

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2005

TITLE OF PAPER : NON-PARAMETRIC METHODS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATORS**

**INSTRUCTIONS : ANSWER ANY THREE QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

ANSWER ANY THREE QUESTIONS:

For all questions, clearly state the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, and the decision & conclusions.

QUESTION ONE.

[20 marks]

A random sample of size 10 is obtained: $X_1 = 0.621$, $X_2 = 0.503$, $X_3 = 0.203$, $X_4 = 0.477$, $X_5 = 0.710$, $X_6 = 0.581$, $X_7 = 0.329$, $X_8 = 0.480$, $X_9 = 0.554$, $X_{10} = 0.382$. Use Kolmogorov Goodness-of Fit test to test the hypothesis that the distribution of the sample data is uniform on the interval from 0 to 1. Use $\alpha = 0.05$ and also calculate the P-value.

QUESTION TWO.

[20 marks]

Twelve MBA graduates are studied to measure the strength of the relationship between their score on the GMAT, which they took prior to entering graduate school, and their grade point average while they were in the MBA program. Their GMAT scores and their GPAs are given below:

Student	1	2	3	4	5	6	7	8	9	10	11	12
GMAT	710	610	640	580	545	560	610	530	560	540	570	560
GPA	4.0	4.0	3.9	3.8	3.7	3.6	3.5	3.5	3.5	3.3	3.2	3.2

Use Kendall's Tau to test whether GMAT scores and GPAs are positively correlated. Use $\alpha = 0.05$ and also calculate the P-value.

QUESTION THREE.

[20 marks]

Four different methods of growing corn were randomly assigned to a large number of different plots of land and the yield per acre was computed for each plot as follows:

Method			
1	2	3	4
83	91	101	78
91	90	100	82
94	81	91	81
89	83	93	77
89	84	96	79
96	83	95	81
91	88	94	80
92	91		81
90	89		
		84	

Using Kruskal-Wallis test. show that the yields are different for those four methods. Consider $\alpha = 0.05$ and also calculate the P-value.

QUESTION FOUR.

[12 + 8 marks]

- a. Total annual precipitation is recorded yearly for 19 years. This record is examined to see if the amount of precipitation is tending to increase or decrease. The precipitation in inches was 45.25, 45.83, 41.77, 36.26, 45.37, 52.25, 35.37, 57.16, 35.37, 58.32, 41.05, 33.72, 45.73, 37.90, 41.72, 36.07, 49.83, 36.24, and 39.90. Indicate the appropriate conclusion with $\alpha = 0.01$ and also calculate the P-value.
- b. An item A is manufactured using certain process. Item B serves the same function as A but manufactured using a new process. The manufacturer wishes to determine whether B is preferred to A by the consumer, so she selects a random sample consisting of 10 consumers, gives each of them one A and one B and asks them to use the items for some period of time. At the end of the allotted period of time the consumers report their preferences to the manufacturer. Eight consumers preferred B to A, 1 preferred A to B, and 1 reported "no preference." Use an appropriate test to find out the manufacturer's decision at 5% level of significance. Also calculate the P-value.

