



UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2007

TITLE OF PAPER: **Applied linear regression**

COURSE CODE : **ST 403**

TIME ALLOWED : **TWO (2) HOURS**

INSTRUCTIONS : **THIS PAPER HAS FIVE QUESTIONS.
ANSWER ANY FOUR (4) QUESTIONS.
EACH QUESTION CARRIES 15 MARKS.**

REQUIREMENTS: **Scientific calculator and statistical table**

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by the Chief Invigilator**

QUESTION ONE

The morning newspaper lists the following used car prices for a foreign compact , with age X measured in years and selling price Y measured in thousands Emalangeni.

X	1	2	3	4	5	6	7	8	9	10
Y	2.45	1.8	2.0	2.0	1.7	1.2	1.15	0.69	0.6	0.47

- (a) Determine the equation of the Least Squares regression line.
- (b) Construct a 95% confidence interval for the slope of the regression line.
- (c) Determine the predicted value for the average selling price of a five-year-old model compact and construct a 95% confidence.

(5+5+5Marks)

QUESTION TWO

The following scores are obtained on a test of dexterity and aggression administered to a random sample of 10 high school seniors:

Student	1	2	3	4	5	6	7	8	9	10
Dexterity	23	29	45	36	49	41	30	15	40	38
Aggression	45	48	16	28	38	21	36	18	31	37

Using Spearman's statistic, test the null hypothesis that the manifestations of dexterity and aggression are independent.

(15Marks)

QUESTION THREE

In a multiple linear regression model $\mathbf{Y} = \mathbf{X}\beta + \mathbf{U}$, if all the assumptions necessary for the least squares method hold, except that $E(\mathbf{U}\mathbf{U}') \neq \sigma^2 I$.

- (a) What happens to the estimates of the parameters by the ordinary least squares method?
- (b) Suggest an alternative estimating procedure and find the estimates of the parameters and the variance-covariance matrix of the estimates.

(5+10Marks)

QUESTION FOUR

- (a) Show that under the first order auto-regressive scheme $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$,

where $u_t \sim NID(0, \sigma_u^2)$; $Cov(\varepsilon_t, \varepsilon_{t-s}) = \rho^s \left(\frac{\sigma_u^2}{1 - \rho^2} \right)$, $s \neq 0$.

(5Marks)

- (b) Given the following results from the regression

model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ell_t$.

t	1	2	3	4	5	6	7	8	9
$Y - \hat{Y}$	-0.418	-0.350	0.507	-0.374	-0.181	0.652	0.256	0.342	-0.434

Obtain the estimate of ρ and test at 5% level of significance for positive correlation given that $n=50$.

(10Marks)

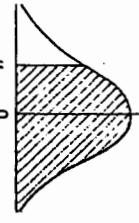
QUESTION FIVE

Four groups of students were subjected to different teaching techniques and tested at the end of specified period of time. As a result of drop-outs from the experimental group (due to sickness, transfer and soon), the number of students varied from group to group. Do the data below present sufficient evidence to indicate a difference in the mean achievement for the four teaching techniques.

STUDENTS' GROUP			
1	2	3	4
65	75	59	94
87	69	78	89
73	83	67	80
79	81	62	88
81	72	83	
69	79	76	
	90		

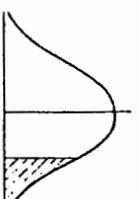
The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-x^2/2} dx$.

the probability that $U > u$, where $U \sim N(0,1)$.



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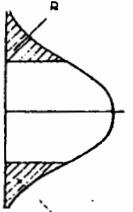
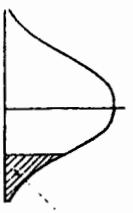
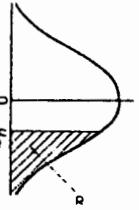


