

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2008

TITLE OF PAPER : DESCRIPTIVE/INFERENTIAL STATISTICS

COURSE CODE : ST230/IDE-ST230-1&2

TIME ALLOWED : 3 (THREE) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ALL THREE (3) QUESTIONS IN
SECTION ONE & ANY FOUR (3) QUESTIONS
IN SECTION TWO. ALL QUESTIONS CARRY
MARKS AS INDICATED WITHIN THE
PARENTHESIS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

SECTION ONE

ANSWER ALL QUESTIONS:

QUESTION ONE.

[20 marks]

State the most correct answer for each of the following:

- 1.1 Which one of the following location measures is not affected by outliers?
 - a. Mean
 - b. Median
 - c. Mode
 - d. Both (b) and (c)
- 1.2 When a distribution is positively skewed, the relationship of the mean, median, and mode from left to right will be
 - a. mean, median, mode
 - b. median, mode, mean
 - c. mean, mode, median
 - d. mode, median, mean
- 1.3 Which of the following statements is true in a Bar Chart?
 - a. Height of the bar is proportional to the frequency
 - b. Width of the bar is arbitrary
 - c. There are equal gaps between bars
 - d. All of the above
- 1.4 What is the value of the mode when all values in the data set are different?
 - a. One.
 - b. Zero.
 - c. There is no mode.
 - d. It cannot be determined unless the data values are given.
- 1.5 The width of the bar in a histogram is proportional to
 - a. cumulative frequency
 - b. frequency
 - c. class interval
 - d. none of the above
- 1.6 The Plumbers Union consists of 1,020 members. A representative group of 48 members were selected and asked questions. The 48 is considered
 - a. the population.
 - b. the parameter.
 - c. a statistic.
 - d. a sample.

- 1.7 Which measure of central location should not be used when a distribution is highly skewed?
- mean
 - mode
 - median
 - All of the above
- 1.8 When data are categorized as, for example, places of residence (rural, suburban, urban), the most appropriate measure of central location is
- mean
 - mode
 - median
 - It cannot be determined unless the data values are given.
- 1.9 Which measure of dispersion can be negative?
- variance.
 - quartile deviation.
 - Both (a) and (b).
 - Neither (a) nor (b).
- 1.10 Which of the following is a discrete categorical random variable?
- Students' major subject
 - Number of Children in a family
 - Number of car sold per month
 - Age of students in UNISWA
- 1.11 Which of the following properties is not true about any derived probability?
- A probability value lies only between 0 and 1.
 - If an event A is certain to occur, then $P(A) = 1$.
 - The sum of probabilities of the set of all exhaustive events cannot exceed one.
 - All of the above.
- 1.12 When two statistically independent events occur together;
- then one event have an influence over the other event.
 - then one event does not have an influence over the other event.
 - then one event may or may not have an influence over the other event.
 - It is not possible to occur together.
- 1.13 Which one is not a property of the normal probability distribution?
- It is smooth bell-shaped curve.
 - It is symmetrical about the central mean value.
 - The tails of the curve are non-asymptotic.
 - All of the above.

- 1.14 A study of absenteeism from the classroom is being conducted. It was found that 106 students were absent from Monday morning classes. This number 106 is called
 - a. an outcome.
 - b. an event.
 - c. a statistic.
 - d. The study does not have complete information to say about the number.
- 1.15 There are five vacant parking places. Five automobiles arrive at the same time. How many different ways they can park?
 - a. 5
 - b. 25
 - c. 120
 - d. 5^5
- 1.16 The value of the Chi-square test statistics cannot be
 - a. zero.
 - b. negative.
 - c. positive.
 - d. determined unless the data values are given.
- 1.17 When a 99% confidence interval is calculated instead of a 95% confidence interval without changing the sample size, the maximum error of the estimate will
 - a. be smaller.
 - b. be larger.
 - c. remain same.
 - d. not be possible to determine.
- 1.18 When the value of α is increased, the probability of committing a type I error
 - a. is decreased.
 - b. is increased.
 - c. Remains unchanged.
 - d. None of the above.
- 1.19 How many outcomes are there in a binomial experiment?
 - a. 1.
 - b. 2.
 - c. 3.
 - d. It varies.
- 1.20 In the standard normal distribution, the area outside the range $Z = -1.0$ to $Z = +1.5$ is:
 - a. 0.7745
 - b. 0.1587
 - c. 0.2255
 - d. 0.0668

QUESTION THREE.

[10 marks]

State which of the following statements are **TRUE** and which are **FALSE**? Write the appropriate answer (complete word **TRUE** or **FALSE**, not **T** or **F**).

