UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2007

TITLE OF PAPER:	**Probability Theory**
COURSE CODE :	**ST 201**
TIME ALLOWED :	**THREE (3) HOURS**
INSTRUCTIONS :	**THIS PAPER HAS SEVEN QUESTIONS.** **ANSWER ANY SIX(6) QUESTIONS.** **EACH QUESTION CARRIES 10 MARKS.**
REQUIREMENTS:	**Scientific calculator and statistical table.**

Please do not open this paper until permission has been granted by the Chief Invigilator

QUESTION ONE

(a) A party of hikers, 3 males and 2 females walk along a mountain path in a single line. The ordering of males and females is observed.

(i) Write out the sample spaces.

(ii) Find the subsets of S which correspond to the following events:

U: a female is the leader

V: a male is at the end

W: females are in the second and fourth positions. (2+3 marks)

(b) In a certain industry, 6 persons (2males and 4 females) are eligible for a promotion to two higher positions. Assume that the persons to be promoted are selected at random. Find the probability that

(i) Exactly one female will be promoted. (3marks)

(ii) No female will be promoted. (2marks)

QUESTION TWO

Out of the 15 puppies handled by a Veterinary doctor, 8 were vaccinated against rabies and 9 against distemper. Of those puppies vaccinated, 5 were vaccinated against both rabies and distemper. One puppy is to be selected at random. Let R and D denote the events that the selected puppies are vaccinated against rabies and distemper respectively. Obtain the following probabilities:

(i) Pr(R) ;(ii) Pr(D) ;(iii)Pr(R and D); (iv) Pr(R or D);(v) Pr(puppy was not vaccinated)

(2+2+2+2+2Marks)

QUESTION THREE

A garden is to be planted with three plants one selected at random from each of the three containers. Each container has different varieties of plants in it, and they are mixed with blooming and none blooming ones. Out of 8 plants in the first container, 3 will produce white blooms and 5 red ones. Of the 12 plants in the second container, 7 will produce white blooms and 5 red ones, and of the 18 plants in the third container, 10 will produce white blooms and 6 red and 2 orange blooms. What is the probability that the garden will contain?

(i) All white blooming plants.

(ii) Two red and one orange blooming plants.

(2+2Marks)

If a container is to be selected at random and one plant selected for a single plot, what is the chance that:

(iii) Container I and a red blooming plant is selected.

(iv) A red blooming plant is selected.

(v) A red blooming plant is selected and is found to be from the first container.

(2+2+2Marks)

QUESTION FOUR

In a game of craps you roll two fair dice. Whether you win or lose depends on the sum of the numbers occurring on top of the dice. Let X be the random variable which equals the sum of the numbers on top of the dice.

(i) What values can X take on? (3Marks)

(ii) What is the probability distribution of these X values? (3marks)

(iii) What is the probability that on a single throw X will not be 2 or 7 or 11?

(4Marks)

QUESTION FIVE

The joint probability density function of X and Y is given as $f(x,y)=\begin{cases} x+y; 0\le x\le 1 \text{ and } 0\le y\le 1 \\ 0, \text{elsewhere} \end{cases}$

Find:

(i) The marginal density function for X (3Marks)

(ii) The marginal density function for Y (3Marks)

(iii) E(Y\X=2/3) (4Marks)

QUESTION SIX

If the finishing times in a long road race are normally distributed with μ=10hours and σ=0.7hours.

(i) What is the probability that a racer finishes between 8.2 and 9.6 hours?

(ii) What is the probability that a racer finishes in less than 8.2 hours?

(5+5Marks)

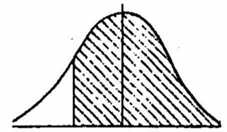
QUESTION SEVEN

(a) A certain examination consists of 12 questions divided into two parts of 6 questions each. How many ways can a student choose any 8 questions if he must attempt 5 questions from the first part? (5Marks)

(b) Let the true proportion of the whole population having a disease is π, if the probabilities of true and false detection remain 0.8 and 0.1 respectively. Find an expression in terms of π for the probability that a person giving a positive test result really has the disease. (5Marks)

The function tabulated is $\frac{1}{\sqrt{2\pi}}\int_u^{\infty} e^{-x^2/2}\,dx$,

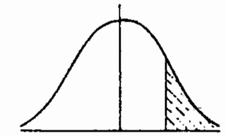
the probability that $U > u$, where $U \sim N(0,1)$.