$-u$	-0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
0.99995	0.99997	0.99997	0.99996	0.99996	0.99996	0.99996	0.99996	0.99995	0.99995	0.99995
0.99992	0.99995	0.99995	0.99995	0.99994	0.99994	0.99994	0.99994	0.99993	0.99993	0.99993
0.99989	0.99988	0.99988	0.99988	0.99987	0.99987	0.99987	0.99986	0.99986	0.99985	0.99984
0.99983	0.99983	0.99982	0.99981	0.99981	0.99980	0.99979	0.99978	0.99978	0.99977	0.99977
0.99976	0.99975	0.99974	0.99974	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99966
0.99965	0.99964	0.99964	0.99962	0.99961	0.99960	0.99958	0.99957	0.99955	0.99953	0.99952
0.99950	0.99948	0.99948	0.99946	0.99944	0.99942	0.99940	0.99938	0.99936	0.99934	0.99931
0.99929	0.99926	0.99924	0.99921	0.99918	0.99916	0.99913	0.99910	0.99906	0.99903	0.99903
0.99900	0.99896	0.99893	0.99889	0.99886	0.99882	0.99878	0.99874	0.99865	0.99865	0.99865
0.99861	0.99856	0.99851	0.99846	0.99841	0.99838	0.99831	0.99825	0.99819	0.99813	0.99813
0.99836	0.99827	0.99820	0.99815	0.99810	0.99805	0.99801	0.99795	0.99781	0.99774	0.99774
0.99807	0.99798	0.99790	0.99782	0.99771	0.99763	0.99757	0.99750	0.99744	0.99737	0.99737
0.99786	0.99778	0.99770	0.99762	0.99752	0.99744	0.99737	0.99730	0.99724	0.99717	0.99717
0.99764	0.99752	0.99743	0.99732	0.99721	0.99711	0.99702	0.99693	0.99683	0.99674	0.99663
0.99750	0.99732	0.99719	0.99699	0.99685	0.99669	0.99653	0.99640	0.99627	0.99617	0.99607
0.99720	0.99696	0.99671	0.99649	0.99621	0.99598	0.99558	0.99513	0.99461	0.99413	0.99396
0.99691	0.99663	0.99632	0.99609	0.99585	0.99553	0.99510	0.99462	0.99417	0.99379	0.99339
0.99651	0.99620	0.99566	0.99492	0.99447	0.99406	0.99340	0.99286	0.99226	0.99180	0.99124
0.99611	0.99534	0.99433	0.99305	0.99266	0.992245	0.99224	0.99202	0.99180	0.99124	0.99066
0.99574	0.99494	0.99413	0.99343	0.99292	0.99256	0.99224	0.99202	0.99180	0.99124	0.99066
0.99543	0.99463	0.99382	0.99305	0.99266	0.99224	0.99202	0.99180	0.99124	0.99066	0.99000
0.99520	0.99450	0.99357	0.99290	0.99250	0.99224	0.99202	0.99180	0.99124	0.99066	0.99000
0.99489	0.99419	0.99324	0.99250	0.99214	0.99174	0.99134	0.99091	0.98951	0.98811	0.98752
0.99449	0.99374	0.99295	0.99225	0.99185	0.99147	0.99107	0.98951	0.98811	0.98752	0.98703
0.99408	0.99348	0.99269	0.99200	0.99161	0.99121	0.99081	0.98931	0.98811	0.98752	0.98703
0.99374	0.99314	0.99234	0.99166	0.99126	0.99086	0.98945	0.98811	0.98752	0.98703	0.98644
0.99349	0.99287	0.99202	0.99134	0.99094	0.98950	0.98811	0.98752	0.98703	0.98644	0.98582
0.99319	0.99257	0.99172	0.99104	0.99066	0.98924	0.98811	0.98752	0.98703	0.98644	0.98582
0.99286	0.99214	0.99134	0.99066	0.98924	0.98811	0.98752	0.98703	0.98644	0.98582	0.98520
0.99256	0.99189	0.99104	0.99036	0.98922	0.98811	0.98752	0.98703	0.98644	0.98582	0.98520
0.99226	0.99164	0.99080	0.98949	0.98811	0.98752	0.98703	0.98644	0.98582	0.98520	0.98463
0.99196	0.99149	0.99066	0.98924	0.98811	0.98752	0.98703	0.98644	0.98582	0.98520	0.98463
0.99166	0.99126	0.99042	0.98900	0.98811	0.98752	0.98703	0.98644	0.98582	0.98520	0.98463
0.99136	0.99090	0.98900	0.98768	0.98634	0.98503	0.98463	0.98324	0.98284	0.98244	0.98204
0.99099	0.98953	0.98814	0.98682	0.98543	0.98414	0.98375	0.98336	0.98296	0.98256	0.98216
0.99061	0.98824	0.98694	0.98556	0.98424	0.98384	0.98344	0.98304	0.98264	0.98224	0.98184
0.98927	0.98793	0.98657	0.98518	0.98381	0.98244	0.98121	0.98089	0.97952	0.97814	0.97674
0.98891	0.98746	0.98608	0.98468	0.98324	0.98185	0.98044	0.97903	0.97765	0.97627	0.97487
0.98859	0.98714	0.98575	0.98436	0.98292	0.98147	0.98006	0.97865	0.97727	0.97587	0.97447
0.98827	0.98674	0.98534	0.98394	0.98254	0.98115	0.97974	0.97834	0.97693	0.97553	0.97413
0.98793	0.98639	0.98492	0.98350	0.98212	0.98071	0.97930	0.97789	0.97648	0.97508	0.97368
0.98754	0.98608	0.98463	0.98324	0.98185	0.98044	0.97893	0.97752	0.97611	0.97470	0.97330
0.98724	0.98574	0.98434	0.98292	0.98153	0.97992	0.97851	0.97710	0.97569	0.97428	0.97287
0.98693	0.98537	0.98394	0.98254	0.98115	0.97974	0.97833	0.97692	0.97551	0.97410	0.97269
0.98662	0.98500	0.98362	0.98223	0.98082	0.97941	0.97799	0.97658	0.97517	0.97376	0.97235
0.98632	0.98487	0.98347	0.98208	0.98067	0.97927	0.97786	0.97645	0.97504	0.97363	0.97222
0.98601	0.98449	0.98302	0.98163	0.97922	0.97781	0.97640	0.97499	0.97358	0.97217	0.97076
0.98570	0.98419	0.98271	0.98132	0.97891	0.97750	0.97609	0.97468	0.97327	0.97186	0.96945
0.98539	0.98391	0.98249	0.98108	0.97868	0.97727	0.97586	0.97445	0.97304	0.97163	0.96922
0.98508	0.98354	0.98214	0.98073	0.97832	0.97691	0.97550	0.97409	0.97268	0.97127	0.96886
0.98477	0.98314	0.98173	0.97932	0.97691	0.97550	0.97409	0.97268	0.97127	0.96986	0.96745
0.98446	0.98274	0.98134	0.97893	0.97652	0.97511	0.97370	0.97229	0.97088	0.96947	0.96706
0.98415	0.98234	0.98092	0.97851	0.97612	0.97471	0.97330	0.97189	0.97048	0.96907	0.96666
0.98384	0.98293	0.98152	0.97911	0.97670	0.97529	0.97388	0.97247	0.97106	0.96965	0.96724
0.98353	0.98252	0.98112	0.97871	0.97630	0.97489	0.97348	0.97207	0.97066	0.96925	0.96684
0.98322	0.98190	0.97949	0.97708	0.97467	0.97326	0.97185	0.97044	0.96893	0.96752	0.96511
0.98291	0.98149	0.97908	0.97667	0.97426	0.97285	0.97144	0.96993	0.96852	0.96711	0.96469
0.98260	0.98090	0.97858	0.97617	0.97376	0.97235	0.97094	0.96953	0.96812	0.96671	0.96430
0.98229	0.97949	0.97708	0.97467	0.97226	0.97085	0.96944	0.96793	0.96652	0.96511	0.96269
0.98198	0.97893	0.97652	0.97411	0.97170	0.96929	0.96788	0.96647	0.96506	0.96365	0.96123
0.98167	0.97743	0.97502	0.97261	0.96920	0.96779	0.96638	0.96497	0.96357	0.96216	0.95975
0.98136	0.97593	0.97352	0.97111	0.96770	0.96629	0.96488	0.96347	0.96206	0.96065	0.95823
0.98105	0.97443	0.97202	0.96961	0.96620	0.96479	0.96338	0.96197	0.96056	0.95915	0.95673
0.98074	0.97293	0.96952	0.96711	0.96460	0.96319	0.96178	0.96037	0.95896	0.95755	0.95513
0.98043	0.97143	0.96802	0.96561	0.96320	0.96179	0.96038	0.95897	0.95756	0.95615	0.95373
0.98012	0.96992	0.96651	0.96410	0.96169	0.96028	0.95887	0.95746	0.95605	0.95464	0.95222
0.97981	0.96843	0.96502	0.96261	0.95920	0.95779	0.95638	0.95497	0.95356	0.95215	0.94973
0.97950	0.96792	0.96451	0.96210	0.95869	0.95728	0.95587	0.95446	0.95305	0.95164	0.94922
0.97919	0.96643	0.96302	0.96061	0.95720	0.95579	0.95438	0.95297	0.95156	0.95015	

The u_α values tabulated are such that $\Pr(U > u_\alpha) = \alpha$, where $U \sim N(0,1)$

ONE-SIDED TEST

TWO-SIDED TEST



$\Pr(T_v > t_v(\alpha)) = \alpha$,
for v degrees of freedom.

$\Pr(T_v > t_v(\alpha) \text{ or } T_v < -t_v(\alpha)) = 2\alpha$,
for v degrees of freedom.