- 2.1 The range provides information about the clustering of data values between minimum and maximum data values.
- 2.2 The standard deviation is not distorted by extreme values in the data.
- 2.3 A frequency distribution is positively skewed when there are a few extremely large data values relative to other data values in the sample.
- 2.4 An index number measures percentage changes from a base period.
- 2.5 Trend analysis is the statistical technique used to isolate underlying long-term movement.
- 2.6 The original time series is a smoother series than a moving average time series values.
- 2.7 The nominal-scaled is the strongest form of data to analyse.
- 2.8 If a person's score on an exam corresponds to the 90th percentile, it means that only 10 students scored more than that person.
- 2.9 The mode is a valid measure of central tendency for all data type.
- 2.10 A frequency polygon may be constructed by connecting the midpoints of the bars of a histogram by straight line.
- 2.11 Classical probability uses a frequency distribution to compute probabilities.
- 2.12 The total number of outcomes in tossing three coins experiment is 6.
- 2.13 A binomial experiment has a fixed number of trials.
- 2.14 In the standard normal distribution, the area to the right plus the area to the left of a Z-score is equal to one.
- 2.15 For a specific confidence interval, the larger the sample size, the smaller the maximum error of estimate will be.
- 2.16 The test values for the chi-square goodness of fit test and the independent test are computed using the same formula.
- 2.17 The degrees of freedom for a 4X3 contingency table are 12.
- 2.18 The range ± 1.96 standard errors give the range within which 95% sample results can be expected to lie.
- 2.19 If the value of the Chi-square Test Statistics is zero, the null hypothesis is true.
- 2.20 The Poisson probability distribution deals with experiments that have only two possible outcomes, a success or a failure.

QUESTION THREE.

[2 + 2 + 2 + 2 + 2 marks]

On a very hot summer day, 10 percent of the production employees at Gulf Steel Company are absent from work. Ten production employees are to be selected at random for a special in-depth study on absenteeism.

- 3.1 What is the random variable in this problem?
- 3.2 Is the random variable discrete or continuous? Why?
- 3.3 What is the probability of selecting 10 production employees at random on a hot summer day and finding that none of them is absent?
- 3.4 Find the average number of employees are absent on a hot summer day.
- 3.5 Which probability distribution represents this type of problem? Why?

SECTION TWO**ANSWER ANY FOUR QUESTIONS:**

(You must show all of your works in order to obtain full marks)

QUESTION FOUR.

[2 + 2 + 2 + 2 + 2 + 5 marks]

- 4.1 A company offer each of its employees a choice of three performance bonus options: a cash bonus option; a profit sharing options; and a share options. The number of employees who selected each bonus option together with their work function is shown in the following table:

Work Function	Bonus Options		
	Cash Bonus	Profit-sharing	Share Options
Admin	28	44	68
Production	56	75	29

- a. What is the probability that an employee selected a cash bonus?
 - b. If income tax must be paid on the cash bonus or the profit sharing option, what is the probability that an employee selected a tax-free bonus option?
 - c. What is the chance that an employee works in production and chose the share option?
 - d. If an employee is in administration, what is the probability that the employee chose the cash bonus option?
 - e. If event **A** = *share option* and event **B** = *an administration employee*, check whether the choice of bonus options is independent of the work function of the employee.
- 4.2 Is there a statistical association between the bonus options and the work function of the employee in the table given in question 4.1? Test at 5% level of significance.

QUESTION FIVE.

[10 + 5 marks]

- 5.1 A motorcycle dealer has recorded the unit prices and quantities sold of three models of a specific type of motorcycle for 2005 and 2006. The quantities sold and unit selling prices for both these years are given in the following table (use 2005 as the base period):

Motorcycle Model	2005		2006	
	Unit Price (in E1000)	Quantity (unit sold)	Unit Price (in E1000)	Quantity (unit sold)
Luxury	25	10	30	7
Executive	15	55	19	58
Standard	12	32	14	40

Compute Laspeyres's price and quantity index for 2006 and interpret both indices for the motorcycle dealer.

- 5.2 The monthly rentals per square metre for office space in 30 buildings (in Emalangen) are:

189	156	250	265	195	300
350	315	290	285	165	178
415	280	212	580	395	360
285	225	230	450	185	193
580	248	460	250	520	300

Present the above data by an appropriate graph and Comment on the shape of the distribution.

QUESTION SIX.

[2 + 3 + 3 + 3 + 3 + 1 marks]

Eighty packages have been randomly selected from a frozen food warehouse, and the age of each package is identified:

Age (in weeks)	Number of packages
0 – under 10	25
10 – under 20	17
20 – under 30	15
30 – under 40	9
40 – under 50	10
50 – under 60	4

- 6.1 Calculate the value of the mean age.
- 6.2 Calculate the value of the median age.
- 6.3 Calculate the variance for the ages of the packages.
- 6.4 Determine the lower quartile age.
- 6.5 75% of the packages are under certain weeks old. What is the maximum age of those packages?
- 6.6 Calculate the quartile deviation of the above data.

QUESTION SEVEN.