−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00	u
0.99997	0.99997	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99995	0.99995	−3.9
0.99995	0.99995	0.99995	0.99994	0.99994	0.99994	0.99994	0.99993	0.99993	0.99993	−3.8
0.99992	0.99992	0.99992	0.99992	0.99991	0.99991	0.99990	0.99990	0.99990	0.99989	−3.7
0.99989	0.99988	0.99988	0.99987	0.99987	0.99986	0.99986	0.99985	0.99985	0.99984	−3.6
0.99983	0.99983	0.99982	0.99981	0.99981	0.99980	0.99979	0.99978	0.99978	0.99977	−3.5
0.99976	0.99975	0.99974	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99966	−3.4
0.99965	0.99964	0.99962	0.99961	0.99960	0.99958	0.99957	0.99965	0.99953	0.99962	−3.3
0.99950	0.99948	0.99946	0.99944	0.99942	0.99940	0.99938	0.99936	0.99934	0.99931	−3.2
0.99929	0.99926	0.99924	0.99921	0.99918	0.99916	0.99913	0.99910	0.99906	0.99903	−3.1
0.99900	0.99896	0.99893	0.99889	0.99886	0.99882	0.99878	0.99874	0.99865	0.99865	−3.0
0.99861	0.99856	0.99851	0.99846	0.99841	0.99836	0.99831	0.99825	0.99819	0.99813	−2.9
0.99807	0.99801	0.99795	0.99788	0.99781	0.99774	0.99767	0.99760	0.99752	0.99744	−2.8
0.99736	0.99728	0.99720	0.99711	0.99702	0.99693	0.99683	0.99674	0.99664	0.99653	−2.7
0.99643	0.99632	0.99621	0.99609	0.99598	0.99585	0.99573	0.99560	0.99547	0.99534	−2.6
0.99520	0.99506	0.99492	0.99477	0.99461	0.99446	0.99430	0.99413	0.99396	0.99379	−2.5
0.99361	0.99343	0.99324	0.99305	0.99286	0.99266	0.99245	0.99224	0.99202	0.99180	−2.4
0.99158	0.99134	0.99111	0.99086	0.99061	0.99036	0.99010	0.98983	0.98956	0.98928	−2.3
0.98899	0.98870	0.98840	0.98809	0.98778	0.98745	0.98713	0.98679	0.98645	0.98610	−2.2
0.98574	0.98537	0.98500	0.98461	0.98422	0.98382	0.98341	0.98300	0.98257	0.98214	−2.1
0.98169	0.98124	0.98077	0.98030	0.97982	0.97932	0.97882	0.97831	0.97778	0.97725	−2.0
0.97670	0.97615	0.97558	0.97500	0.97441	0.97381	0.97320	0.97257	0.97193	0.97128	−1.9
0.97062	0.96995	0.96926	0.96856	0.96784	0.96712	0.96638	0.96562	0.96485	0.96407	−1.8
0.96327	0.96246	0.96164	0.96080	0.95994	0.95907	0.95818	0.95728	0.95637	0.95543	−1.7
0.95449	0.95352	0.95254	0.95154	0.95053	0.94950	0.94845	0.94738	0.94630	0.94520	−1.6
0.94408	0.94295	0.94179	0.94062	0.93943	0.93822	0.93699	0.93574	0.93448	0.93319	−1.5
0.93189	0.93056	0.92922	0.92785	0.92647	0.92507	0.92364	0.92220	0.92073	0.91924	−1.4
0.91774	0.91621	0.91466	0.91308	0.91149	0.90988	0.90824	0.90658	0.90490	0.90320	−1.3
0.90147	0.89973	0.89796	0.89617	0.89435	0.89251	0.89065	0.88877	0.88686	0.88493	−1.2
0.88298	0.88100	0.87900	0.87698	0.87493	0.87286	0.87076	0.86864	0.86650	0.86433	−1.1
0.86214	0.85993	0.85769	0.85543	0.85314	0.85083	0.84850	0.84614	0.84375	0.84134	−1.0
0.83891	0.83646	0.83398	0.83147	0.82894	0.82639	0.82381	0.82121	0.81859	0.81594	−0.9
0.81327	0.81057	0.80785	0.80511	0.80234	0.79955	0.79673	0.79389	0.79103	0.78814	−0.8
0.78524	0.78230	0.77935	0.77637	0.77337	0.77035	0.76731	0.76424	0.76115	0.75804	−0.7
0.75490	0.75175	0.74857	0.74537	0.74215	0.73891	0.73565	0.73237	0.72907	0.72575	−0.6
0.72240	0.71904	0.71566	0.71226	0.70884	0.70540	0.70194	0.69847	0.69497	0.69146	−0.5
0.68793	0.68439	0.68082	0.67724	0.67364	0.67003	0.66640	0.66276	0.65910	0.65542	−0.4
0.65173	0.64803	0.64431	0.64058	0.63683	0.63307	0.62930	0.62552	0.62172	0.61791	−0.3
0.61409	0.61026	0.60642	0.60257	0.59871	0.59483	0.59095	0.58706	0.58317	0.57926	−0.2
0.57535	0.57142	0.56750	0.56356	0.55962	0.55567	0.55172	0.54776	0.54380	0.53983	−0.1
0.53586	0.53188	0.52790	0.52392	0.51994	0.51595	0.51197	0.50798	0.50399	0.50000	−0.0