α	u_α	α	u_α	α	u_α	α	u_α
0.50	0.0000	0.34	0.41246	0.18	0.91537	0.025	1.98000
0.49	0.02507	0.33	0.43991	0.17	0.95416	0.020	2.05375
0.48	0.05015	0.32	0.46770	0.16	0.99446	0.010	2.30635
0.47	0.07527	0.31	0.49585	0.15	1.03643	0.009	2.36862
0.46	0.10004	0.30	0.52440	0.14	1.08032	0.008	2.40891
0.45	0.12566	0.29	0.55338	0.13	1.12639	0.007	2.46726
0.44	0.15097	0.28	0.58284	0.12	1.17459	0.006	2.51214
0.43	0.17637	0.27	0.61281	0.11	1.22653	0.005	2.57583
0.42	0.20189	0.26	0.64335	0.10	1.28155	0.004	2.63207
0.41	0.22754	0.25	0.67449	0.09	1.34076	0.003	2.74778
0.40	0.25335	0.24	0.70630	0.08	1.40567	0.002	2.87816
0.39	0.27932	0.23	0.73885	0.07	1.47579	0.001	3.08023
0.38	0.30548	0.22	0.77219	0.06	1.55477	0.0005	3.28053
0.37	0.33185	0.21	0.80642	0.05	1.64485	0.0001	3.71902
0.36	0.35846	0.20	0.84162	0.04	1.75069	0.00005	3.89060
0.35	0.38632	0.19	0.87790	0.03	1.88079	0.00001	4.28648

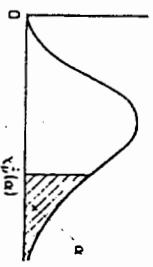
Table 6 ORDINATES OF THE STANDARDISED NORMAL DISTRIBUTION

The function tabulated is $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$.



u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.39894	0.39695	0.39104	0.38139	0.36827	0.35207	0.33322	0.31225	0.28969	0.26669
1.0	0.24197	0.19419	0.17137	0.14973	0.12592	0.11082	0.09405	0.07895	0.06562	0.05199
2.0	0.05399	0.04347	0.03239	0.02172	0.01753	0.01358	0.01042	0.00792	0.00595	0.00443
3.0	0.00443	0.00327	0.00238	0.00172	0.00123	0.00087	0.00061	0.00042	0.00029	0.00013
4.0	0.00013	0.00009	0.00006	0.00004	0.00002	0.00001	0.00000	0.00000	0.00000	0.00000

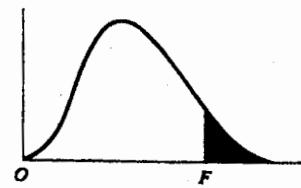
The values tabulated are $\chi_{\nu}^2(\alpha)$, where
 $\Pr(\chi_{\nu}^2 > \chi_{\nu}^2(\alpha)) = \alpha$, for ν degrees of freedom.



For $\nu > 30$ take $\chi_{\nu}^2(\alpha) = \nu \left[1 - \frac{1}{9\nu} + u_{\alpha} \sqrt{\frac{1}{9\nu}} \right]$ where u_{α} is such that $\Pr(U > u_{\alpha}) = \alpha$, and $U \sim N(0,1)$.

ν	α	0.995	0.990	0.975	0.950	0.900	0.750	0.500	0.00
392704, 10 ⁻¹⁰	157988, 10 ⁻⁹	982069, 10 ⁻⁸	393214, 10 ⁻⁷	0.0157908	0.1015308	0.454936	1	1	
0.0100251	0.0201007	0.0506356	0.102587	0.210721	0.575364	1.38629	2	2	
0.0717218	0.14832	0.215795	0.351846	0.584374	1.212534	2.36597	3	3	
0.206989	0.297109	0.484419	0.710723	1.063623	1.92256	3.35669	4	4	
0.411742	0.554298	0.831212	1.145476	1.61031	2.67480	4.35146	5	5	
0.675727	0.872090	1.23734	1.63538	2.20413	3.45460	5.34812	6	6	
0.989256	1.239043	1.68987	2.16735	2.83311	4.25485	6.34581	7	7	
1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	8	8	
1.73493	2.08790	2.70039	3.2511	4.16816	5.89883	8.34283	9	9	
2.15886	2.56821	3.24697	3.94030	4.86518	6.73720	9.34182	10	10	
2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.3410	11	11	
3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.3403	12	12	
3.56503	4.10592	5.00875	5.89186	7.04150	9.29907	12.3398	13	13	
4.07467	4.66043	5.62873	6.57063	7.78853	10.1653	13.3393	14	14	
4.60092	5.22335	6.26214	7.26094	8.54676	11.0365	14.3389	15	15	
5.14221	5.81221	6.90766	7.91224	9.31224	11.9172	15.3385	16	16	
5.68722	6.40776	7.56419	8.67176	10.0852	12.7919	16.3382	17	17	
6.24840	7.01491	8.23075	9.39046	10.8649	13.6753	17.3379	18	18	
6.84397	7.63273	8.90652	10.1170	11.65652	14.5620	18.3377	19	19	
7.43384	8.26040	9.59078	10.85078	12.4426	15.4518	19.3374	20	20	
8.03365	8.89720	10.28293	11.5913	13.2396	16.3444	20.3372	21	21	
8.64272	9.54249	10.9823	12.3380	14.0415	17.2396	21.3370	22	22	
9.26043	10.19867	11.6886	13.0905	14.8480	18.1373	22.3369	23	23	
9.88623	10.8564	12.4012	13.8484	15.6587	19.0373	23.3367	24	24	
10.5197	11.5240	13.1197	14.6714	18.4734	19.9393	24.3366	25	25	
11.1602	12.1981	13.8439	15.3792	17.2919	20.3434	25.3365	26	26	
11.8076	12.8785	14.5734	16.1514	18.1139	21.7494	26.3363	27	27	
12.4613	13.5647	15.3079	16.9279	18.9392	22.6572	27.3362	28	28	
13.1211	14.2565	16.0471	17.7084	19.7677	23.5686	28.3361	29	29	
13.7867	14.9535	16.7908	18.4927	20.5592	24.4776	29.3360	30	30	
20.7065	22.1843	24.4330	26.5093	29.0506	32.3363	39.3363	40	40	
27.9907	29.7667	32.3574	34.7643	37.6886	42.9421	49.3349	50	50	
35.5345	37.4849	40.4817	43.1880	46.4589	52.2838	59.3347	60	60	
43.2752	45.4417	48.7576	51.7393	55.3289	61.6983	69.3345	70	70	
51.1719	53.5401	57.1532	60.3915	64.2778	71.1445	79.3343	80	70	
59.1963	61.7641	65.6466	69.1260	73.2911	80.6247	88.1303	80	80	
67.3276	70.0649	74.2219	77.9295	82.3881	90.1332	99.3341	90	90	
							100	100	
							109.141	109.141	