TABLE A1 Normal Distribution^a

<i>p</i>	Selected values		$Z_{0.0001} = -3.7190$	$Z_{0.0005} = -3.2905$	$Z_{0.025} = -1.9600$	$Z_{0.05} = -1.6449$	$Z_{0.9999} = 3.7190$	$Z_{0.9995} = 3.2905$	$Z_{0.975} = 1.9600$	$Z_{0.95} = 1.6449$	$Z_{0.001} = -3.0902$	$Z_{0.002} = -2.8782$	$Z_{0.003} = -2.7478$	$Z_{0.004} = -2.6521$	$Z_{0.005} = -2.5758$	$Z_{0.006} = -2.5121$	$Z_{0.007} = -2.4573$	$Z_{0.008} = -2.4089$	$Z_{0.009} = -2.3656$																																	
	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.23	0.24																		
0.00	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-2.7478	-2.6521	-2.5758	-2.5121	-2.4573	-2.4089	-2.3656	-2.3233	-2.2810	-2.2387	-2.1964	-2.1541	-2.1118	-2.0695	-2.0272	-1.9849	-1.9426	-1.9003	-1.8580	-1.8157	-1.7734	-1.7311	-1.6888	-1.6465	-1.6042	-1.5619	-1.5196	-1.4773	-1.4350	-1.3927	-1.3494	-1.3071	-1.2648	-1.2225	-1.1700	-1.1275	-1.0850	-1.0425	-1.0000	-9.5370						
0.01	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-2.2571	-2.2262	-2.1973	-2.1701	-2.1444	-2.1201	-2.0969	-2.0749	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.02	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-2.0141	-1.9954	-1.9774	-1.9501	-1.9244	-1.8981	-1.8718	-1.8455	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.03	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.8522	-1.8384	-1.8250	-1.8078	-1.7815	-1.7552	-1.7289	-1.7026	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.04	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.7279	-1.7169	-1.7060	-1.6897	-1.6634	-1.6471	-1.6208	-1.5945	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.05	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.6258	-1.6164	-1.6072	-1.5909	-1.5746	-1.5583	-1.5320	-1.5141	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.06	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.5301	-1.5220	-1.5137	-1.5055	-1.4973	-1.4890	-1.4798	-1.4696	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.07	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.4611	-1.4538	-1.4466	-1.4394	-1.4322	-1.4250	-1.4178	-1.4106	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.08	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.3917	-1.3852	-1.3787	-1.3722	-1.3657	-1.3592	-1.3527	-1.3462	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.09	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.3285	-1.3225	-1.3165	-1.3106	-1.3047	-1.3000	-1.2939	-1.2878	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.10	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.2702	-1.2646	-1.2591	-1.2536	-1.2481	-1.2426	-1.2371	-1.2316	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.11	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.2160	-1.2107	-1.2055	-1.2004	-1.1952	-1.1901	-1.1850	-1.1799	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.12	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.1650	-1.1601	-1.1552	-1.1503	-1.1455	-1.1407	-1.1359	-1.1311	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.13	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.1170	-1.1123	-1.1077	-1.1031	-1.1085	-1.1047	-1.1009	-1.1061	-2.3233	-2.2810	-2.2387	-2.2024	-2.1661	-2.1318	-2.0955	-2.0592	-2.0229	-1.9866	-1.9503	-1.9140	-1.8777	-1.8414	-1.8051	-1.7688	-1.7325	-1.6965	-1.6602	-1.6239	-1.5876	-1.5513	-1.5150	-1.4787	-1.4424	-1.4061	-1.3698	-1.3335	-1.2972	-1.2609	-1.2246	-1.1883	-1.1520	-1.1157	-1.0794	-1.0431	-1.0068	-9.5370
0.14	-3.7190	-3.2905	-1.9600	-1.6449	-3.0902	-2.8782	-1.0714	-1.0669	-1.0625	-1.0581	-1.0537	-1.0494	-1.0450	-1.0407	-2.3233	-2.2810	-2.2387	-2.2024</td																																		

Table A1 (Continued)

<i>p</i>	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

<i>p</i>	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3408	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4118	1.4187	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4758	1.4833	1.4909	1.4985	1.5063	1.5141	1.5220	1.5301	1.5382	1.5464
0.94	1.5548	1.5632	1.5718	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

SOURCE: Generated by R. L. Iman. Used with permission.

*The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

TABLE A2 Chi-Squared Distribution^a

<i>k</i>	<i>p</i> = 0.750	0.900	0.950	0.975	0.990	0.995	0.999
1	1.323	2.706	3.841	5.024	6.635	7.879	10.83
2	2.773	4.605	5.991	7.378	9.210	10.60	13.82
3	4.08	6.251	7.815	9.348	11.34	12.84	16.27
4	5.385	7.779	9.488	11.14	13.28	14.86	18.47
5	6.626	9.236	11.07	12.83	15.09	16.75	20.51
6	7.841	10.64	12.59	14.45	16.81	18.55	22.46
7	9.037	12.02	14.07	16.01	18.48	20.28	24.32
8	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	28.24	33.20	36.42	39.37	42.98	45.56	51.18
25	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	31.53	36.74	40.11	43.19	46.96	49.64	55.49
28	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	77.58	85.53	90.53	95.02	100.4	104.2	112.3
80	88.13	96.38	101.9	106.6	112.3	116.3	124.8
90	98.65	107.5	113.1	118.1	124.1	128.3	137.2
100	109.1	118.5	124.3	129.6	135.8	140.2	149.4
<i>Z_p</i>	0.675	1.282	1.645	1.960	2.326	2.576	3.090