[2 + 2 + 3 + 3 + 5 marks]

- 7.1 The manager of a local gym has determined that the *length of time* patrons spend at the gym is a normally distributed variable with a mean of 80 minutes and a standard deviation of 20 minutes.
- What proportion of patrons spend more than two hours at the gym?
 - What proportion of patrons spend less than one hour at the gym?
 - What is the least amount of time spent by 60% of the patrons at the gym?
- 7.2 The average age of passenger cars in use in South Africa is 7.0 years. For a simple random sample of 34 vehicles observed in the employee parking area of a large manufacturing plant, the average age is 8.3 years, with a standard deviation of 3.1 years.
- Construct a 99% confidence interval for the mean age of passenger cars of the plant's employees.
 - Using the 10% level of significance, can we conclude that the average age of cars driven to work by the plant's employees is greater than the national average? Explain your decision

QUESTION EIGHT.

[1 + 4 + 2 + 3 + 5 marks]

- 8.1 The training manager of a company that assembles and exports pool pumps wants to know if there is a link between the number of hours spent by assembly workers in training and their productivity (average daily output) on the job. A random sample of 10 assembly workers was selected and their performances evaluated and the following results were computed from those data:

$$\sum x = 311, \quad \sum y = 535, \quad \sum xy = 17240, \quad \sum x^2 = 10213, \quad \text{and} \quad \sum y^2 = 29649$$

- Identify the dependent variable (y) and the independent variable (x).
 - Find the best fitted regression line.
 - Estimate the average daily output of an assembly worker who has received 25 hours of training.
 - Compute the value of the coefficient of correlation. Interpret its meaning and advise the training manager.
- 8.2 An association of coffee producers has estimated that 70% of adult workers drink coffee at least occasionally while on the job. If 5 adult workers are randomly selected, what is the probability that no more than 3 of them drink coffee at least occasionally while on the job? At least 2 of them drink coffee at least occasionally while on the job?

QUESTION NINE.

[7 + 8 marks]

The number of claims per quarter on household policies submitted to an insurance company is as follows:

Year	Q1	Q2	Q3	Q4
2003	84	53	60	75
2004	81	57	51	73
2005	69	37	40	77
2006	73	46	39	63

9.1 Find the trend for the above data.

9.2 De-seasonalise the claims data.

QUESTION TEN.

[4 + 5 + 1 + 2 + 3 marks]

10.1 Medical researchers monitoring two groups of physicians over a 6-year period found that, of 3429 doctors who took aspirin daily, 148 died from heart attack or stroke during this period. For 1710 doctors who received a placebo instead of aspirin, 79 deaths were recorded.

- Construct an interval estimate for the difference between two proportions of deaths with a confidence level of 95%.
- At the 0.01 level of significance, does this study indicate that taking aspirin is effective in reducing the likelihood of a heart attack? State, clearly, all the steps required from hypothesis to conclusion.
- Will you suggest the heart disease patients to take aspirin daily? Explain.

10.2 A motor spares dealer sells, on average, 4 car batteries per week.

- What is the probability that the dealer will sell exactly 2 batteries in the current week?
- If the dealer has 3 batteries in stock at the beginning of a given week, what is the probability that the dealer will run out of stock in that week?

a	p																a
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99				
0	.818	.358	.122	.012	.001	.000	.000	.000	.000	.000	.000	.000	.000	0			
1	.983	.736	.392	.069	.008	.001	.000	.000	.000	.000	.000	.000	.000	1			
2	.999	.925	.677	.206	.035	.004	.000	.000	.000	.000	.000	.000	.000	2			
3	1.000	.984	.867	.411	.107	.016	.001	.000	.000	.000	.000	.000	.000	3			
4	1.000	.997	.957	.630	.238	.051	.006	.000	.000	.000	.000	.000	.000	4			
5	1.000	1.000	.989	.804	.416	.126	.021	.002	.000	.000	.000	.000	.000	5			
6	1.000	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000	.000	6			
7	1.000	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000	.000	7			
8	1.000	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000	.000	8			
9	1.000	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000	.000	9			
10	1.000	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000	.000	10			
11	1.000	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000	.000	11			
12	1.000	1.000	1.000	1.000	1.000	.999	.868	.584	.228	.032	.000	.000	.000	12			
13	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000	.000	13			
14	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000	.000	14			
15	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003	.000	15			
16	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016	.000	16			
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075	.001	17			
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264	.017	18			
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642	.182	19			