ν	α	0.250	0.100	0.050	0.025	0.010	0.005	0.001
1	1.32330	2.70554	3.84148	5.02389	6.63490	7.87944	10.828	
2	2.77259	4.60517	5.99146	7.37776	9.21034	10.5986	13.816	
3	4.10834	6.25139	7.81473	9.34840	11.3449	12.8382	16.266	
4	5.38827	7.77944	9.48773	11.1433	13.2767	14.8603	18.467	
5	6.62668	9.23636	11.0705	12.8325	15.0863	16.7496	20.515	
6	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476	22.458	
7	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777	24.322	
8	10.2189	13.3816	15.5073	17.5345	20.0902	21.9550	26.125	
9	11.3888	14.68337	16.9190	19.0728	21.6660	23.5894	27.877	
10	12.5489	15.9872	18.3070	20.4832	23.2093	25.1882	29.588	
11	13.7007	17.2750	19.5751	21.9200	24.7250	26.7568	31.264	
12	14.8454	21.0261	23.3367	26.2170	28.2995	32.909		
13	15.9493	24.7871	27.5871	30.1910	33.4087	37.7185		
14	17.1169	21.6049	24.7884	27.1718	30.1435	33.1914	36.123	
15	18.2451	22.3071	24.9958	27.4884	30.5779	32.8013	37.697	
16	19.3689	23.5418	26.2962	28.8454	31.9999	34.2672	39.252	
17	20.4887	24.8700	27.5871	30.2784	34.0803	37.1565	42.312	
18	21.6049	25.9894	28.8693	31.5053	34.8523	38.5823	43.820	
19	22.7178	27.2036	30.1435	32.8523	36.1909	38.5823		
20	23.8277	28.4120	31.4104	34.1696	37.5662	39.9688	45.315	
21	24.9348	29.6151	32.6706	35.4789	38.9322	41.4011	48.977	
22	26.0893	30.8133	33.9244	36.7807	40.2894	42.7957	48.268	
23	27.1413	32.0069	35.1725	38.0756	41.6384	44.1813	48.728	
24	28.2412	33.1982	36.4150	39.3841	42.9798	45.5585	51.179	
25	29.3369	34.3816	37.6525	40.6485	44.3141	46.9279	52.618	
26	30.4346	35.5532	38.8851	41.9232	45.8417	48.2889	54.052	
27	31.5784	37.4712	40.1133	43.1945	46.9628	49.6449	55.476	
28	32.6205	37.9159	41.3371	44.4608	48.2782	50.9334	56.892	
29	33.7109	39.0875	42.5570	45.7223	48.5879	52.3386	58.301	
30	34.7997	40.2560	43.7730	46.9792	50.8922	53.6770	59.703	
31	35.9363	41.9051	55.3785	58.3417	63.6907	66.7660	73.402	
32	37.6886	42.9421	49.3349	50	52	56.1336	68.661	
33	38.7686	43.1880	46.9449	52.1420	57.1671	64.9800	74.3970	
34	40.4817	44.4589	50.9334	54.9877	60.9815	70.0819	83.2977	
35	42.4817	46.4589	52.2838	57.9075	64.9815	71.9517	91.9517	
36	43.1880	47.4817	53.3445	58.3345	65.9322	70.9322	85.9322	
37	44.4589	48.4589	54.3345	59.3345	67.9322	72.9322	88.9322	
38	46.4589	49.4589	55.3345	60.3345	68.9322	74.9322	91.9322	
39	48.4589	50.4589	56.3345	61.3345	69.9322	76.9322	93.9322	
40	50	51.8051	55.7585	57.1420	63.6907	66.7660	73.402	
41	52	53.1336	56.1336	57.4322	64.9815	67.9322	74.3970	
42	54	54.1336	57.4322	58.7322	66.9800	69.9322	77.9322	
43	56	56.1336	58.7322	59.7322	68.9815	70.9322	79.9322	
44	58	58.1336	60.7322	61.7322	69.9815	71.9322	80.9322	
45	60	60	62.1336	63.1336	70.9819	72.9322	81.9322	
46	62	62.1336	64.1336	65.1336	72.9819	74.9322	83.9322	
47	64	64.1336	66.1336	67.1336	74.9819	76.9322	85.9322	
48	66	66.1336	68.1336	69.1336	76.9819	78.9322	87.9322	
49	68	68.1336	70.1336	71.1336	78.9819	80.9322	89.9322	
50	70	70	72.1336	73.1336	80.9819	82.9322	91.9322	
51	72	72.1336	74.1336	75.1336</td				

Table B-4 *F* distribution5 percent (Roman type) and 1 percent (italic type) points for the distribution of *F*

