UNIVERSITY OF SWAZILAND

EXAMINATION PAPER 2006

TITLE OF PAPER	: SAMPLE SURVEY THEORY
COURSE CODE	: ST 306 *(OLD PROGRAMME)*
TIME ALLOWED	: TWO (3) HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES FORMULA SHEET ATTACHED
INSTRUCTIONS	: ANSWER QUESTION ONE AND ANY OTHER THREE QUESTIONS

Question 1

a) A local radio station carries out regular polls of its listeners on items of current interest. In one such poll listeners were asked to telephone the station and just answer yes or no to the following question:
Do you think dogs should be allowed in public places only if on the lead?
The poll was carried out between 8 am and 9 am one morning. At 8.30 am the announcer said that the percentage yes vote was 63%. When the poll closed at 9 am he announced that the percentage yes vote was 52%.

i) What additional number should have been announced at the end of the poll in order to assess how accurate this percentage is? Explain briefly why this number is needed. (5)

ii) List three problems associated with this method of polling and suggest why each problem might cause misleading conclusions to be drawn. (9)

iii) If respondents could have been asked one question about themselves when they telephoned, suggest a suitable question which might be relevant to their response, and explain how conclusions from the survey could have been extended using the extra information. (6)

b) The city council for a region has 3000 staff in a town-centre complex in a large city. The central canteen for this complex at present has a main self-service area for hot meals, and a salad bar. The canteen is not open to the general public but is intended for council staff, who pay prices that are slightly subsidised by the council. The head of the canteen service intends to carry out a survey. He sends out a questionnaire with a prepaid reply envelope to a stratified sample of staff. The strata are the four council departments listed:

Department	*Number of staff*
Education	1100
Social Services	900
Chief Executive's	320
Environment and Resources	680

i) Using simple *proportional allocation*, how many staff should be sampled from each department to give a total sample of 450? (5)

ii) Why might stratified random sampling be an improvement on simple random sampling of the entire population? (5)

iii) The head of the canteen service has heard that a "multi-stage" survey can be cheaper than any other sampling design. Discuss situations in which this may be true, and whether it is likely to be relevant in the present instance. (5)

iv) Assume that the aims of the survey are both to improve the service to existing customers and to extend the customer base. State, giving reasons, your recommendation for a sampling method to be used to meet these aims. (5)

Question 2

A simple random sample of 1 in 20 households in a small town provided the following data about the availability of cars and the number of adults in households.

		Adults in household (x_i)					
		1	2	3	4	5	Total
Number of cars (y_i) in household	0	58	127	9	6	0	200
	1	68	140	27	4	1	240
	2	4	30	5	8	3	50
	3	0	3	4	2	1	10
Total		130	300	45	20	5	500

Note: summing over all 500 households in the sample, $\sum x_i y_i = 795$

Obtain point estimates and approximate 95% confidence intervals for the following:

(a) the total number of cars in the town's households,
(b) the ratio of cars per adult in the town's households,
(c) the proportion of households with 1 or more cars per adult.

(20)

Question 3

A region has 3510 farms which cluster naturally into 90 different "villages". In any village, x denotes the total number of farms and y the total number of cattle. A simple random sample of 15 villages (clusters) was selected, giving the data shown in the table.

Village (cluster) i	Number of farms x_i	Number of cattle y_i	Mean number of cattle per farm $z_i = y_i/x_i$
1	35	418	11.94
2	25	402	16.08
3	48	360	7.50
4	30	394	13.13
5	70	515	7.36
6	55	910	16.55
7	66	600	9.09
8	18	316	17.56
9	30	288	9.60
10	32	350	10.94
11	64	784	12.25
12	24	290	12.08
13	48	795	16.56
14	40	478	11.95
15	82	906	11.05
Total	667	7806	

a) Estimate the mean number of cattle per farm in the region as a whole in three different ways:
(i) using $\bar{z}$, the simple mean of the cluster means;
(ii) using the cluster sample ratio estimate of y to x;
(iii) using the cluster sample total.