(c) $n = 25$

a	p																a
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.99				
0	.778	.277	.072	.004	.000	.000	.000	.000	.000	.000	.000	.000	.000	0			
1	.974	.642	.271	.027	.002	.000	.000	.000	.000	.000	.000	.000	.000	1			
2	.998	.873	.537	.098	.009	.000	.000	.000	.000	.000	.000	.000	.000	2			
3	1.000	.966	.764	.234	.033	.002	.000	.000	.000	.000	.000	.000	.000	3			
4	1.000	.993	.902	.421	.090	.009	.000	.000	.000	.000	.000	.000	.000	4			
5	1.000	.999	.967	.617	.193	.029	.002	.000	.000	.000	.000	.000	.000	5			
6	1.000	1.000	.991	.780	.341	.074	.007	.000	.000	.000	.000	.000	.000	6			
7	1.000	1.000	.998	.891	.512	.154	.022	.001	.000	.000	.000	.000	.000	7			
8	1.000	1.000	1.000	.953	.677	.274	.054	.004	.000	.000	.000	.000	.000	8			
9	1.000	1.000	1.000	.983	.811	.425	.115	.013	.000	.000	.000	.000	.000	9			
10	1.000	1.000	1.000	.994	.902	.586	.212	.034	.002	.000	.000	.000	.000	10			
11	1.000	1.000	1.000	.998	.956	.732	.345	.078	.006	.000	.000	.000	.000	11			
12	1.000	1.000	1.000	1.000	.983	.846	.500	.154	.017	.000	.000	.000	.000	12			
13	1.000	1.000	1.000	1.000	.994	.922	.655	.268	.044	.002	.000	.000	.000	13			
14	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.006	.000	.000	.000	14			
15	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.017	.000	.000	.000	15			
16	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.047	.000	.000	.000	16			
17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.109	.002	.000	17			
18	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.220	.009	.000	.000	18			
19	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.383	.033	.001	.000	19			
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.579	.098	.007	.000	20			
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.766	.236	.034	.000	21			
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.902	.463	.127	.002	22			
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.973	.729	.358	.026	23			
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.928	.723	.222	24			

Table 1. Binomial Probabilities

Tabulated values are $P(Y \leq a) = \sum_{y=0}^a p(y)$. (Computations are rounded at third decimal place.)

(a) $n = 5$

a	p										a
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	
0	.951	.774	.590	.328	.168	.078	.031	.010	.002	.000	0
1	.999	.977	.919	.737	.528	.337	.188	.087	.031	.007	1
2	1.000	.999	.991	.942	.837	.683	.500	.317	.163	.058	2
3	1.000	1.000	1.000	.993	.969	.913	.812	.663	.472	.263	3
4	1.000	1.000	1.000	1.000	.998	.990	.969	.922	.832	.672	4

(b) $n = 10$

a	p										a
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	0
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	1
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	2
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	3
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	4
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	5
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	6
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	7
8	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.624	8
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	9

(c) $n = 15$

a	p										a
	0.01	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	
0	.860	.463	.206	.035	.005	.000	.000	.000	.000	.000	0
1	.990	.829	.549	.167	.035	.005	.000	.000	.000	.000	1
2	1.000	.964	.816	.398	.127	.027	.004	.000	.000	.000	2
3	1.000	.995	.944	.648	.297	.091	.018	.002	.000	.000	3
4	1.000	.999	.987	.836	.515	.217	.059	.009	.001	.000	4
5	1.000	1.000	.998	.939	.722	.403	.151	.034	.004	.000	5
6	1.000	1.000	1.000	.982	.869	.610	.304	.095	.015	.001	6
7	1.000	1.000	1.000	.996	.950	.787	.500	.213	.050	.004	7
8	1.000	1.000	1.000	.999	.985	.905	.696	.390	.131	.018	8
9	1.000	1.000	1.000	1.000	.996	.966	.849	.597	.278	.061	9
10	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.164	10
11	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.352	11
12	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.652	.184	12
13	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.833	.451	13
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.794	14

Table 2. Table of e^{-x}

x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0	1.000000	2.60	.074274	5.10	.006097
0.10	.904837	2.70	.067206	5.20	.005517
0.20	.818731	2.80	.060810	5.30	.004992
0.30	.740818	2.90	.055023	5.40	.004517
0.40	.670320	3.00	.049787	5.50	.004087
0.50	.606531	3.10	.045049	5.60	.003698
0.60	.548812	3.20	.040762	5.70	.003346
0.70	.496585	3.30	.036883	5.80	.003028
0.80	.449329	3.40	.033373	5.90	.002739
0.90	.406570	3.50	.030197	6.00	.002479
1.00	.367879	3.60	.027324	6.10	.002243
1.10	.332871	3.70	.024724	6.20	.002029
1.20	.301194	3.80	.022371	6.30	.001836
1.30	.272532	3.90	.020242	6.40	.001661
1.40	.246597	4.00	.018316	6.50	.001503
1.50	.223130	4.10	.016573	6.60	.001360
1.60	.201897	4.20	.014996	6.70	.001231
1.70	.182684	4.30	.013569	6.80	.001114
1.80	.165299	4.40	.012277	6.90	.001008
1.90	.149569	4.50	.011109	7.00	.000912
2.00	.135335	4.60	.010052	7.10	.000825
2.10	.122456	4.70	.009095	7.20	.000747
2.20	.110803	4.80	.008230	7.30	.000676
2.30	.100259	4.90	.007447	7.40	.000611
2.40	.090718	5.00	.006738	7.50	.000553
2.50	.082085				