Degrees of freedom for denominator (<i>v</i> ₂)	Degrees of freedom for numerator (<i>v</i> ₁)																								
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
1	161 4052	200 4999	216 5403	225 5625	230 5764	234 5859	237 5928	239 5981	241 6022	242 6056	243 6082	244 6106	245 6142	246 6169	248 6208	249 6234	250 6258	251 6286	252 6302	253 6323	253 6334	254 6352	254 6361	254 6366	
2	18.51 98.49	19.00 99.01	19.16 99.17	19.25 99.25	19.30 99.30	19.33 99.33	19.36 99.36	19.37 99.37	19.38 99.38	19.39 99.39	19.40 99.40	19.41 99.41	19.42 99.42	19.43 99.43	19.44 99.44	19.45 99.45	19.46 99.46	19.47 99.47	19.47 99.48	19.47 99.49	19.47 99.49	19.48 99.49	19.49 99.49	19.49 99.49	19.50 99.50
3	10.13 36.12	9.55 30.81	9.28 29.46	9.12 28.71	9.01 28.24	8.94 27.91	8.88 27.67	8.84 27.49	8.81 27.34	8.78 27.23	8.76 27.13	8.74 27.05	8.71 26.93	8.69 26.83	8.66 26.69	8.64 26.50	8.62 26.42	8.60 26.30	8.58 26.27	8.57 26.23	8.56 26.18	8.54 26.14	8.54 26.12	8.53 26.12	
4	7.71 21.20	6.94 18.00	6.59 16.49	6.39 15.58	6.26 15.53	6.16 15.22	6.09 14.94	6.04 14.66	6.00 14.54	5.96 14.45	5.93 14.37	5.91 14.25	5.87 14.02	5.84 13.83	5.81 13.63	5.78 13.43	5.74 13.24	5.71 13.07	5.68 12.91	5.66 12.82	5.65 12.72	5.64 12.62	5.63 12.52		
5	6.61 16.26	5.79 13.27	5.41 12.06	5.19 11.39	5.05 10.87	4.95 10.67	4.88 10.45	4.82 10.37	4.78 10.25	4.74 10.15	4.70 9.96	4.68 9.89	4.64 9.77	4.60 9.68	4.56 9.55	4.53 9.47	4.50 9.38	4.46 9.29	4.42 9.24	4.40 9.13	4.38 9.07	4.37 9.02	4.36 9.02		
6	5.59 13.74	5.14 10.92	4.76 9.78	4.53 9.15	4.39 8.75	4.28 8.47	4.21 8.26	4.15 8.10	4.10 7.98	4.06 7.87	4.03 7.79	4.00 7.72	3.96 7.60	3.92 7.52	3.87 7.39	3.84 7.32	3.81 7.23	3.77 7.14	3.75 7.09	3.72 7.02	3.69 6.99	3.68 6.86	3.67 6.80		
7	5.39 12.25	4.74 9.55	4.35 8.45	4.12 7.85	3.97 7.46	3.87 7.19	3.79 7.00	3.73 6.84	3.68 6.71	3.63 6.62	3.60 6.54	3.57 6.47	3.52 6.35	3.49 6.27	3.44 6.15	3.41 6.07	3.38 5.98	3.34 5.90	3.32 5.85	3.29 5.78	3.28 5.75	3.25 5.70	3.24 5.67	3.23 5.65	
8	5.32 11.26	4.46 8.65	4.07 7.59	3.84 7.02	3.69 6.63	3.58 6.37	3.50 6.19	3.44 6.03	3.39 5.91	3.34 5.82	3.31 5.74	3.28 5.67	3.23 5.56	3.20 5.48	3.15 5.38	3.12 5.21	3.08 5.11	3.05 5.06	3.03 5.00	3.00 4.96	2.98 4.91	2.96 4.86	2.95 4.81	2.93 4.71	
9	5.12 10.56	4.26 8.02	3.86 6.99	3.63 6.42	3.48 6.06	3.37 5.80	3.29 5.62	3.23 5.47	3.18 5.35	3.13 5.26	3.10 5.20	3.07 5.11	3.02 5.00	2.98 4.92	2.93 4.80	2.90 4.73	2.86 4.64	2.82 4.56	2.80 4.51	2.77 4.45	2.76 4.42	2.73 4.36	2.72 4.33	2.71 4.31	
10	4.96 10.04	4.10 7.56	3.71 6.55	3.48 5.99	3.33 5.64	3.22 5.39	3.14 5.31	3.07 5.06	3.02 4.95	2.97 4.85	2.94 4.78	2.91 4.71	2.86 4.60	2.82 4.52	2.77 4.42	2.74 4.33	2.70 4.25	2.67 4.17	2.64 4.12	2.61 4.05	2.59 4.02	2.56 3.96	2.55 3.93	2.54 3.91	
11	4.84 9.85	3.98 7.29	3.59 6.22	3.36 5.67	3.20 5.32	3.09 5.07	3.01 4.88	2.95 4.74	2.90 4.63	2.96 4.54	2.82 4.46	2.79 4.40	2.74 4.39	2.70 4.21	2.65 4.10	2.61 4.02	2.57 3.94	2.53 3.86	2.50 3.80	2.47 3.74	2.45 3.70	2.42 3.66	2.41 3.62	2.40 3.60	
12	4.75 9.33	3.88 6.93	3.49 5.95	3.26 5.42	3.11 5.06	3.00 4.82	2.92 4.65	2.85 4.50	2.80 4.39	2.76 4.30	2.72 4.22	2.69 4.16	2.64 4.05	2.60 3.96	2.54 3.86	2.50 3.78	2.46 3.70	2.42 3.62	2.40 3.56	2.36 3.49	2.35 3.46	2.32 3.41	2.31 3.38	2.30 3.36	
13	4.67 9.07	3.80 6.70	3.41 5.74	3.18 5.20	3.02 4.86	2.92 4.42	2.84 4.44	2.77 4.30	2.72 4.19	2.67 4.10	2.63 4.02	2.60 3.96	2.55 3.85	2.51 3.78	2.46 3.67	2.42 3.59	2.38 3.51	2.34 3.42	2.32 3.37	2.28 3.30	2.26 3.27	2.24 3.21	2.22 3.18	2.21 3.16	
14	4.60 8.86	3.74 6.51	3.34 5.56	3.11 5.03	2.96 4.69	2.85 4.46	2.77 4.28	2.70 4.14	2.65 4.03	2.60 3.94	2.56 3.86	2.53 3.80	2.50 3.70	2.45 3.62	2.40 3.52	2.36 3.43	2.31 3.34	2.27 3.26	2.24 3.21	2.21 3.18	2.19 3.15	2.16 3.12	2.14 3.09	2.13 3.06	
15	4.54 8.69	3.68 6.36	3.29 5.42	3.06 4.89	2.90 4.58	2.79 4.32	2.70 4.14	2.64 4.00	2.59 3.89	2.55 3.80	2.51 3.73	2.48 3.67	2.43 3.56	2.39 3.48	2.33 3.36	2.29 3.28	2.25 3.20	2.21 3.12	2.18 3.07	2.15 3.00	2.12 2.97	2.10 2.92	2.08 2.89	2.07 2.87	
16	4.49 8.53	3.63 6.23	3.24 5.29	3.01 4.77	2.85 4.44	2.74 4.20	2.66 4.03	2.59 3.89	2.54 3.76	2.49 3.69	2.45 3.61	2.42 3.55	2.37 3.43	2.33 3.37	2.28 3.28	2.24 3.20	2.20 3.10	2.16 3.01	2.13 2.96	2.10 2.89	2.07 2.80	2.07 2.77	2.07 2.75		
17	4.45 8.40	3.59 6.11	3.20 5.18	2.81 4.67	2.70 4.34	2.62 4.20	2.55 3.93	2.50 3.79	2.45 3.68	2.41 3.59	2.38 3.52	2.33 3.45	2.29 3.35	2.23 3.27	2.22 3.26	2.19 3.08	2.15 3.00	2.11 2.92	2.08 2.86	2.04 2.79	2.02 2.76	1.99 2.70	1.97 2.67	1.96 2.65	
18	4.41 8.28	3.55 6.01	3.16 5.09	2.93 4.54	2.77 4.25	2.66 4.01	2.58 3.85	2.51 3.71	2.46 3.60	2.41 3.51	2.37 3.44	2.34 3.37	2.29 3.27	2.19 3.19	2.15 3.07	2.11 3.00	2.07 2.91	2.04 2.83	2.00 2.78	2.00 2.71	1.98 2.68	1.95 2.62	1.93 2.59	1.92 2.57	
19	4.38 8.18	3.52 5.93	3.13 5.01	2.90 4.50	2.74 4.17	2.63 3.94	2.55 3.77	2.48 3.63	2.43 3.52	2.38 3.43	2.34 3.36	2.31 3.30	2.26 3.19	2.21 3.12	2.15 3.00	2.11 2.92	2.07 2.84	2.02 2.76	2.00 2.70	1.96 2.63	1.94 2.60	1.91 2.54	1.90 2.51	1.88 2.49	
20	4.35 8.10	3.49 5.83	3.10 4.43	2.87 4.20	2.66 3.87	2.52 3.72	2.45 3.56	2.40 3.45	2.35 3.37	2.31 3.30	2.28 3.23	2.23 3.13	2.18 3.05	2.12 2.94	2.08 2.84	2.04 2.76	1.99 2.69	1.96 2.63	1.92 2.56	1.89 2.53	1.87 2.47	1.85 2.44	1.84 2.42		
21	4.32 8.02	3.47 5.78	3.07 4.47	2.84 4.04	2.68 3.81	2.55 3.63	2.49 3.45	2.42 3.31	2.37 3.24	2.32 3.17	2.28 3.07	2.25 3.07	2.20 2.99	2.15 2.88	2.10 2.80	2.05 2.72	2.00 2.63	1.96 2.58	1.93 2.51	1.89 2.47	1.87 2.42	1.85 2.38	1.84 2.36		
22	4.30 7.94	3.44 5.72	3.05 4.82	2.82 4.32	2.66 3.99	2.55 3.76	2.47 3.59	2.40 3.45	2.35 3.37	2.30 3.30	2.26 3.23	2.23 3.13	2.18 3.04	2.12 2.94	2.08 2.84	2.04 2.76	1.98 2.67	1.93 2.61	1.89 2.54	1.85 2.46	1.81 2.42	1.78 2.37	1.76 2.33		
23	4.28 7.88	3.42 5.66	3.03 4.76	2.80 4.36	2.64 3.94	2.53 3.71	2.45 3.54	2.38 3.41	2.32 3.30	2.28 3.21	2.24 3.14	2.20 3.07	2.14 3.07	2.10 2.97	2.04 2.89	2.00 2.78	1.96 2.62	1.91 2.53	1.88 2.46	1.84 2.41	1.82 2.37	1.77 2.32	1.76 2.28		
24	4.26 7.82	3.40 5.61	3.01 4.72	2.78 4.22	2.62 3.90	2.51 3.67	2.43 3.50	2.36 3.36	2.30 3.25	2.26 3.17	2.22 3.09	2.18 3.03	2.13 2.93	2.09 2.85	2.02 2.74	1.98 2.66	1.94 2.49	1.89 2.44	1.86 2.36	1.82 2.32	1.78 2.27	1.74 2.23	1.73 2.21		
25	4.24 7.77	3.38 5.57	2.99 4.64	2.76 4.18	2.60 3.86	2.49 3.63	2.41 3.46	2.34 3.32	2.28 3.21	2.24 3.13	2.20 3.05	2.16 2.99	2.11 2.89												