For $k > 100$ use the approximation $w_p = \frac{1}{2}(z_p + \sqrt{2k - 1})^2$, or the more accurate $w_p = k \left(1 - \frac{2}{9k} + z_p \sqrt{\frac{2}{9k}}\right)$, where z_p is the value from the standardized normal distribution shown in the bottom of the table.

SOURCE: Abridged from Table 8, Vol. I of Pearson and Hartley (1976), with permission from the Biometrika Trustees.

^aThe entries in this table are quantiles w_p of a chi-squared random variable W with k degrees of freedom, selected so $P(W \leq w_p) = p$ and $P(W > w_p) = 1 - p$.

TABLE A3 Binomial Distribution^a

<i>n</i>	<i>y</i>	<i>p</i> = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
1	0	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500
2	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	2	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025
4	3	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975
5	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	5	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585
7	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	7	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562
9	8	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931
10	9	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688
11	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	11	0.9999	0.9997	0.9990	0.9947	0.9898	0.9815	0.9716	0.9643	0.9531
13	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	13	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
15	14	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
16	15	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
17	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	17	0.9993	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
19	18	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
20	19	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
21	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	21	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
23	22	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
24	23	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
25	24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
26	25	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
27	26	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
28	27	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
29	28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	29	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
40	39	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
50	49	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
60	59	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
70	69	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
80	79	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
90	89	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
100	99	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
110	109	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
120	119	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
130	129	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
140	139	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
150	149	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
160	159	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
170	169	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
180	179	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
190	189	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
200	199	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308
210	209	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9815
220	219	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447
230	229	1.0000	0.9							

TABLE A3 (Continued)

TABLE A3 (Continued)

<i>n</i>	<i>y</i>	$p = 0.50$	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
8	0	0.0039	0.0017	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0352	0.0181	0.0085	0.0036	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.1445	0.0885	0.0498	0.0253	0.0113	0.0042	0.0012	0.0002	0.0000	0.0000	0.0000
3	0.3633	0.2604	0.1737	0.1061	0.0580	0.0273	0.0104	0.0029	0.0004	0.0000	0.0000
4	0.6367	0.5330	0.4059	0.2936	0.1941	0.1138	0.0563	0.0214	0.0050	0.0004	0.0000
5	0.8555	0.7799	0.6846	0.5722	0.4482	0.3215	0.2031	0.1052	0.0381	0.0058	0.0000
6	0.9648	0.9368	0.8936	0.8309	0.7447	0.6329	0.4967	0.3428	0.1869	0.0572	0.0000
7	0.9961	0.9916	0.9832	0.9681	0.9424	0.8999	0.8322	0.7275	0.5695	0.3366	0.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0.0020	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0195	0.0091	0.0038	0.0014	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0898	0.0498	0.0250	0.0112	0.0043	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000
3	0.2539	0.1658	0.0994	0.0536	0.0253	0.0100	0.0031	0.0006	0.0001	0.0000	0.0000
4	0.5000	0.3786	0.2666	0.1717	0.0988	0.0489	0.0196	0.0056	0.0009	0.0000	0.0000
5	0.7461	0.6386	0.5174	0.3911	0.2703	0.1657	0.0856	0.0339	0.0083	0.0006	0.0000
6	0.9102	0.8505	0.7682	0.6627	0.5373	0.3993	0.2618	0.1409	0.0530	0.0084	0.0000
7	0.9805	0.9615	0.9295	0.8789	0.8040	0.6997	0.5638	0.4005	0.2252	0.0712	0.0000
8	0.9980	0.9954	0.9899	0.9793	0.9596	0.9249	0.8658	0.7684	0.6126	0.3698	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0107	0.0045	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0547	0.0274	0.0123	0.0048	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.1719	0.1020	0.0548	0.0260	0.0106	0.0035	0.0009	0.0001	0.0000	0.0000	0.0000
4	0.3770	0.2616	0.1662	0.0949	0.0473	0.0197	0.0064	0.0014	0.0001	0.0000	0.0000
5	0.6230	0.4956	0.3669	0.2485	0.1503	0.0781	0.0328	0.0099	0.0016	0.0001	0.0000
6	0.8281	0.7340	0.6177	0.4862	0.3504	0.2241	0.1209	0.0500	0.0128	0.0010	0.0000
7	0.9453	0.9004	0.8327	0.7384	0.6172	0.4744	0.3222	0.1798	0.0702	0.0115	0.0000
8	0.9893	0.9767	0.9536	0.9140	0.8507	0.7560	0.6242	0.4557	0.2639	0.0861	0.0000
9	0.9990	0.9975	0.9940	0.9865	0.9718	0.9437	0.8926	0.8031	0.6513	0.4013	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0059	0.0022	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0327	0.0148	0.0059	0.0020	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.1133	0.0610	0.0793	0.0122	0.0043	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
4	0.2744	0.1738	0.0994	0.0501	0.0216	0.0076	0.0020	0.0003	0.0000	0.0000	0.0000
5	0.5000	0.3669	0.2465	0.1487	0.0782	0.0343	0.0117	0.0027	0.0003	0.0000	0.0000
6	0.7756	0.6029	0.4672	0.3317	0.2103	0.1146	0.0504	0.0159	0.0028	0.0001	0.0000
7	0.8867	0.8089	0.7037	0.5744	0.4304	0.2867	0.1611	0.0694	0.0185	0.0016	0.0000
8	0.9673	0.9348	0.8811	0.7999	0.6873	0.5448	0.3826	0.2212	0.0896	0.0152	0.0000
9	0.9941	0.9861	0.9698	0.9394	0.8870	0.8029	0.6779	0.5078	0.3026	0.1019	0.0000
10	0.9995	0.9986	0.9964	0.9912	0.9802	0.9578	0.9141	0.8327	0.6862	0.4312	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE A3 (Continued)