Table 3. Poisson Probabilities

$$P(Y \leq a) = \sum_{y=0}^a e^{-\lambda} \frac{\lambda^y}{y!}$$

λ	0	1	2	3	4	5	6	7	8	9
0.0	0.02	0.980	1.000							
0.04	0.04	0.961	0.999	1.000						
0.06	0.06	0.942	0.998	1.000						
0.08	0.08	0.923	0.997	1.000						
0.10	0.10	0.905	0.995	1.000						
0.15	0.15	0.861	0.990	0.999	1.000					
0.20	0.20	0.819	0.982	0.999	1.000					
0.25	0.25	0.779	0.974	0.998	1.000					
0.30	0.30	0.741	0.963	0.996	1.000					
0.35	0.35	0.705	0.951	0.994	1.000					
0.40	0.40	0.670	0.938	0.992	0.999	1.000				
0.45	0.45	0.638	0.925	0.989	0.999	1.000				
0.50	0.50	0.607	0.910	0.986	0.998	1.000				
0.55	0.55	0.577	0.894	0.982	0.988	1.000				
0.60	0.60	0.549	0.878	0.977	0.997	1.000				
0.65	0.65	0.522	0.861	0.972	0.996	0.999	1.000			
0.70	0.70	0.497	0.844	0.966	0.994	0.999	1.000			
0.75	0.75	0.472	0.827	0.959	0.993	0.999	1.000			
0.80	0.80	0.449	0.809	0.953	0.991	0.999	1.000			
0.85	0.85	0.427	0.791	0.945	0.989	0.998	1.000			
0.90	0.90	0.407	0.772	0.937	0.987	0.998	1.000			
0.95	0.95	0.387	0.754	0.929	0.981	0.997	1.000			
1.00	1.00	0.368	0.736	0.920	0.981	0.996	0.999	1.000		
1.1	1.1	0.333	0.699	0.900	0.974	0.995	0.999			
1.2	1.2	0.301	0.663	0.879	0.966	0.992	0.998	1.000		
1.3	1.3	0.273	0.627	0.857	0.957	0.989	0.998	1.000		
1.4	1.4	0.247	0.592	0.833	0.946	0.986	0.997	0.999	1.000	
1.5	1.5	0.223	0.558	0.809	0.934	0.981	0.996	0.999	1.000	
1.6	1.6	0.202	0.525	0.783	0.921	0.976	0.994	0.999	1.000	
1.7	1.7	0.183	0.493	0.757	0.907	0.970	0.992	0.998	1.000	
1.8	1.8	0.165	0.463	0.731	0.891	0.964	0.990	0.997	0.999	1.000
1.9	1.9	0.150	0.434	0.704	0.875	0.956	0.987	0.997	0.999	1.000
2.0	2.0	0.135	0.406	0.677	0.857	0.947	0.983	0.995	0.999	1.000

Table 3. (Continued)

λ	α	0	1	2	3	4	5	6	7	8	9
6.2	0.002	0.015	0.054	0.134	0.259	0.414	0.574	0.716	0.826	0.902	
6.4	0.002	0.012	0.046	0.119	0.235	0.384	0.542	0.687	0.803	0.886	
6.6	0.001	0.010	0.040	0.105	0.213	0.355	0.511	0.658	0.780	0.869	
6.8	0.001	0.009	0.034	0.093	0.192	0.327	0.480	0.628	0.755	0.850	
7.0	0.001	0.007	0.030	0.082	0.173	0.301	0.450	0.599	0.729	0.830	
7.2	0.001	0.006	0.025	0.072	0.156	0.276	0.420	0.569	0.703	0.810	
7.4	0.001	0.005	0.022	0.063	0.140	0.253	0.392	0.539	0.676	0.788	
7.6	0.001	0.004	0.019	0.055	0.125	0.231	0.365	0.510	0.648	0.765	
7.8	0.000	0.004	0.016	0.048	0.112	0.210	0.338	0.481	0.620	0.741	
8.0	0.000	0.003	0.014	0.042	0.100	0.191	0.313	0.453	0.593	0.717	
8.5	0.000	0.002	0.009	0.030	0.074	0.150	0.256	0.386	0.523	0.653	
9.0	0.000	0.001	0.006	0.021	0.055	0.116	0.207	0.324	0.456	0.587	
9.5	0.000	0.001	0.004	0.015	0.040	0.089	0.165	0.269	0.392	0.522	
10.0	0.000	0.000	0.003	0.010	0.029	0.067	0.130	0.220	0.333	0.458	
10	0.949	0.975	0.989	0.995	0.998	0.999	1.000				
6.2	0.949	0.975	0.989	0.995	0.998	0.999	1.000				
6.4	0.939	0.969	0.986	0.994	0.997	0.999	1.000				
6.6	0.927	0.963	0.982	0.992	0.997	0.999	1.000				
6.8	0.915	0.955	0.978	0.990	0.996	0.998	0.999	1.000			
7.0	0.901	0.947	0.973	0.987	0.994	0.998	0.999	1.000			
7.2	0.887	0.937	0.967	0.984	0.993	0.997	0.999	1.000			
7.4	0.871	0.926	0.961	0.980	0.991	0.996	0.998	0.999	1.000		
7.6	0.854	0.915	0.954	0.976	0.989	0.995	0.998	0.999	1.000		
7.8	0.835	0.902	0.945	0.971	0.986	0.993	0.997	0.999	1.000		
8.0	0.816	0.888	0.936	0.966	0.983	0.992	0.996	0.998	0.999	1.000	
8.5	0.763	0.849	0.909	0.949	0.973	0.986	0.993	0.997	0.999	0.999	
9.0	0.706	0.803	0.876	0.926	0.959	0.978	0.989	0.995	0.998	0.999	
9.5	0.645	0.752	0.836	0.898	0.940	0.967	0.982	0.991	0.996	0.998	
10.0	0.583	0.697	0.792	0.864	0.917	0.951	0.973	0.986	0.993	0.997	
20	21	22									
8.5	1.000										
9.0	1.000										
9.5	0.999	1.000									
10.0	0.998	0.999	1.000								