Table B-4 (Continued)
5 percent (Roman type) and 1 percent (italic type) points for the distribution of F

Degrees of freedom for denominator (v_2)	Degrees of freedom for numerator (v_1)																							
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
26	4.22 7.72	3.37 5.53	2.89 4.64	2.74 4.44	2.59 3.82	2.47 3.59	2.39 3.42	2.32 3.29	2.27 3.17	2.22 3.09	2.18 3.02	2.15 2.96	2.10 2.86	2.05 2.77	1.99 2.66	1.95 2.58	1.90 2.50	1.85 2.41	1.82 2.36	1.78 2.38	1.76 2.25	1.72 2.19	1.70 2.15	1.69 2.13
27	4.21 7.68	3.35 5.49	2.86 4.60	2.73 4.11	2.57 3.79	2.46 3.56	2.37 3.39	2.30 3.26	2.25 3.14	2.20 3.06	2.16 2.98	2.13 2.93	2.08 2.83	2.03 2.74	1.97 2.63	1.93 2.55	1.88 2.47	1.84 2.38	1.80 2.33	1.76 2.25	1.74 2.21	1.71 2.16	1.68 2.12	1.67 2.10
28	4.20 7.64	3.34 5.45	2.85 4.57	2.71 4.07	2.56 3.76	2.44 3.53	2.36 3.36	2.29 3.23	2.24 3.11	2.19 3.03	2.15 2.95	2.12 2.90	2.06 2.80	2.02 2.71	1.96 2.60	1.91 2.52	1.87 2.44	1.81 2.35	1.78 2.30	1.75 2.22	1.72 2.18	1.69 2.13	1.67 2.09	1.65 2.06
29	4.18 7.60	3.33 5.32	2.83 4.34	2.70 4.04	2.54 3.73	2.43 3.50	2.35 3.33	2.28 3.20	2.22 3.00	2.18 2.92	2.14 2.87	2.10 2.77	2.05 2.68	2.00 2.57	1.94 2.49	1.90 2.41	1.85 2.32	1.80 2.27	1.77 2.19	1.73 2.15	1.71 2.10	1.68 2.06	1.65 2.03	
30	4.17 7.56	3.32 5.39	2.82 4.51	2.69 4.02	2.53 3.70	2.42 3.47	2.34 3.30	2.27 3.17	2.21 3.06	2.16 2.98	2.12 2.90	2.09 2.84	2.04 2.74	1.99 2.66	1.93 2.55	1.89 2.47	1.84 2.38	1.79 2.29	1.76 2.24	1.72 2.16	1.69 2.13	1.66 2.07	1.64 2.03	
32	4.15 7.50	3.30 5.34	2.80 4.46	2.67 3.97	2.51 3.66	2.40 3.42	2.32 3.25	2.25 3.01	2.19 2.94	2.14 2.86	2.10 2.80	2.07 2.70	2.02 2.62	1.97 2.42	1.91 2.34	1.86 2.25	1.82 2.15	1.76 2.05	1.74 2.02	1.69 2.08	1.67 2.02	1.64 1.98		
34	4.13 7.44	3.28 5.29	2.88 4.42	2.65 3.93	2.49 3.61	2.38 3.38	2.30 3.21	2.23 3.08	2.17 2.97	2.12 2.89	2.08 2.82	2.05 2.76	2.00 2.66	1.95 2.58	1.89 2.47	1.84 2.38	1.80 2.30	1.74 2.21	1.71 2.15	1.67 2.08	1.64 2.04	1.59 1.94		
36	4.11 7.39	3.26 5.25	2.86 4.38	2.63 3.89	2.48 3.58	2.36 3.35	2.28 3.18	2.21 3.04	2.15 2.94	2.10 2.86	2.06 2.78	2.03 2.72	1.99 2.62	1.93 2.54	1.87 2.43	1.82 2.35	1.78 2.26	1.69 2.22	1.65 2.04	1.62 2.00	1.59 1.90	1.56 1.87		
38	4.10 7.35	3.25 5.21	2.85 4.34	2.62 3.86	2.46 3.54	2.35 3.32	2.26 3.15	2.19 3.02	2.14 2.91	2.09 2.82	2.05 2.75	2.02 2.69	1.96 2.59	1.92 2.51	1.85 2.30	1.80 2.32	1.76 2.22	1.71 2.14	1.67 2.08	1.63 2.00	1.60 1.97	1.57 1.86		
40	4.08 7.31	3.23 5.18	2.84 4.31	2.61 3.83	2.45 3.51	2.34 3.29	2.25 3.12	2.18 2.99	2.12 2.88	2.07 2.80	2.04 2.73	2.00 2.66	1.95 2.49	1.90 2.37	1.84 2.37	1.79 2.29	1.74 2.20	1.71 2.11	1.69 2.05	1.66 1.97	1.61 1.94	1.59 1.84		
42	4.07 7.27	3.22 5.15	2.83 4.29	2.59 3.80	2.44 3.49	2.32 3.36	2.24 3.10	2.17 2.96	2.11 2.86	2.06 2.77	2.02 2.70	1.99 2.64	1.94 2.54	1.89 2.46	1.82 2.35	1.78 2.26	1.73 2.17	1.68 2.06	1.64 2.02	1.60 1.94	1.57 1.80	1.54 1.78		
44	4.06 7.24	3.21 5.12	2.82 4.26	2.58 3.78	2.43 3.46	2.31 3.24	2.23 3.07	2.16 2.94	2.10 2.84	2.05 2.75	2.01 2.68	1.98 2.62	1.92 2.52	1.88 2.44	1.81 2.32	1.76 2.25	1.72 2.06	1.66 2.00	1.63 1.92	1.58 1.88	1.56 1.75	1.52 1.75		
46	4.05 7.21	3.20 5.10	2.81 4.24	2.57 3.76	2.42 3.44	2.30 3.22	2.22 3.05	2.14 2.92	2.09 2.82	2.04 2.73	2.00 2.66	1.97 2.60	1.91 2.50	1.87 2.42	1.80 2.40	1.75 2.22	1.71 2.13	1.65 2.04	1.62 1.98	1.57 1.90	1.54 1.86	1.51 1.72		
