{sys})}$$

$$\mu = \frac{\sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} M_i} = \frac{\sum_{i=1}^{N} y_i}{M} \qquad \mu = \left(\frac{N}{M}\right)\sum_{i=1}^{N}\frac{y_i}{N}$$

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m}$$

$$\widehat{\mathrm{var}}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2\,(\overline{y}_i - \hat{\mu}_{c(a)})^2$$

$$\widehat{\mathrm{var}}(\widehat{\mu}_{c(a)}) = \frac{(N-n)n}{N(n-1)} \sum_{i=1}^{n} \left(\frac{M_i}{m}\right)^2 (\overline{y}_i - \widehat{\mu}_{c(a)})^2$$

$$\widehat{\mu}_{c(b)} = \frac{N}{M} \frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM} \sum_{i=1}^{n} y_i$$

$$\widehat{\mathrm{var}}(\widehat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2} \sum_{i=1}^{n} (y_i - \overline{y})^2 = \frac{(N-n)N}{nM^2} s_u^2$$

$$\widehat{\mathrm{var}}(\widehat{\mu}_{c(b)}) = \frac{(N-n)n}{(n-1)Nm^2} \sum_{i=1}^{n} (y_i - \overline{y})^2 = \frac{(N-n)n}{Nm^2} s_u^2$$

$$\widehat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n}$$

$$\mathrm{var}(\widehat{p}_c) = \left(\frac{N-n}{nN}\right) \sum_{i=1}^{N} \frac{(p_i - p)^2}{N-1} = \left(\frac{1-f}{n}\right) \sum_{i=1}^{N} \frac{(p_i - p)^2}{N-1}$$

$$\widehat{\mathrm{var}}(\widehat{p}_c) = \left(\frac{N-n}{nN}\right) \sum_{i=1}^{n} \frac{(p_i - \widehat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right) \sum_{i=1}^{n} \frac{(p_i - \widehat{p}_c)^2}{n-1}$$

$$\widehat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i}$$

$$\mathrm{var}(\widehat{p}_c) \approx = \left(\frac{1-f}{n\overline{M}^2}\right) \frac{\sum_{i=1}^{N} (y_i - pM_i)^2}{N-1}$$

$$\widehat{\mathrm{var}}(\widehat{p}_c) \approx \left(\frac{1-f}{n\overline{m}^2}\right) \frac{\sum_{i=1}^{n} (y_i - p_c M_i)^2}{n-1}$$

$$= \left(\frac{1-f}{n\overline{m}^2}\right) \frac{\sum_{i=1}^{n} y_i^2 - 2p_c \sum_{i=1}^{n} y_i M_i + p_c^2 \sum_{i=1}^{n} M_i^2}{n-1}$$

$$\widehat{y}_i = \frac{M_i}{m_i} \sum_{j=1}^{m_i} y_{ij} = M_i \overline{y}_i \quad \widehat{\tau} = \frac{N}{n} \sum_{i=1}^{n} \widehat{y}_i \quad \widehat{\mu}_1 = \frac{\sum_{i=1}^{n} \widehat{y}_i}{n}$$

$$\mathrm{var}(\hat{\tau}) = N(N-n)\frac{\sigma_u^2}{n} + \frac{N}{n}\sum_{i=1}^{N} M_i(M_i - m_i)\frac{\sigma_i^2}{m_i}$$

$$\sigma_u^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \mu_1)^2 \quad \sigma_i^2 = \frac{1}{M_i - 1}\sum_{j=1}^{M_i}(y_{ij} - \mu_{i\cdot})^2$$

$$\widehat{\mathrm{var}}(\hat{\tau}) = N(N-n)\frac{s_u^2}{n} + \frac{N}{n}\sum_{i=1}^{n} M_i(M_i - m_i)\frac{s_i^2}{m_i}$$

$$s_u^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\hat{y}_i - \hat{\mu}_1)^2 \quad s_i^2 = \frac{1}{m_i - 1}\sum_{j=1}^{m_i}(y_{ij} - \overline{y}_i)^2$$

$$\hat{\tau} = \frac{M}{nl}\sum_{i=1}^{n}\sum_{j=1}^{l} y_{ij} = \frac{M}{n}\sum_{i=1}^{n}\overline{y}_i$$

$$\hat{\mu} = \frac{1}{nl}\sum_{i=1}^{n}\sum_{j=1}^{l} y_{ij} = \frac{1}{n}\sum_{i=1}^{n}\overline{y}_i$$

$$\widehat{\mathrm{var}}(\hat{\tau}) = N(N-n)\frac{s_u^2}{n} + \frac{M(L-l)}{nl}\sum_{i=1}^{n} s_i^2$$

$$\widehat{\mathrm{var}}(\hat{\mu}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n} + \frac{L-l}{Mnl}\sum_{i=1}^{n} s_i^2$$

$$\hat{\tau} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\tau})} \qquad \hat{\mu} \pm t^*\sqrt{\widehat{\mathrm{var}}(\hat{\mu})}$$