<i>n</i>	<i>y</i>	$p = 0.05$	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0000
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0000
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0000
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.0000
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7673	0.6652	0.5269	0.0000
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.0000
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.0000
8	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9241	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	0	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004
1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0000
2	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0000
3	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0000
4	0.9997	0.9935	0.9658	0.9099	0.7940	0.6543	0.5050	0.3530	0.2279	0.0000
5	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.0000
6	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.0000
7	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.0000
8	1.0000	1.0000	0.9999	0.9990	0.9960	0.9874	0.9679	0.9302	0.8777	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002
1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0000
2	0.96									

TABLE A3
(Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

TABLE A3 (Continued)

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TABLE A3 (Continued)

<i>n</i>	<i>y</i>	<i>p</i> = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000	0.0000
9	0.5290	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000	0.0000
10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000	0.0000
11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000	0.0000
12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000	0.0000
13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002	0.0002
14	0.9904	0.9720	0.9304	0.8250	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020	0.0020
15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132	0.0132
16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665	0.0665
17	1.0000	0.9998	0.9992	0.9869	0.9650	0.9171	0.8015	0.5797	0.2453	0.0411	0.0411
18	1.0000	0.9999	0.9997	0.9889	0.9558	0.9856	0.9544	0.8649	0.6226	0.0000	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.889	0.889
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000	0.0000
10	0.5881	0.4086	0.2447	0.1218	0.0640	0.0139	0.0026	0.0002	0.0000	0.0000	0.0000
11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000	0.0000
12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000	0.0000
13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000	0.0000
14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003	0.0000
15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026	0.0000
16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159	0.0000
17	1.0000	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755	0.0000
18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642	0.0000
19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415	0.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* Y has the binomial distribution with parameters n and p . The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95. For n larger than 20, the r th quantile Y_r of a binomial random variable may be approximated using $Y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the r th quantile of a standard normal random variable, obtained from Table A1.