48	4.04 7.19	3.19 5.08	2.80 4.22	2.56 3.74	2.41 3.42	2.30 3.20	2.21 3.04	2.14 2.90	2.08 2.80	2.03 2.71	1.99 2.64	1.96 2.58	1.90 2.48	1.86 2.40	1.79 2.28	1.74 2.20	1.70 2.11	1.64 2.02	1.61 1.96	1.56 1.88	1.53 1.84	1.50 1.70		
50	4.03 7.17	3.18 5.06	2.79 4.20	2.56 3.72	2.40 3.41	2.29 3.18	2.20 3.02	2.13 2.88	2.07 2.78	2.02 2.70	1.98 2.62	1.95 2.56	1.90 2.46	1.85 2.39	1.78 2.26	1.74 2.18	1.69 2.10	1.65 2.00	1.63 1.94	1.55 1.86	1.52 1.82	1.48 1.76		
55	4.02 7.12	3.17 5.01	2.78 4.16	2.54 3.68	2.38 3.37	2.27 3.15	2.18 2.98	2.11 2.85	2.05 2.75	2.00 2.66	1.97 2.59	1.93 2.53	1.88 2.43	1.83 2.35	1.76 2.23	1.72 2.15	1.67 2.04	1.61 1.96	1.58 1.90	1.52 1.82	1.50 1.71	1.46 1.64		
60	4.00 7.08	3.15 4.94	2.76 4.13	2.52 3.65	2.37 3.34	2.25 3.12	2.17 2.95	2.10 2.82	2.04 2.72	1.99 2.63	1.95 2.56	1.92 2.50	1.86 2.40	1.81 2.32	1.75 2.20	1.70 2.12	1.65 2.03	1.59 1.93	1.56 1.87	1.50 1.74	1.48 1.60			
65	3.99 7.04	3.14 4.95	2.75 4.10	2.51 3.62	2.36 3.31	2.24 3.09	2.15 2.93	2.08 2.79	2.02 2.70	1.98 2.61	1.94 2.54	1.90 2.47	1.85 2.30	1.73 2.17	1.68 2.09	1.63 2.00	1.57 1.84	1.54 1.76	1.49 1.64	1.46 1.60				
70	3.98 7.01	3.13 4.92	2.74 4.08	2.50 3.60	2.35 3.29	2.32 3.07	2.14 2.91	2.07 2.77	2.01 2.67	1.97 2.59	1.93 2.51	1.89 2.45	1.84 2.35	1.79 2.28	1.72 2.15	1.67 2.07	1.62 1.98	1.56 1.82	1.53 1.74	1.47 1.63	1.45 1.56			
80	3.96 6.96	3.11 4.88	2.72 4.04	2.48 3.56	2.33 3.25	2.21 3.04	2.12 2.87	2.05 2.74	1.99 2.64	1.95 2.55	1.91 2.48	1.88 2.41	1.82 2.32	1.77 2.22	1.70 2.11	1.65 2.03	1.54 1.84	1.51 1.76	1.45 1.68	1.41 1.68				
100	3.94 6.90	3.09 4.82	2.70 3.98	2.46 3.51	2.30 3.20	2.19 2.99	2.10 2.82	2.03 2.69	1.97 2.59	1.92 2.51	1.88 2.43	1.85 2.36	1.79 2.26	1.75 2.19	1.68 2.06	1.63 1.98	1.57 1.79	1.51 1.73	1.48 1.64	1.42 1.63				
125	3.92 6.84	3.07 4.78	2.68 3.94	2.44 3.47	2.29 3.17	2.17 2.95	2.08 2.79	2.01 2.65	1.95 2.56	1.90 2.47	1.86 2.40	1.83 2.33	1.77 2.23	1.72 2.03	1.65 1.94	1.55 1.85	1.49 1.75	1.45 1.68	1.39 1.59	1.36 1.54	1.31 1.40			
150	3.91 6.81	3.06 4.75	2.67 3.91	2.43 3.44	2.27 3.13	2.16 2.92	2.07 2.76	2.00 2.62	1.94 2.53	1.89 2.44	1.85 2.37	1.82 2.30	1.76 2.20	1.71 2.12	1.64 1.91	1.59 1.83	1.54 1.72	1.47 1.66	1.43 1.51	1.37 1.33				
200	3.89 6.76	3.04 4.71	2.65 3.88	2.41 3.41	2.26 3.11	2.14 2.90	2.05 2.73	1.98 2.60	1.92 2.50	1.87 2.42	1.83 2.34	1.78 2.24	1.72 2.17	1.67 2.09	1.61 1.97	1.55 1.88	1.49 1.71	1.45 1.62	1.35 1.48	1.32 1.39	1.28 1.33			
400	3.86 6.70	3.02 4.66	2.62 3.83	2.39 3.36	2.23 3.06	2.12 2.85	2.03 2.69	1.96 2.55	1.90 2.46	1.85 2.37	1.81 2.29	1.78 2.22	1.72 2.04	1.67 1.92	1.54 1.84	1.49 1.74	1.42 1.64	1.37 1.57	1.32 1.42	1.28 1.34				
1000	3.85 6.66	3.00 4.62	2.61 3.80	2.38 3.34	2.22 3.04	2.10 2.82	2.02 2.66	1.95 2.53	1.89 2.43	1.84 2.34	1.80 2.26	1.76 2.20	1.70 2.09	1.65 2.01	1.58 1.89	1.47 1.81	1.41 1.71	1.36 1.62	1.32 1.54	1.28 1.38	1.28 1.33			
	3.84 6.66	2.99 4.60	2.60 3.78	2.37 3.32	2.21 3.02	2.09 2.80	2.01 2.64	1.94 2.51	1.88 2.41	1.83 2.32	1.79 2.24	1.75 2.18	1.69 2.07	1.64 1.99	1.57 1.87	1.52 1.79	1.46 1.69	1.40 1.59	1.35 1.52	1.28 1.42	1.24 1.36	1.17 1.25	1.11 1.11	

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Table B-5 Durbin-Watson statistic (Savin-White tables) Durbin-Watson statistic: 1 percent significance points of d_L and d_U