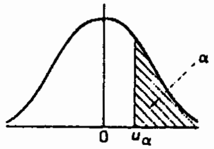
The function tabulated is $\frac{1}{\sqrt{2\pi}}\int_u^{\infty} e^{-x^2/2}\,dx$,

the probability that $U > u$, where $U \sim N(0,1)$.



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

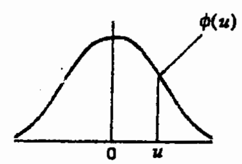
The u_α values tabulated are such that $\Pr(U > u_\alpha) = \alpha$, where $U \sim N(0,1)$



α	u_α	α	u_α	α	u_α	α	u_α
0.50	0.00000	0.34	0.41246	0.18	0.91537	0.025	1.96000
0.49	0.02507	0.33	0.43991	0.17	0.95416	0.020	2.05375
0.48	0.05015	0.32	0.46770	0.16	0.99446	0.010	2.32635
0.47	0.07527	0.31	8.49585	0.15	1.03643	0.009	2.36562
0.46	0.10004	0.30	0.52440	0.14	1.08032	0.008	2.40891
0.45	0.12566	0.29	0.55338	0.13	1.12639	0.007	2.46726
0.44	0.15097	0.28	0.58284	0.12	1.17499	0.006	2.51214
0.43	0.17637	0.27	0.61281	0.11	1.22653	0.005	2.57583
0.42	0.20189	0.26	0.64335	0.10	1.28155	0.004	2.65207
0.41	0.22754	0.25	0.67449	0.09	1.34076	0.003	2.74778
0.40	0.25335	0.24	0.70630	0.08	1.40507	0.002	2.87816
0.39	0.27932	0.23	0.73885	0.07	1.47579	0.001	3.09023
0.38	0.30548	0.22	0.77219	0.06	1.55477	0.0005	3.29053
0.37	0.33185	0.21	0.80642	0.05	1.64485	0.0001	3.71902
0.36	0.35846	0.20	0.84162	0.04	1.75069	0.00005	3.89060
0.35	0.38532	0.19	0.87790	0.03	1.88079	0.00001	4.26489

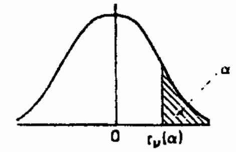
Table 6 ORDINATES OF THE STANDARDISED NORMAL DISTRIBUTION

The function tabulated is $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$.



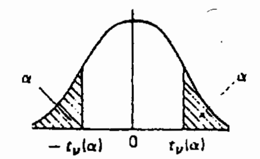
u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.39894	0.39695	0.39104	0.38139	0.36827	0.35207	0.33322	0.31225	0.28969	0.26609
1.0	0.24197	0.21785	0.19419	0.17137	0.14973	0.12952	0.11092	0.09405	0.07895	0.06562
2.0	0.05399	0.04398	0.03547	0.02833	0.02239	0.01753	0.01358	0.01042	0.00792	0.00595
3.0	0.00443	0.00327	0.00238	0.00172	0.00123	0.00087	0.00061	0.00042	0.00029	0.00020
4.0	0.00013	0.00009	0.00006	0.00004	0.00002	0.00002	0.00001	0.00001	0.00000	0.00000

ONE-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)) = \alpha$,
for ν degrees of freedom.

TWO-SIDED TEST



$\Pr(T_\nu > t_\nu(\alpha)$ or $T_\nu < -t_\nu(\alpha)) = 2\alpha$,
for ν degrees of freedom.

ν	$\alpha = 0.4$ $2\alpha = 0.8$	0.25 0.5	0.1 0.2	0.05 0.1	0.025 0.05	0.01 0.02	0.005 0.01	0.0025 0.005	0.001 0.002	0.0005 0.001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.320	318.310	636.620
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291