TABLE A4 Exact Confidence Intervals for the Binomial Parameter p

<i>n</i>	<i>y</i>	90%		95%		99%	
		Lower	Upper	Lower	Upper	Lower	Upper
1	0	0.00	0.95	0.00	0.975	0.00	0.995
2	0	0.05	1.00	0.025	0.975	0.013	0.997
3	0	0.225	1.00	0.100	0.987	0.051	0.999
4	0	0.549	1.00	0.277	0.999	0.122	1.000
5	0	1.00	0.451	0.522	0.999	0.226	0.653
6	0	1.513	0.549	0.393	0.999	0.171	0.815
7	0	2.063	0.582	0.447	0.999	0.229	0.746
8	0	2.618	0.618	0.524	0.999	0.347	0.666
9	0	3.153	0.647	0.478	0.999	0.459	0.586
10	0	3.797	0.682	0.595	0.999	0.559	0.696
11	0	4.337	0.718	0.626	0.999	0.641	0.744
12	0	4.877	0.757	0.659	0.999	0.734	0.815
13	0	5.418	0.791	0.711	0.999	0.824	0.899
14	0	5.959	0.824	0.733	0.999	0.911	0.977
15	0	6.500	0.857	0.747	0.999	0.993	1.000
16	0	7.041	0.882	0.812	0.999	1.000	1.000
17	0	7.572	0.907	0.842	0.999	1.000	1.000
18	0	8.103	0.932	0.871	0.999	1.000	1.000
19	0	8.634	0.957	0.911	0.999	1.000	1.000
20	0	9.165	0.982	0.959	0.999	1.000	1.000

* Y has the binomial distribution with parameters n and p . The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95. For n larger than 20, the r th quantile Y_r of a binomial random variable may be approximated using $Y_r = np + z_r \sqrt{np(1-p)}$, where z_r is the r th quantile of a standard normal random variable, obtained from Table A1.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes*

Sample Sizes	$W_{1,0}$	$W_{1,5}$	$W_{4,0}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0667	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2164	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0460	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2482	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6764	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

SOURCE: Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.

*The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A10 Quantiles of Spearman's ρ^*

n	$\rho = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000	0.8000	0.9000	0.9000	0.9429
5	0.7000	0.7114	0.8286	0.8857	0.8929	0.9643
6	0.6000	0.6786	0.7500	0.8571	0.8926	0.9286
7	0.5357	0.6190	0.7143	0.8095	0.8571	0.9000
8	0.5000	0.6190	0.6833	0.7667	0.8167	0.9000
9	0.4667	0.5833	0.6833	0.7333	0.7818	0.8667
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5849	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5541	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4255	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4379	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For $n > 30$ the approximate quantiles of ρ may be obtained from

$$w_p \equiv \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glazier and Winter (1961), with corrections, with permission from the *Biometrika Trustees*.

*The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_r - N_s$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$\rho = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2777)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3505)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE AII (Continued)