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
	d_L	d_U								
6	0.390	1.142	—	—	—	—	—	—	—	—
7	0.435	1.036	0.294	1.676	—	—	—	—	—	—
8	0.497	1.003	0.345	1.489	0.229	2.102	—	—	—	—
9	0.554	0.986	0.408	1.389	0.279	1.875	0.183	2.433	—	—
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690
11	0.653	1.010	0.519	1.297	0.394	1.640	0.286	2.030	0.193	2.453
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.826	0.294	2.150
14	0.776	1.054	0.660	1.254	0.547	1.490	0.441	1.757	0.343	2.049
15	0.811	1.070	0.700	1.252	0.591	1.464	0.488	1.704	0.391	1.967
16	0.844	1.086	0.737	1.252	0.631	1.446	0.532	1.663	0.437	1.900
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.630	0.480	1.847
18	0.902	1.118	0.805	1.259	0.708	1.422	0.613	1.604	0.522	1.803
19	0.928	1.132	0.835	1.265	0.742	1.415	0.630	1.584	0.561	1.767
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.590	1.737
21	0.975	1.161	0.890	1.277	0.803	1.408	0.718	1.554	0.633	1.712
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691
23	1.018	1.187	0.938	1.291	0.854	1.407	0.777	1.534	0.698	1.673
24	1.037	1.199	0.960	1.298	0.882	1.407	0.805	1.528	0.728	1.658
25	1.055	1.211	0.981	1.305	0.904	1.409	0.831	1.523	0.754	1.645
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635
27	1.089	1.233	1.019	1.319	0.949	1.413	0.878	1.515	0.808	1.626
28	1.104	1.244	1.037	1.325	0.969	1.415	0.900	1.513	0.832	1.618
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611
30	1.133	1.263	1.070	1.339	1.006	1.421	0.941	1.511	0.877	1.606
31	1.147	1.273	1.085	1.345	1.023	1.425	0.960	1.510	0.897	1.601
32	1.160	1.282	1.100	1.352	1.040	1.428	0.979	1.510	0.917	1.597
33	1.172	1.291	1.114	1.358	1.055	1.432	0.994	1.510	0.936	1.594
34	1.184	1.299	1.128	1.364	1.070	1.435	1.012	1.511	0.954	1.591
35	1.195	1.307	1.140	1.370	1.085	1.439	1.028	1.512	0.971	1.589
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584
40	1.246	1.344	1.196	1.398	1.148	1.457	1.098	1.518	1.048	1.584
41	1.258	1.376	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584
50	1.324	1.403	1.285	1.446	1.245	1.491	1.209	1.538	1.164	1.587
55	1.356	1.427	1.320	1.466	1.284	1.504	1.247	1.548	1.209	1.592
60	1.383	1.449	1.350	1.484	1.317	1.520	1.283	1.558	1.249	1.598
65	1.407	1.468	1.377	1.500	1.346	1.534	1.315	1.568	1.283	1.604
70	1.429	1.485	1.400	1.513	1.372	1.566	1.343	1.578	1.313	1.611
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.340	1.617
80	1.466	1.515	1.441	1.541	1.416	1.568	1.398	1.595	1.364	1.624
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.630
90	1.496	1.540	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.403	1.666
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693
200	1.664	1.684	1.633	1.693	1.643	1.704	1.633	1.715	1.623	1.725

Table B-5 (Continued)

n	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18	k=19	k=20
	d_L	d_U								
16	0.060	3.446	—	—	—	—	—	—	—	—
17	0.084	3.286	0.053	3.306	—	—	—	—	—	—
18	0.113	3.146	0.075	3.358	0.047	3.557	—	—	—	—
19	0.145	3.023	0.102	3.227	0.067	3.420	0.043	3.601	—	—
20	0.178	2.914	0.131	3.109	0.079	3.297	0.061	3.474	0.038	3.639
21	0.212	2.817	0.162	3.004	0.119	3.185	0.084	3.358	0.055	3.521
22	0.246	2.729	0.194	2.909	0.148	3.084	0.109	3.252	0.077	3.412
23	0.281	2.651	0.227	2.822	0.178	2.991	0.136	3.155	0.100	3.311
24	0.315	2.580	0.260	2.744	0.209	2.904	0.163	3.065	0.125	3.218
25	0.348	2.517	0.292	2.674	0.240	2.829	0.194	2.982	0.152	3.131
26	0.381	2.460	0.324	2.610	0.272	2.758	0.224	2.906	0.180	3.050
27	0.413	2.405	0.356	2.552	0.303	2.694	0.253	2.856	0.208	2.976
28	0.444	2.363	0.387	2.499	0.333	2.635	0.283	2.772	0.237	2.907
29	0.474	2.321	0.417	2.451	0.363	2.582	0.313	2.713	0.266	2.843
30	0.503	2.283	0.447	2.407	0.393	2.533	0.342	2.659	0.294	2.905
31	0.531	2.248	0.475	2.367	0.422	2.487	0.371	2.609	0.322	2.730
32	0.558	2.216	0.503	2.330	0.450	2.446	0.399	2.563	0.350	2.680
33	0.585	2.187	0.530	2.296	0.477	2.408	0.426	2.520	0.377	2.633
34	0.610	2.160	0.534	2.266	0.503	2.373	0.452	2.481	0.408	2.590
35	0.634	2.136	0.538	2.237	0.529	2.340	0.478	2.444	0.430	2.550
36	0.658	2.113	0.605	2.210	0.554	2.310	0.504	2.410	0.455	2.512
37	0.680	2.092	0.628	2.186	0.578	2.282	0.528	2.379	0.480	2.477
38	0.702	2.073	0.651	2.164	0.601	2.256	0.552	2.350	0.504	2.445
39	0.723	2.055	0.673	2.143	0.623	2.232	0.575	2.323	0.528	2.414
40	0.744	2.039	0.694	2.123	0.645	2.210	0.597	2.297	0.551	2.386
45	0.835	1.972	0.790	2.084	0.744	2.118	0.700	2.193	0.655	2.269
50	0.913	1.925	0.871	1.987	0.829	2.051	0.787	2.116	0.746	2.182
55	0.979	1.891	0.940	1.945	0.902	2.002	0.863	2.059	0.825	2.117
60	1.037	1.865	1.001	1.914	0.963	1.964	0.923	2.015	0.893	2.067
65	1.087	1.845	1.033	1.889	1.020	1.934	0.964	1.993	0.953	2.027
70	1.131	1.831	1.099	1.870	1.068	1.911	1.037	1.953	1.005	1.995
75	1.170	1.819	1.141	1.856	1.111	1.893	1.082	1.931	1.052	1.976

Table B-5 (Continued)

n	d_L	d_U	k'=1	d_L	d_U	k'=2	d_L	d_U	k'=3	d_L	d_U	k'=4	d_L	d_U	k'=5	d_L	d_U	k'=6	d_L	d_U	k'=7	d_L	d_U	k'=8	d_L	d_U	k'=9	d_L	d_U	k'=10	d_L	d_U					
6	0.610	1.400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—							
7	0.700	1.356	0.467	1.896	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—							
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—							
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—							
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—								
11	0.927	1.324	0.758	1.604	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—							
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—						
13	1.018	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.445	2.390	0.328	2.692	0.230	2.985	0.147	3.266	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—					
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—				
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.908	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589	0.369	2.783	0.290	2.974	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.338	2.885	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.504	2.613	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	1.944	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	2.027	0.941	2.085	0.861	2.181	0.795	2.281	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.086	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—		
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.131	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.200	0.821	2.293	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.949	2.098	0.877	2.180	0.811	2.280	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39	1.435	1.540	1.382	1.597	1.323	1.656	1.273	1.720	1.220	1.789	1.157</																										