Table 3. (Continued)

λ	α	0	1	2	3	4	5	6	7	8	9
2.2	0.111	0.355	0.623	0.819	0.928	0.975	0.993	0.998	1.000		
2.4	0.091	0.308	0.570	0.779	0.904	0.964	0.988	0.997	0.999	1.000	
2.6	0.074	0.267	0.518	0.736	0.877	0.951	0.983	0.995	0.999	1.000	
2.8	0.061	0.231	0.469	0.692	0.848	0.935	0.976	0.992	0.998	0.999	
3.0	0.050	0.199	0.423	0.647	0.815	0.916	0.966	0.988	0.996	0.999	
3.2	0.041	0.171	0.380	0.603	0.781	0.895	0.955	0.983	0.994	0.998	
3.4	0.033	0.147	0.340	0.558	0.744	0.871	0.942	0.977	0.992	0.997	
3.6	0.027	0.126	0.303	0.515	0.706	0.844	0.927	0.969	0.988	0.996	
3.8	0.022	0.107	0.269	0.473	0.668	0.816	0.909	0.960	0.984	0.994	
4.0	0.018	0.092	0.238	0.433	0.629	0.785	0.889	0.949	0.979	0.992	
4.2	0.015	0.078	0.210	0.395	0.590	0.753	0.867	0.936	0.972	0.989	
4.4	0.012	0.066	0.185	0.359	0.551	0.720	0.844	0.921	0.964	0.985	
4.6	0.010	0.056	0.163	0.326	0.513	0.686	0.818	0.905	0.955	0.980	
4.8	0.008	0.048	0.143	0.294	0.476	0.651	0.791	0.887	0.944	0.975	
5.0	0.007	0.040	0.125	0.265	0.440	0.616	0.762	0.867	0.932	0.968	
5.2	0.006	0.034	0.109	0.238	0.406	0.581	0.732	0.845	0.918	0.960	
5.4	0.005	0.029	0.095	0.213	0.373	0.546	0.702	0.822	0.903	0.951	
5.6	0.004	0.024	0.082	0.191	0.342	0.512	0.670	0.797	0.886	0.941	
5.8	0.003	0.021	0.072	0.170	0.313	0.478	0.638	0.771	0.867	0.929	
6.0	0.002	0.017	0.062	0.151	0.285	0.446	0.606	0.744	0.847	0.916	
10	1.000										
3.0	1.000										
3.2	1.000										
3.4	0.999	1.000									
3.6	0.999	1.000									
3.8	0.998	0.999	1.000								
4.0	0.997	0.999	1.000								
4.2	0.996	0.999	1.000								
4.4	0.994	0.998	0.999	1.000							
4.6	0.992	0.997	0.999	1.000							
4.8	0.990	0.996	0.999	1.000							
5.0	0.986	0.995	0.998	0.999	1.000						
5.2	0.982	0.993	0.997	0.999	1.000						
5.4	0.977	0.990	0.996	0.999	1.000						
5.6	0.972	0.988	0.995	0.998	0.999	1.000					
5.8	0.965	0.984	0.993	0.997	0.999	1.000					
6.0	0.957	0.980	0.991	0.996	0.999	0.999	1.000				

Table 3. (Continued)

