

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2008

TITLE OF PAPER : LINEAR STATISTICAL METHODS

COURSE CODE : ST204

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATORS**

**INSTRUCTIONS : 1. THIS PAPER CONSISTS OF FIVE
QUESTIONS.
2. ANSWER ANY FOUR QUESTIONS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION 1

1. A researcher wants to compare the effectiveness of two different teaching methods for three different class sizes. The data on the student's score on a standardized test were collected and analyzed using SPSS. The following ANOVA table is part of the output from that analysis.

Source of variation	Sum of squares	Degrees of freedom	Mean squares	F-ratio
Between treatments	220	5		
Factor <i>A</i>	120	1	120	24
Factor <i>B</i>	20	2	10	2
<i>A</i> × <i>B</i> Interaction	80	2	40	8
Within treatments	120	24	5	
Total	340	29		

- (a) What are the treatments in this experiment? How many? [4 marks]
- (b) Which one is factor *A*? Which one is factor *B*? [4 marks]
- (c) How many scores were used in the experiment? [4 marks]
- (d) Using the 1% level of significance, examine the significance of
- (i) the teaching method. [4 marks]
 - (ii) class size [4 marks]
 - (iii) the interaction between class size and the teaching method. [5 marks]

QUESTION 2

2. Five automobiles were used in an experiment designed to compare the mileage per gallon obtained by 3 competing brands of gasoline and the following data resulted.

Automobile	Gasoline		
	A	B	C
1	21	23	22
2	25	26	24
3	29	30	29
4	32	34	32
5	18	24	22

- (a) Identify the dependent variable, factor studied and factor levels. [5 marks]
- (b) Complete the computation of the ANOVA table and conduct the F test. Clearly state all the steps in the test including the conclusions. [20 marks]

QUESTION 3

3. To see whether a widely used food preservative contributes to the hyperactivity of preschool children, a dietician chose a random sample of 10 four year-olds known to be fairly hyperactive from various nursery schools and observed their behaviour 45 minutes after they had eaten measured amounts of food containing the preservative. In the table that follows, X is the amount of food consumed containing the preservative (in grams) and Y is a subjective rating of hyperactivity (on a scale of 1 to 20) based on a child's restlessness and interaction with other children.

X	36	82	45	49	21	24	58	73	85	52
Y	6	14	5	13	5	8	14	11	18	6

- (a) Find the least squares regression line. [5 marks]
- (b) Use the equation in (a) to estimate the hyperactivity rating of a child that had 65 grams of food with the preservative 45 minutes earlier. [2 marks]
- (c) Interpret the estimated values of β_0 and β_1 . [4 marks]
- (d) Find the coefficient of determination and interpret the result. [5 marks]
- (e) Compute σ^2 . [5 marks]
- (f) Test the hypothesis $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$ at a significance level $\alpha = 0.05$ [4 marks]

QUESTION 4

4. The following table shows how many weeks six persons have worked at an automobile inspection station and the number of cars each one inspected between noon and 2 P.M. on a given day.

Number of weeks employed	Number of cars inspected
X	Y
2	13
7	21
9	23
1	14
5	15
12	21

- (a) Find the least squares regression line. [7 marks]
- (b) Compute σ^2 . [5 marks]
- (c) Test the null hypothesis $\beta_1 = 1.5$ against the alternative hypothesis $\beta < 1.5$ at the 0.05 level of significance. [3 marks]
- (d) Construct a 95% confidence interval for the slope β_1 . [5 marks]
- (e) Construct a 99% confidence interval for the mean hyperactivity rating of a four year old at one of the nursery schools 45 minutes after he or she has eaten 60 grams of food containing the preservative. [5 marks]

QUESTION 5

5. (a) What is Simple Linear Regression Model? Discuss the main purposes of fitting this model. [5 marks]
- (b) Find the point estimators of two regression coefficients using Method of Least Squares. [5 marks]
- (c) Discuss Regression Analysis and Analysis of Variance in terms of their similarities and differences. [5 marks]
- (d) Define a two-factor ANOVA Model and state its important features. [5 marks]
- (e) Suppose there is only one case for each treatment. What would be the new form of the model defined in question (b) ? Explain [5 marks]

END OF EXAMINATION

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P(t(\nu) \leq t(A; \nu)) = A$