(8)

b) Estimate the variance of each of the three estimators. Comment on the properties of the estimators. (12)

Question 4

Wildland managers want to estimate the total number of caribou in the Nelchina herd located in south central Alaska. The density of caribou differs dramatically in different types of habitat. A preliminary aerial survey has identified the area used by the herd, and divided it into six strata based on habitat type.
The organiser has decided to divide the area into sub-areas called quadrats, each 4 km2. The main survey will be conducted by selecting a simple random sample of quadrats from each stratum; the number of caribou, y, in the quadrats will be counted from an aerial photograph.
Estimates of the means and standard deviations of the measurements, y, in each stratum based on the preliminary survey of 211 quadrats are as follows.

Stratum (h)	N_h	n_h	$\bar{y}_h$	s_h	$N_h s_h$
1	400	98	24.1	74.7	29880
2	40	10	25.6	63.7	2548
3	100	37	267.6	589.5	58950
4	40	6	179.0	151.0	6040
5	70	39	293.7	351.5	24605
6	120	21	33.2	99.0	11880
Total	770	211			133903

$\sum N_h s_h^2 = 47882186$

(a) Discuss briefly the merits of using stratified sampling for this survey. (4)

(b) Based on the results of the preliminary aerial survey, estimate the total number of caribou in the herd and obtain an estimate of the standard error for your estimator. (5)

(c) For the main survey, the managers wish to estimate the total number of caribou to within d animals with 95% probability (i.e. the width of the interval is $2d$). You may assume that the formula for the total sample size n is;

$$n = \frac{\sum_h N_h^2 S_h^2 / w_h}{V + \sum_h N_h S_h^2}.$$

Define N_h, S_h, w_h and V as used in this formula. (2)

(d) Define *optimal* allocation. Discuss briefly why you would choose an optimal allocation rather than proportional allocation for this survey. You may assume that the cost of sampling any unit is constant. (3)

(e) Use optimal allocation to calculate the total sample size and the allocations n_h needed to estimate the total population of caribou to within 8000 animals with 95% probability. Calculate the standard error for your estimator of the population total. (6)

Question 5

A wholesale food distributor in a large city wants to assess the demand for a new product based on mean monthly sales. He plans to sell this product in a sample of stores he services. He only services four large chains in the city. Hence, for administrative convenience, he decides to use stratified random sampling with each chain as a stratum. A stratified random sample of 20 stores yields the following sales figures after a month:

Stratum (Chain) 1	2	3	4
$N_1 = 24$	$N_2 = 36$	$N_3 = 30$	$N_4 = 30$
$n_1 = 4$	$n_2 = 6$	$n_3 = 5$	$n_4 = 5$
$\bar{y}_1 = 99.3$	$\bar{y}_2 = 100.0$	$\bar{y}_3 = 98.0$	$\bar{y}_4 = 100.0$
$s_1 = 9.00$	$s_2 = 7.46$	$s_3 = 6.28$	$s_4 = 10.61$
94	91	108	92
90	99	96	110
103	93	100	94
110	105	93	91
	111	93	113
	101		

a) Explain what is meant by *stratification with proportional allocation*, and verify that this has been used to construct the above stratum sample sizes.

(5)

b) Estimate the mean monthly sales and obtain an estimate of the standard error of your estimator. Construct an approximate 95% confidence interval for the population mean.

(5)

c) Suppose instead that a simple random sample of 20 stores from the population of 120 stores had been selected, with the same responses as given in the table. Had this been done, the estimate for the population standard deviation would have been 7.75. Construct an approximate 95% confidence interval for the population mean.

(2)

d) Compare the efficiencies of your estimators in parts (b) and (c). Suggest why stratified random sampling gives a less precise estimate of the population mean than simple random sampling in this case.

(4)

e) Advise the wholesaler on how to select appropriate strata for a stratified random sample when the objective of stratification is to produce estimators with small variance.

(4)

$$s^2 = \sum_{i=1}^{n} \frac{(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{V}(\hat{\mu})_{srs} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{V}(\hat{\tau})_{srs} = N^2\hat{V}(\hat{\mu})_{srs}$$

$$\hat{p}_{srs} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{srs} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{pps} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i}\right) \qquad \hat{V}(\hat{\tau})_{pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{\pi_i} - \hat{\tau}_{pps}\right)^2$$

$$\hat{\mu}_{pps} = \frac{1}{N}\hat{\tau}_{pps} \qquad \hat{V}(\hat{\mu})_{pps} = \frac{1}{N^2}\hat{V}(\hat{\tau})_{pps}$$