λ	0	1	2	3	4	5	6	7	8	9
10.5	0.000	0.000	0.002	0.007	0.021	0.050	0.102	0.179	0.279	0.397
11.0	0.000	0.000	0.001	0.005	0.015	0.038	0.079	0.143	0.232	0.341
11.5	0.000	0.000	0.001	0.003	0.011	0.028	0.060	0.114	0.191	0.289
12.0	0.000	0.000	0.001	0.002	0.008	0.020	0.046	0.090	0.155	0.242
12.5	0.000	0.000	0.000	0.002	0.005	0.015	0.035	0.070	0.125	0.201
13.0	0.000	0.000	0.000	0.001	0.004	0.011	0.026	0.054	0.100	0.166
13.5	0.000	0.000	0.000	0.001	0.003	0.008	0.019	0.041	0.079	0.135
14.0	0.000	0.000	0.000	0.000	0.002	0.006	0.014	0.032	0.062	0.109
14.5	0.000	0.000	0.000	0.000	0.001	0.004	0.010	0.024	0.048	0.088
15.0	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.018	0.037	0.070
10	0.521	0.639	0.742	0.825	0.888	0.932	0.960	0.978	0.988	0.994
10.5	0.521	0.639	0.742	0.825	0.888	0.932	0.960	0.978	0.988	0.994
11.0	0.460	0.579	0.689	0.781	0.854	0.907	0.944	0.968	0.982	0.991
11.5	0.402	0.520	0.633	0.733	0.815	0.878	0.924	0.954	0.974	0.986
12.0	0.347	0.462	0.576	0.682	0.772	0.844	0.899	0.937	0.963	0.979
12.5	0.297	0.406	0.519	0.628	0.725	0.806	0.869	0.916	0.948	0.969
13.0	0.252	0.353	0.463	0.573	0.675	0.764	0.835	0.890	0.930	0.957
13.5	0.211	0.304	0.409	0.518	0.623	0.718	0.798	0.861	0.908	0.942
14.0	0.176	0.260	0.358	0.464	0.570	0.669	0.756	0.827	0.883	0.923
14.5	0.145	0.220	0.311	0.413	0.518	0.619	0.711	0.790	0.853	0.901
15.0	0.118	0.185	0.268	0.363	0.466	0.568	0.664	0.749	0.819	0.875
20	0.997	0.999	0.999	1.000						
10.5	0.997	0.999	0.999	1.000						
11.0	0.995	0.998	0.999	1.000						
11.5	0.992	0.996	0.998	0.999	1.000					
12.0	0.988	0.994	0.997	0.999	0.999	1.000				
12.5	0.983	0.991	0.995	0.998	0.999	0.999	1.000			
13.0	0.975	0.986	0.992	0.996	0.998	0.999	1.000			
13.5	0.965	0.980	0.989	0.994	0.997	0.998	0.999	1.000		
14.0	0.952	0.971	0.983	0.991	0.995	0.997	0.999	0.999	1.000	
14.5	0.936	0.960	0.976	0.986	0.992	0.996	0.998	0.999	0.999	1.000
15.0	0.917	0.947	0.967	0.981	0.989	0.994	0.997	0.998	0.999	1.000

Table 3. (Continued)

λ	14	15	16	17	18	19	20	21	22	23
16	0.368	0.467	0.566	0.659	0.742	0.812	0.868	0.911	0.942	0.963
17	0.281	0.371	0.468	0.564	0.655	0.736	0.805	0.861	0.905	0.937
18	0.208	0.287	0.375	0.469	0.562	0.651	0.731	0.799	0.855	0.899
19	0.150	0.215	0.292	0.378	0.469	0.561	0.647	0.725	0.793	0.849
20	0.105	0.157	0.221	0.297	0.381	0.470	0.559	0.644	0.721	0.787
21	0.072	0.111	0.163	0.227	0.302	0.384	0.471	0.558	0.640	0.716
22	0.048	0.077	0.117	0.169	0.232	0.306	0.387	0.472	0.556	0.637
23	0.031	0.052	0.082	0.123	0.175	0.238	0.310	0.389	0.472	0.555
24	0.020	0.034	0.056	0.087	0.128	0.180	0.243	0.314	0.392	0.473
25	0.012	0.022	0.038	0.060	0.092	0.134	0.185	0.247	0.318	0.394
24	0.978	0.987	0.993	0.996	0.998	0.999	0.999	1.000		
16	0.978	0.987	0.993	0.996	0.998	0.999	0.999	1.000		
17	0.959	0.975	0.985	0.991	0.995	0.997	0.999	0.999	1.000	
18	0.932	0.955	0.972	0.983	0.990	0.994	0.997	0.998	0.999	1.000
19	0.893	0.927	0.951	0.969	0.980	0.988	0.993	0.996	0.998	0.999
20	0.843	0.888	0.922	0.948	0.966	0.978	0.987	0.992	0.995	0.997
21	0.782	0.838	0.883	0.917	0.944	0.963	0.976	0.985	0.991	0.994
22	0.712	0.777	0.832	0.877	0.913	0.940	0.959	0.973	0.983	0.989
23	0.635	0.708	0.772	0.827	0.873	0.908	0.936	0.956	0.971	0.981
24	0.554	0.632	0.704	0.768	0.823	0.868	0.904	0.932	0.953	0.969
25	0.473	0.553	0.629	0.700	0.763	0.818	0.863	0.900	0.929	0.950
34	0.999	1.000								
19	0.999	1.000								
20	0.999	0.999	1.000							
21	0.997	0.998	0.999	0.999	1.000					
22	0.994	0.996	0.998	0.999	0.999	1.000				
23	0.988	0.993	0.996	0.997	0.999	0.999	1.000			
24	0.979	0.987	0.992	0.995	0.997	0.998	0.999	0.999	1.000	
25	0.966	0.978	0.985	0.991	0.991	0.997	0.998	0.999	0.999	1.000