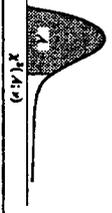
ν	A					
	.50	.70	.80	.85	.90	.95
1	0.333	0.727	1.376	1.963	3.078	6.314
2	0.289	0.617	1.061	1.386	1.886	4.303
3	0.277	0.584	0.978	1.250	1.638	3.182
4	0.271	0.569	0.941	1.190	1.533	2.776
5	0.267	0.559	0.920	1.154	1.476	2.571
6	0.265	0.553	0.906	1.134	1.440	2.447
7	0.263	0.549	0.896	1.119	1.415	2.365
8	0.262	0.546	0.889	1.108	1.397	2.306
9	0.261	0.543	0.883	1.100	1.383	2.262
10	0.260	0.542	0.879	1.095	1.372	2.228
11	0.260	0.540	0.876	1.092	1.363	2.201
12	0.259	0.539	0.873	1.088	1.356	2.179
13	0.259	0.537	0.870	1.085	1.350	2.160
14	0.258	0.536	0.868	1.082	1.345	2.145
15	0.258	0.535	0.866	1.079	1.341	2.131
16	0.258	0.535	0.865	1.071	1.337	2.120
17	0.257	0.534	0.863	1.069	1.333	2.110
18	0.257	0.534	0.862	1.067	1.330	2.101
19	0.257	0.533	0.861	1.065	1.328	2.093
20	0.257	0.533	0.860	1.064	1.327	2.086
21	0.257	0.532	0.859	1.063	1.323	2.080
22	0.256	0.532	0.858	1.061	1.321	2.074
23	0.256	0.532	0.858	1.060	1.319	2.069
24	0.256	0.531	0.857	1.059	1.318	2.064
25	0.256	0.531	0.856	1.058	1.316	2.060
26	0.256	0.531	0.856	1.058	1.315	2.056
27	0.256	0.531	0.855	1.057	1.314	2.052
28	0.256	0.530	0.855	1.056	1.313	2.048
29	0.256	0.530	0.854	1.055	1.311	2.045
30	0.256	0.530	0.854	1.055	1.310	2.042
40	0.255	0.529	0.851	1.050	1.303	1.994
60	0.254	0.527	0.848	1.045	1.296	1.960
120	0.254	0.526	0.845	1.041	1.289	1.940
∞	0.253	0.524	0.842	1.036	1.282	1.924

TABLE A.3 (continued) Percentiles of the *t* Distribution

ν	A					
	.98	.985	.99	.9925	.995	.9975
1	15.895	21.203	31.821	42.434	63.657	127.322
2	4.849	5.643	6.965	8.073	9.923	14.089
3	3.482	3.886	4.541	5.047	5.841	7.453
4	2.999	3.298	3.747	4.088	4.604	5.988
5	2.757	3.003	3.365	3.634	4.032	4.773
6	2.612	2.829	3.143	3.372	3.707	4.317
7	2.517	2.715	2.998	3.203	3.499	4.029
8	2.449	2.634	2.886	3.083	3.355	3.833
9	2.398	2.574	2.821	2.998	3.250	3.690
10	2.359	2.527	2.764	2.932	3.169	3.581
11	2.328	2.491	2.718	2.879	3.106	3.497
12	2.303	2.464	2.681	2.836	3.054	3.428
13	2.282	2.444	2.650	2.800	3.012	3.372
14	2.264	2.428	2.620	2.771	2.977	3.326
15	2.249	2.397	2.602	2.746	2.947	3.286
16	2.235	2.382	2.583	2.724	2.921	3.252
17	2.224	2.368	2.567	2.704	2.898	3.222
18	2.214	2.358	2.552	2.689	2.878	3.197
19	2.205	2.346	2.539	2.678	2.861	3.174
20	2.197	2.336	2.528	2.661	2.843	3.153
21	2.189	2.328	2.518	2.649	2.831	3.135
22	2.183	2.320	2.508	2.639	2.819	3.119
23	2.177	2.313	2.500	2.629	2.807	3.104
24	2.172	2.307	2.492	2.620	2.797	3.091
25	2.167	2.301	2.485	2.612	2.787	3.078
26	2.162	2.296	2.479	2.605	2.779	3.067
27	2.158	2.291	2.473	2.598	2.771	3.057
28	2.154	2.286	2.467	2.592	2.763	3.047
29	2.150	2.282	2.462	2.586	2.756	3.038
30	2.147	2.278	2.457	2.581	2.750	3.030
40	2.123	2.250	2.423	2.542	2.704	2.971
60	2.099	2.223	2.390	2.504	2.660	2.915
120	2.076	2.203	2.358	2.468	2.617	2.860
∞	2.054	2.170	2.326	2.432	2.576	2.807