$$\hat{\mu}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{\mu})_{sys} = \frac{s^2}{n}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{sys} = N\hat{\mu}_{sys} \qquad \hat{V}(\hat{\tau})_{sys} = N^2\hat{V}(\hat{\mu})_{sys}$$

$$\hat{p}_{sys} = \sum_{i=1}^{n} \frac{y_i}{n} \qquad \hat{V}(\hat{p})_{sys} = \frac{\hat{p}(1-\hat{p})}{(n-1)}\left(\frac{N-n}{N}\right)$$

$$\hat{\mu}_{rsys} = \sum_{i=1}^{ns} \frac{\hat{\mu}_i}{ns} \qquad \hat{V}(\hat{\mu})_{rsys} = \left(\frac{N-n}{N}\right)\sum_{i=1}^{ns} \frac{(\hat{\mu}_i - \hat{\mu}_{rsys})^2}{ns(ns-1)}$$

$$\hat{\tau}_{rsys} = N\hat{\mu}_{rsys} \qquad \hat{V}(\hat{\tau})_{rsys} = N^2\hat{V}(\hat{\mu})_{rsys}$$

$$\hat{\mu}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\frac{s_i^2}{n_i}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{V}(\hat{\tau})_{str} = N^2\hat{V}(\hat{\mu})_{str}$$

$$\hat{p}_{str} = \frac{1}{N}\sum_{i=1}^{L} N_i\hat{p}_i \qquad \hat{V}(\hat{p})_{str} = \frac{1}{N^2}\sum_{i=1}^{L} N_i^2\left(\frac{N_i - n_i}{N_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}_i)}{n_i - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{i=1}^{L} w_i\bar{y}_i \qquad \hat{V}(\hat{\mu})_{pstr} = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{i=1}^{L} w_i s_i^2 + \frac{1}{n^2}\sum_{i=1}^{L}(1 - w_i)s_i^2$$

$$r = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$$

$$\hat{V}(r) = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - r x_i)^2}{(n-1)}$$

$$\hat{\rho} = \frac{\hat{cov}(x,y)}{s_x s_y}$$

$$\hat{V}(r) = \frac{1-(n/N)}{n}\left(\frac{1}{\mu_x^2}\right)(s_y^2 + r^2 s_x^2 - 2r\hat{\rho} s_x s_y)$$

$$\hat{\tau}_{ratio} = r\tau_x$$

$$\hat{V}(\hat{\tau})_{ratio} = \tau_x^2 \hat{V}(r)$$

$$\hat{\mu}_{ratio} = r\mu_x$$

$$\hat{V}(\hat{\mu})_{ratio} = \mu_x^2 \hat{V}(r)$$

$$Y_i = \beta_0 + \beta_1(X_i) + \varepsilon_i$$

$$\sum_{i=1}^{n}(y_i - rx_i)^2 = \sum_{i=1}^{n} y_i^2 + r^2 \sum_{i=1}^{n} x_i^2 - 2r \sum_{i=1}^{n} y_i x_i$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$b_1 = \hat{\rho}(s_y/s_x)$$

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\rho} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}$$

$$\hat{\mu}_{reg} = \bar{y} + b_1(\mu_x - \bar{x})$$

$$\hat{V}(\hat{\mu})_{reg} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n(n-1)}$$

$$\hat{y}_i = b_0 + b_1(x_i)$$

$$\hat{V}(\hat{\mu})_{reg} \approx \left(\frac{N-n}{N}\right)\frac{MSE}{n}$$

$$\hat{\mu}_{diff} = \bar{y} + (\mu_x - \bar{x})$$

$$\hat{V}(\hat{\mu})_{diff} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n(n-1)}$$

$$\sum_{i=1}^{n}(d_i - \bar{d})^2 = \sum_{i=1}^{n} d_i^2 - n\bar{d}^2$$

$$\hat{RE}\left(\frac{E1}{E2}\right) = \frac{\hat{V}(E2)}{\hat{V}(E1)}$$

$$\hat{\mu}_{cts1} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}$$

$$\hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\frac{\sum_{i=1}^{n}(y_i - \bar{y} m_i)^2}{n\bar{M}^2(n-1)}$$