<i>n</i>	<i>p</i> = 0.900	0.950	0.975	0.990	0.995
38	[0.03 (0.1465)]	[3.1 (0.1863)]	[5.5 (0.2205)]	[8.3 (0.2603)]	[20.3 (0.2888)]
39	[0.07 (0.1444)]	[3.7 (0.1849)]	[6.1 (0.2173)]	[9.1 (0.2578)]	[21.1 (0.2848)]
40	[0.10 (0.1372)]	[4.2 (0.1821)]	[6.8 (0.2154)]	[9.8 (0.2538)]	[22.0 (0.2821)]
41	[0.14 (0.1390)]	[4.6 (0.1780)]	[7.4 (0.2122)]	[10.6 (0.2512)]	[22.8 (0.2780)]
42	[0.19 (0.1382)]	[5.1 (0.1754)]	[8.1 (0.2102)]	[11.3 (0.2474)]	[23.5 (0.2729)]
43	[0.23 (0.1362)]	[5.7 (0.1739)]	[8.7 (0.2071)]	[12.1 (0.2447)]	[24.5 (0.2713)]
44	[0.28 (0.1353)]	[6.2 (0.1712)]	[9.4 (0.2051)]	[12.8 (0.2410)]	[25.2 (0.2664)]
45	[0.32 (0.1333)]	[6.8 (0.1697)]	[20.0 (0.2020)]	[23.6 (0.2383)]	[26.2 (0.2446)]
46	[0.35 (0.1304)]	[7.3 (0.1671)]	[20.7 (0.2000)]	[24.5 (0.2367)]	[27.1 (0.2367)]
47	[0.41 (0.1304)]	[7.9 (0.1656)]	[21.3 (0.1970)]	[25.3 (0.2340)]	[27.9 (0.2381)]
48	[0.44 (0.1277)]	[8.6 (0.1649)]	[22.0 (0.1950)]	[26.0 (0.2305)]	[28.8 (0.2353)]
49	[0.50 (0.1276)]	[9.0 (0.1616)]	[22.8 (0.1939)]	[26.8 (0.2279)]	[29.6 (0.2577)]
50	[0.53 (0.1249)]	[9.7 (0.1608)]	[23.3 (0.1902)]	[27.7 (0.2261)]	[30.5 (0.2490)]
51	[0.59 (0.1247)]	[20.3 (0.1592)]	[24.1 (0.1890)]	[28.5 (0.2235)]	[31.5 (0.2571)]
52	[0.62 (0.1222)]	[20.8 (0.1569)]	[24.8 (0.1870)]	[29.4 (0.2217)]	[32.4 (0.2443)]
53	[0.68 (0.1219)]	[21.4 (0.1553)]	[25.6 (0.1858)]	[30.2 (0.2192)]	[33.4 (0.2424)]
54	[0.73 (0.1209)]	[22.1 (0.1544)]	[26.3 (0.1838)]	[31.1 (0.2173)]	[34.3 (0.2397)]
55	[0.77 (0.1192)]	[22.7 (0.1529)]	[26.9 (0.1811)]	[31.9 (0.2148)]	[35.3 (0.2377)]
56	[0.82 (0.1182)]	[23.2 (0.1506)]	[27.6 (0.1792)]	[32.8 (0.2130)]	[36.2 (0.2351)]
57	[0.86 (0.1165)]	[24.0 (0.1504)]	[28.4 (0.1779)]	[33.6 (0.2105)]	[37.2 (0.2338)]
58	[0.91 (0.1155)]	[24.5 (0.1482)]	[29.1 (0.1760)]	[34.5 (0.2087)]	[38.1 (0.2305)]
59	[0.97 (0.1151)]	[25.1 (0.1467)]	[29.9 (0.1748)]	[35.5 (0.2075)]	[39.1 (0.2285)]
60	[0.202 (0.1141)]	[25.8 (0.1458)]	[30.6 (0.1729)]	[36.4 (0.2056)]	[40.2 (0.2271)]

For n greater than 60, approximate quantiles of T may be obtained from

$$w_1 \equiv x_1 \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

Critical regions correspond to values of T greater than (or less than) but not including the appropriate quantile. Note that the median of T is 0. Quantiles for r are obtained by dividing the quantiles of T by $n(n - 1)/2$.

SOURCE. Adapted from Table I, Best (1974), with permission from the author.

TABLE A12 Quantiles of the Wilcoxon Signed Ranks Test Statistic

TABLE A12 (Continued)

	$\frac{n(n+1)}{2}$									
	$w_{0.05}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n+1)/4] + z_p \sqrt{n(n+1)(2n+1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

Source: Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T^* < w_p) \leq p$ and $P(T^* > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^* less than (or greater than) but not including the appropriate quantile.

$w_p = n(n+1)/2 - w_{1-p}$

TABLE A13 Quantiles of the Kolmogorov Test Statistic*

	One-Sided Test									
	$p = 0.90$	0.95	0.975	0.99	0.995	$p = 0.80$	0.90	0.95	0.99	0.995
Two-Sided Test	$p = 0.80$	0.90	0.95	0.98	0.99	$p = 0.80$	0.90	0.95	0.98	0.99
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235

Approximation
for $n > 40$

$$\frac{1.07}{\sqrt{n}}$$

$$\frac{1.22}{\sqrt{n}}$$

$$\frac{1.36}{\sqrt{n}}$$

$$\frac{1.52}{\sqrt{n}}$$

$$\frac{1.63}{\sqrt{n}}$$

Source: Adapted from Table I of Miller (1956). Used with permission of the American Statistical Association.

The entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^ , and T^- as defined by Equation 6.1.1 for two-tailed tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantities in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)/n$ is used instead of \sqrt{n}/n in the denominator.