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; v)$ where $P(\chi^2(v) \leq \chi^2(A; v)) = A$



v	A									
	.005	.010	.025	.050	.100	.500	.950	.975	.990	.995
1	0.0043	0.0086	0.0175	0.0358	0.0718	1.385	3.841	5.024	6.635	7.879
2	0.0100	0.0200	0.0400	0.0798	0.1583	2.770	3.000	3.579	4.605	5.024
3	0.0777	0.1533	0.3177	0.5844	1.2128	2.366	2.748	3.219	4.102	4.353
4	0.2048	0.4045	0.7879	1.3858	2.366	1.928	2.204	2.643	3.357	3.572
5	0.4114	0.8538	1.6258	2.8331	4.779	1.753	2.008	2.435	3.141	3.357
6	0.6758	1.3721	2.6758	4.5508	8.553	1.626	1.854	2.278	2.971	3.179
7	0.9893	2.000	3.828	6.344	14.454	1.535	1.771	2.190	2.878	3.098
8	1.3721	2.733	5.024	8.328	21.920	1.460	1.721	2.109	2.793	2.998
9	1.7348	3.572	6.635	10.557	30.578	1.398	1.676	2.030	2.719	2.928
10	2.160	4.540	8.538	13.442	40.529	1.346	1.635	1.962	2.658	2.877
11	2.601	5.578	10.828	16.919	52.336	1.299	1.597	1.900	2.605	2.832
12	3.076	6.708	13.579	21.026	66.210	1.257	1.563	1.849	2.559	2.791
13	3.571	8.031	16.812	26.217	82.357	1.219	1.532	1.807	2.519	2.753
14	4.077	9.488	20.537	32.671	100.425	1.185	1.503	1.772	2.483	2.719
15	4.602	11.141	25.000	39.597	120.409	1.154	1.476	1.743	2.450	2.688
16	5.142	12.996	30.191	47.223	142.565	1.126	1.451	1.718	2.420	2.660
17	5.700	15.085	36.191	55.939	167.156	1.100	1.428	1.694	2.392	2.635
18	6.268	17.339	42.929	65.779	194.154	1.076	1.406	1.672	2.367	2.612
19	6.854	19.758	50.423	76.759	223.503	1.054	1.385	1.651	2.344	2.590
20	7.433	22.367	58.579	88.996	265.044	1.033	1.365	1.631	2.322	2.569
21	8.013	25.188	67.329	102.557	318.913	1.013	1.346	1.612	2.301	2.549
22	8.594	28.223	76.759	117.584	385.418	0.994	1.328	1.593	2.281	2.530
23	9.176	31.466	86.929	134.214	465.042	0.976	1.311	1.575	2.262	2.511
24	9.759	34.910	97.978	152.542	558.286	0.959	1.294	1.557	2.243	2.493
25	10.333	38.579	109.945	172.610	665.779	0.943	1.278	1.540	2.225	2.475
26	10.908	42.478	122.901	194.481	788.146	0.928	1.262	1.523	2.207	2.457
27	11.484	46.610	136.847	218.200	925.979	0.913	1.247	1.506	2.190	2.440
28	12.061	50.988	151.777	243.884	1079.913	0.899	1.232	1.490	2.173	2.423
29	12.639	55.615	167.697	271.677	1250.610	0.885	1.217	1.474	2.156	2.406
30	13.218	60.484	184.601	301.639	1439.810	0.871	1.202	1.458	2.140	2.390
35	14.454	70.420	223.981	371.534	2053.941	0.857	1.187	1.442	2.123	2.373
40	15.985	82.357	274.881	451.554	2833.710	0.843	1.172	1.426	2.106	2.356
45	17.708	96.379	339.000	542.934	3796.413	0.829	1.157	1.410	2.089	2.339
50	19.600	112.578	418.381	646.979	5066.499	0.815	1.142	1.394	2.072	2.322
60	24.433	145.916	549.340	844.584	7878.461	0.799	1.126	1.377	2.055	2.305
70	29.191	184.984	727.901	1096.000	10960.461	0.783	1.110	1.360	2.038	2.288
80	33.900	229.984	959.000	1426.000	14626.461	0.767	1.094	1.343	2.021	2.271
90	38.579	280.984	1236.000	1846.000	19026.461	0.751	1.078	1.326	2.004	2.254
100	43.281	337.984	1559.000	2366.000	24226.461	0.735	1.062	1.309	1.987	2.237