$$\hat{V}(\hat{\mu})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)(s_y^2 + \hat{\mu}_{cts1}^2 s_m^2 - 2\hat{\mu}_{cts1}\hat{\rho} s_y s_m)$$

$$\hat{\tau}_{cts1(1)} = M\hat{\mu}_{cts1}$$

$$\hat{V}(\hat{\tau})_{cts1(1)} = M^2 \hat{V}(\hat{\mu})_{cts1}$$

$$\hat{\tau}_{cts1(2)} = N\bar{y}_t = N\left(\frac{\sum_{i=1}^{n} y_i}{n}\right)$$

$$\hat{V}(\hat{\tau})_{cts1(2)} = \left(\frac{N-n}{N}\right)\left(\frac{N^2}{n}\right)\frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)}$$

$$\bar{m} = \frac{\sum_{i=1}^{n} m_i}{n}$$

$$\sum_{i=1}^{n}(y_i - \bar{y}m_i)^2 = \sum_{i=1}^{n} y_i^2 + \bar{y}^2 \sum_{i=1}^{n} m_i^2 - 2\bar{y}\sum_{i=1}^{n} y_i m_i$$

$$\hat{p}_{cts1} = \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i}$$

$$\hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)\frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$

$$\Pi_i = \frac{m_i}{M}$$

$$\hat{V}(\hat{p})_{cts1} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)\left(s_a^2 + \hat{p}^2 s_m^2 - 2\hat{p}\hat{\rho} s_a s_m\right)$$

$$\hat{\tau}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\Pi_i}$$

$$\sum_{i=1}^{n}(a_i - \hat{p}m_i)^2 = \sum_{i=1}^{n} a_i^2 + \hat{p}^2\sum_{i=1}^{n} m_i^2 - 2\hat{p}\sum_{i=1}^{n} a_i m_i$$

$$\hat{\tau}_{cts1,pps} = \frac{M}{n}\sum_{i=1}^{n}\bar{y}_i$$

$$\hat{V}(\hat{\tau})_{cts1,pps} = \frac{M^2}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\tau})^2$$

$$\hat{\mu}_{cts1,pps} = \frac{1}{n}\sum_{i=1}^{n}\bar{y}_i$$

$$\hat{V}(\hat{\mu})_{cts1,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu})^2$$

$$\hat{\mu}_{cts2} = \left(\frac{N}{M}\right)\frac{\sum_{i=1}^{n} M_i\bar{y}_i}{n}$$

$$\hat{V}(\hat{\mu})_{cts2} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_b^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_b^2 = \frac{\sum_{i=1}^{n}(M_i\bar{y}_i - \bar{M}\hat{\mu})^2}{n-1}$$

$$s_i^2 = \frac{\sum_{j=1}^{m_i}(y_{ij}-\bar{y}_i)^2}{m_i-1}$$

$$\hat{\tau}_{cts2} = M\hat{\mu}_{cts2}$$

$$\hat{V}(\hat{\tau})_{cts2} = M^2\hat{V}(\hat{\mu})_{cts2}$$

$$\hat{\mu}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i}$$

$$\hat{V}(\hat{\mu})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{s_i^2}{m_i}\right)$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\bar{y}_i - \hat{\mu}_{cts2,r})^2}{n-1}$$

$$s_r^2 = \frac{\sum_{i=1}^{n} M_i^2(\hat{p}_i - \hat{p}_{cts2,r})^2}{n-1}$$

$$\hat{p}_{cts2,ratio} = \frac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i}$$

$$\hat{V}(\hat{p})_{cts2,ratio} = \left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)s_r^2 + \frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{\hat{p}_i(1-\hat{p}}{m_i-1}\right.$$

$\hat{\mu}_{cts2,pps} = \frac{1}{n}\sum_{i=1}^{n} \bar{y}_i$ $\hat{V}(\hat{\mu})_{cts2,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu}_{cts2,pps})^2$

$\hat{\tau}_{cts2,pps} = M\hat{\mu}_{cts2,pps}$ $\hat{V}(\hat{\tau}) = M^2\hat{V}(\hat{\mu})_{cts2,pps}$

n for μ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for τ (SRS): $n = \frac{N\sigma^2}{(N-1)(B^2/4N^2)+\sigma^2}$

n for p (SRS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

n for μ (SYS): $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for p (SYS): $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