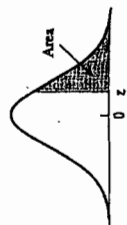


Table 4. Normal curve areas
Standard normal probability in right-hand tail
(for negative values of z areas are found by symmetry)

z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0015									
3.5	.000233									
4.0	.0000317									
4.5	.00000340									
5.0	.000000287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

Table 5. Percentage points of the t distributions



t	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	d.f.
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
inf.	1.282	1.645	1.960	2.326	2.576	inf.

From "Table of Percentage Points of the t -Distribution," Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300. Reproduced by permission of Professor E. S. Pearson.

Table 6. Percentage points of the χ^2 distributions

d.f.	$\chi^2_{0.995}$	$\chi^2_{0.990}$	$\chi^2_{0.975}$	$\chi^2_{0.950}$	$\chi^2_{0.900}$
1	0.0000393	0.0001571	0.0009821	0.0039321	0.0157908
2	0.01000251	0.0201007	0.0506356	0.102587	0.210720
3	0.0717121	0.114832	0.215795	0.351846	0.584375
4	0.206990	0.297110	0.484419	0.710721	1.063623
5	0.411740	0.554300	0.831211	1.145476	1.61031
6	0.675727	0.872085	1.237347	1.63539	2.20413
7	0.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100	67.3276	70.0648	74.2219	77.9295	82.3581

Table 6. (Continued)

$\chi^2_{0.100}$	$\chi^2_{0.050}$	$\chi^2_{0.025}$	$\chi^2_{0.010}$	$\chi^2_{0.005}$	d.f.
2.70554	3.84146	5.02389	6.63490	7.87944	1
4.60517	5.99147	7.37776	9.21034	10.5966	2
6.25139	7.81473	9.34840	11.3449	12.8381	3
7.77944	9.48773	11.1433	13.2767	14.8602	4
9.23635	11.0705	12.8325	15.0863	16.7496	5
10.6446	12.5916	14.4494	16.8119	18.5476	6
12.0170	14.0671	16.0128	18.4753	20.2777	7
13.3616	15.5073	17.5346	20.0902	21.9550	8
14.6837	16.9190	19.0228	21.6660	23.5893	9
15.9871	18.3070	20.4831	23.2093	25.1882	10
17.2750	19.6751	21.9200	24.7250	26.7569	11
18.5494	21.0261	23.3367	26.2170	28.2995	12
19.8119	22.3621	24.7356	27.6883	29.8194	13
21.0642	23.6848	26.1190	29.1413	31.3193	14
22.3072	24.9958	27.4884	30.5779	32.8013	15
23.5418	26.2962	28.8454	31.9999	34.2672	16
24.7690	27.5871	30.1910	33.4087	35.7185	17
25.9894	28.8693	31.5264	34.8053	37.1564	18
27.2036	30.1435	32.8523	36.1908	38.5822	19
28.4120	31.4104	34.1696	37.5662	39.9968	20
29.6151	32.6705	35.4789	38.9321	41.4010	21
30.8133	33.9244	36.7807	40.2894	42.7956	22
32.0069	35.1725	38.0757	41.6384	44.1813	23
33.1963	36.4151	39.3641	42.9798	45.5585	24
34.3816	37.6525	40.6465	44.3141	46.9278	25
35.5631	38.8852	41.9232	45.6417	48.2899	26
36.7412	40.1133	43.1944	46.9630	49.6449	27
37.9159	41.3372	44.4607	48.2782	50.9933	28
39.0875	42.5569	45.7222	49.5879	52.3356	29
40.2560	43.7729	46.9792	50.8922	53.6720	30
51.8050	55.7585	59.3417	63.6907	66.7659	40
63.1671	67.5048	71.4202	76.1539	79.4900	50
74.3970	79.0819	83.2976	88.3794	91.9517	60
85.5271	90.5312	95.0231	100.425	104.215	70
96.5782	101.879	106.629	112.329	116.321	80
107.565	113.145	118.136	124.116	128.299	90
118.498	124.342	129.561	135.807	140.169	100

From "Tables of the Percentage Points of the χ^2 -Distribution," *Biometrika*, Vol. 32 (1941), pp. 188-189, by Catherine M. Thompson. Reproduced by permission of Professor E. S. Pearson.