Source: Biometrika table published by Frank C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," Biometrika 27 (1941), pp. 188-90.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; n_1, n_2)$ where $P(F(n_1, n_2) \leq F(A; n_1, n_2)) = A$



$$F(A; n_1, n_2) = F | - A; n_1, n_2$$



TABLE A-4 (continued) Percentiles of the F Distribution

Den. of A	Numerator of								
	1	2	3	4	5	6	7	8	9
1 .50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	51.6	52.8	53.6	54.2	54.7	55.1	55.4
.95	181	200	216	225	230	233	235	237	238
.975	648	700	744	770	788	800	809	815	819
.99	4,072	4,000	3,463	3,226	3,139	3,098	3,072	3,054	3,042
.995	15,312	20,000	21,612	22,500	23,006	23,317	23,515	23,632	23,699
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2 .50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.33	9.00	9.16	9.24	9.28	9.33	9.35	9.38	9.39
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3 .50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.34	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
.975	17.4	16.0	15.4	15.2	14.9	14.6	14.6	14.5	14.5
.99	44.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
.995	35.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.3	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4 .50	0.549	0.822	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.44	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
.95	7.7	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
.995	31.3	26.3*	24.5	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5 .50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.34	3.32	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6 .50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.10	3.05	3.01	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975	8.91	7.48	6.78	6.33	6.09	5.92	5.79	5.70	5.62
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	8.00
.995	18.4	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7 .50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
.95	5.59	4.78	4.35	4.12	3.97	3.87	3.79	3.73	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.99	8.88	8.81
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.5

TABLE A-4 (continued) Percentiles of the F Distribution

Den. of A	Numerator of											
	10	12	15	20	24	30	60	120	∞			
1 .50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20			
.90	40.2	40.7	41.2	41.7	42.0	42.3	42.8	43.1	43.3			
.95	66.2	67.4	68.4	69.4	70.0	70.5	71.3	71.7	72.0			
.975	146.2	148.2	150.0	151.7	153.0	154.0	155.5	156.1	156.4			
.99	5,056	5,106	5,175	5,250	5,317	5,381	5,431	5,468	5,500			
.995	24,224	24,435	24,650	24,870	25,094	25,321	25,550	25,780	25,999			
.999	605,650	610,670	615,700	620,910	625,300	629,900	633,970	638,620				
2 .50	1.34	1.36	1.38	1.40	1.41	1.43	1.43	1.44	1.44			
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.48	9.48	9.48			
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5			
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5			
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5			
.995	199	199	199	199	199	199	199	199	200			
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5			
3 .50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27			
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13			
.95	8.79	8.76	8.70	8.66	8.64	8.62	8.57	8.55	8.51			
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9			
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1			
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8			
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5			
4 .50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19			
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76			
.95	5.96	5.91	5.84	5.79	5.77	5.75	5.69	5.67	5.61			
.975	8.94	8.75	8.66	8.56	8.50	8.46	8.31	8.26	8.26			
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.6	13.5	13.5			
.995	20.4	20.2	20.2	20.0	20.0	19.8	19.5	19.5	19.3			
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1			
5 .50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15			
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11			
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37			
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02			
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02			
.995	13.6	13.4	13.1	12.9	12.8	12.4	12.3	12.3	12.1			
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8			
6 .50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12			
.90	2.80	2.80	2.80	2.82	2.82	2.80	2.76	2.74	2.72			
.95	4.08	4.00	3.94	3.87	3.84	3.81	3.76	3.70	3.67			
.975	4.98	4.78	4.67	4.57	4.52	4.46	4.36	4.35	4.31			
.99	7.26	7.02	6.86	6.70	6.63	6.53	6.36	6.29	6.24			
.995	10.2	9.91	9.75	9.59	9.49	9.36	9.00	8.97	8.88			
.999	18.4	18.0	17.8	17.1	16.9	16.7	16.0	16.0	15.7			
7 .50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10			
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.50	2.47	2.47			
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.29	3.23			
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.24	4.14			
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65			
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.05			
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7			