$k \leq \frac{N}{n}$ $k' = k(ns)$

n for μ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

n for τ (STR): $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4N^2)+\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR μ:

$n_i = n\left(\frac{N_i\sigma_i/\sqrt{c_i}}{\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}}\right)$ $n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{i=1}^{L} N_i\sigma_i\sqrt{c_i}\right)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_i}\right)$ $n = \frac{\left(\sum_{i=1}^{L} N_i\sigma_i\right)^2}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i}{N}\right)$ $n = \frac{\sum_{i=1}^{L} N_i\sigma_i^2}{N^2(B^2/4)+(1/N)\sum_{i=1}^{L} N_i\sigma_i^2}$

$\hat{\mu}_{cts2,pps} = \frac{1}{n}\sum_{i=1}^{n} \bar{y}_i$ $\quad\quad$ $\hat{V}(\hat{\mu})_{cts2,pps} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\bar{y}_i - \hat{\mu}_{cts2,pps})^2$

$\hat{\tau}_{cts2,pps} = M\hat{\mu}_{cts2,pps}$ $\quad\quad$ $\hat{V}(\hat{\tau}) = M^2\hat{V}(\hat{\mu})_{cts2,pps}$

n for μ (SRS): $\quad\quad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for τ (SRS): $\quad\quad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4N^2)+\sigma^2}$

n for p (SRS): $\quad\quad$ $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

n for μ (SYS): $\quad\quad$ $n = \frac{N\sigma^2}{(N-1)(B^2/4)+\sigma^2}$

n for p (SYS): $\quad\quad$ $n = \frac{Np(1-p)}{(N-1)(B^2/4)+p(1-p)}$

$k \leq \frac{N}{n}$ $\quad\quad$ $k' = k(ns)$

n for μ (STR): $\quad\quad$ $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

n for τ (STR): $\quad\quad$ $n = \frac{\sum_{i=1}^{L} N_i^2(\sigma_i^2/w_i)}{N^2(B^2/4N^2)+\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR μ:

$n_i = n\left(\frac{N_i\sigma_i/\sqrt{c_i}}{\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}}\right)$ $\quad\quad$ $n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{i=1}^{L} N_i\sigma_i\sqrt{c_i}\right)}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i\sigma_i}{\sum_{k=1}^{L} N_k\sigma_i}\right)$ $\quad\quad$ $n = \frac{\left(\sum_{i=1}^{L} N_i\sigma_i\right)^2}{N^2(B^2/4)+\sum_{i=1}^{L} N_i\sigma_i^2}$

$n_i = n\left(\frac{N_i}{N}\right)$ $\quad\quad$ $n = \frac{\sum_{i=1}^{L} N_i\sigma_i^2}{N^2(B^2/4)+(1/N)\sum_{i=1}^{L} N_i\sigma_i^2}$

Allocations for STR τ: change $N^2(B^2/4)$ to $N^2(B^2/4N^2)$

Allocations for STR p:

$$n_i = n\left(\frac{N_i\sqrt{p_i(1-p_i)/c_i}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{i=1}^{L} N_i^2 p_i(1-p_i)/w_i}{N^2(B^2/4)+\sum_{i=1}^{L} N_i p_i(1-p_i)}$$

n for μ (ratio): $$n = \frac{N\sigma^2}{N(B^2/4)+\sigma^2}$$

n for τ (ratio): $$n = \frac{N\sigma^2}{N(B^2/4N^2)+\sigma^2}$$

n for μ (CTS1): $$n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$$

n for τ (CTS1(1)): $$n = \frac{N\sigma_c^2}{N(B^2/4N^2)+\sigma_c^2}$$

n for τ (CTS1(2)): $$n = \frac{N\sigma_t^2}{N(B^2/4N^2)+\sigma_t^2}$$

$$s_c^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}m_i)^2}{(n-1)} \text{ with } \bar{y} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i} \qquad s_t^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y}_t)^2}{(n-1)} \text{ with } \bar{y}_t = \frac{\sum_{i=1}^{n} y_i}{n}$$

n for p (CTS1): $$n = \frac{N\sigma_c^2}{N(B^2M^2/4)+\sigma_c^2}$$

$$s_c^2 = \frac{\sum_{i=1}^{n}(a_i-\hat{p}m_i)^2}{(n-1)}$$