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
8	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
30	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
95	5.32	4.46	4.07	3.86	3.69	3.58	3.50	3.44	3.39
975	7.57	6.08	5.42	5.05	4.82	4.65	4.53	4.45	4.38
995	11.7	8.63	7.39	6.70	6.31	6.07	5.92	5.83	5.76
999	14.7	10.3	8.90	8.00	7.57	7.27	7.07	6.96	6.88
9	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
30	3.16	3.01	2.82	2.69	2.61	2.55	2.51	2.47	2.44
95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
995	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
999	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54
10	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
30	3.39	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
995	10.0	7.38	6.35	5.59	5.14	4.88	4.70	4.58	4.49
999	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
12	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
30	3.11	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
975	6.35	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
995	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
999	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
15	0.478	0.726	0.825	0.878	0.911	0.933	0.949	0.960	0.970
30	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
995	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
999	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
20	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
30	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.44	2.38
975	6.01	4.58	3.96	3.61	3.38	3.21	3.09	2.99	2.91
995	8.41	6.09	5.17	4.64	4.21	3.97	3.79	3.66	3.56
999	10.8	7.95	6.73	6.05	5.62	5.32	5.09	4.91	4.78
24	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
30	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
95	4.36	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
975	6.02	4.52	3.72	3.38	3.15	2.99	2.87	2.78	2.70
995	8.35	6.06	5.12	4.52	4.09	3.85	3.67	3.56	3.46
999	10.6	7.55	6.39	5.65	5.23	4.93	4.70	4.52	4.40

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	40	60	120	∞	
8	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.08	1.09	
30	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29	2.28	
95	3.15	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93	2.91	
975	4.30	4.20	4.10	4.00	3.95	3.88	3.78	3.73	3.67	3.65	
995	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86	4.84	
999	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95	5.93	
9	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08	1.08	
30	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16	2.16	
95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71	2.71	
975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33	3.33	
995	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31	4.31	
999	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19	5.19	
10	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07	1.07	
30	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.04	2.04	
95	3.02	2.95	2.88	2.77	2.74	2.70	2.61	2.54	2.48	2.48	
975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08	3.08	
995	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91	3.91	
999	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64	4.64	
12	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	0.989	
30	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90	1.90	
95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30	2.30	
975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72	2.72	
995	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36	3.36	
999	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90	3.90	
15	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05	1.05	
30	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76	1.76	
95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07	2.07	
975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40	2.40	
995	3.80	3.67	3.52	3.37	3.29	3.21	3.03	2.96	2.87	2.87	
999	4.62	4.43	4.27	4.07	3.98	3.89	3.68	3.57	3.46	3.46	
20	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03	1.03	
30	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61	1.61	
95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84	1.84	
975	2.77	2.68	2.57	2.46	2.41	2.35	2.23	2.16	2.09	2.09	
995	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42	2.42	
999	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69	2.69	
24	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.03	1.03	1.03	
30	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53	1.53	
95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73	1.73	
975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94	1.94	
995	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21	2.21	
999	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43	2.43	
28	0.954	0.964	0.974	0.984	0.994	1.00	1.01	1.02	1.03	1.03	
30	1.84	1.79	1.74	1.69	1.66	1.63	1.57	1.53	1.50	1.50	
95	2.21	2.14	2.07	1.98	1.94	1.89	1.79	1.73	1.66	1.66	
975	2.60	2.49	2.38	2.27	2.21	2.15	2.00	1.91	1.81	1.81	
995	3.12	2.97	2.82	2.67	2.58	2.49	2.28	2.17	2.05	2.05	
999	3.54	3.37	3.19	2.99	2.89	2.78	2.56	2.43	2.31	2.31	

TABLE A.3 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
30	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.90	2.88	2.49	2.28	2.16	2.08	2.02	1.98	1.95	1.93
.95	4.17	3.52	3.22	3.02	2.92	2.87	2.83	2.80	2.77
.975	5.57	4.18	3.79	3.57	3.47	3.42	3.39	3.37	3.35
.99	7.26	5.13	4.62	4.32	4.22	4.18	4.16	4.15	4.14
.995	9.18	6.32	5.74	5.42	5.32	5.28	5.26	5.25	5.24
.999	13.1	8.77	7.93	7.62	7.53	7.50	7.48	7.47	7.46
60	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975	5.39	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.99	7.04	4.98	4.13	3.65	3.34	3.12	2.93	2.82	2.72
.995	8.89	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.29	2.22
.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.69	2.62
.995	8.18	5.54	4.50	3.92	3.53	3.28	3.09	2.93	2.81
.999	11.4	7.32	5.78	4.93	4.32	4.04	3.77	3.55	3.38
∞	0.455	0.693	0.789	0.840	0.870	0.891	0.907	0.918	0.927
.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.95	3.87	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.995	7.98	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df										
	10	12	15	20	24	30	60	120	∞		
30	0.935	0.866	0.828	0.809	0.794	0.784	0.774	0.767	0.762	0.758	
.90	1.82	1.69	1.62	1.57	1.54	1.52	1.51	1.50	1.49	1.48	
.95	2.31	2.09	2.01	1.93	1.89	1.84	1.82	1.81	1.80	1.79	
.975	2.98	2.64	2.56	2.45	2.41	2.37	2.35	2.34	2.33	2.32	
.99	3.94	3.18	3.01	2.82	2.73	2.63	2.62	2.61	2.60	2.59	
.995	4.24	4.00	3.75	3.49	3.36	3.22	3.22	3.22	3.22	3.22	
.999	4.24	4.00	3.75	3.49	3.36	3.22	3.22	3.22	3.22	3.22	
60	0.945	0.856	0.807	0.788	0.773	0.763	0.754	0.748	0.743	0.739	
.90	1.71	1.66	1.60	1.54	1.51	1.48	1.48	1.48	1.47	1.46	
.95	1.99	1.92	1.84	1.75	1.70	1.65	1.63	1.62	1.61	1.60	
.975	2.27	2.17	2.06	1.94	1.88	1.82	1.80	1.79	1.78	1.77	
.99	2.63	2.50	2.35	2.20	2.12	2.03	2.02	2.01	2.00	1.99	
.995	2.90	2.74	2.57	2.39	2.29	2.19	2.18	2.17	2.16	2.15	
.999	3.14	3.12	3.08	2.83	2.69	2.53	2.53	2.53	2.53	2.53	
120	0.939	0.850	0.801	0.782	0.767	0.757	0.748	0.742	0.737	0.733	
.90	1.63	1.60	1.53	1.46	1.43	1.41	1.40	1.39	1.38	1.37	
.95	1.91	1.83	1.72	1.64	1.61	1.57	1.55	1.54	1.53	1.52	
.975	2.18	2.09	1.97	1.82	1.76	1.69	1.66	1.65	1.64	1.63	
.99	2.47	2.34	2.19	2.01	1.93	1.86	1.84	1.83	1.82	1.81	
.995	2.74	2.62	2.47	2.27	2.19	2.09	2.08	2.07	2.06	2.05	
.999	3.02	3.02	2.78	2.53	2.40	2.26	2.26	2.26	2.26	2.26	
∞	0.934	0.845	0.796	0.777	0.762	0.752	0.743	0.737	0.732	0.728	
.90	1.60	1.55	1.49	1.42	1.38	1.34	1.32	1.31	1.30	1.29	
.95	1.83	1.75	1.67	1.57	1.52	1.46	1.42	1.42	1.41	1.40	
.975	2.03	1.94	1.83	1.71	1.64	1.57	1.53	1.52	1.51	1.50	
.99	2.32	2.18	2.04	1.88	1.79	1.70	1.67	1.66	1.65	1.64	
.995	2.52	2.36	2.19	2.00	1.90	1.79	1.75	1.74	1.73	1.72	
.999	2.96	2.74	2.51	2.27	2.13	1.99	1.95	1.94	1.93	1.92	

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