University of Swaziland

Final Examination, December 2016

B.A.S.S. I , B.Comm I, D.Comm I (IDE), B. Ed

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Course Code : MAT107/MAT121/MS101

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
    a. **SECTION A(COMPULSORY): 40 MARKS**
       Answer ALL QUESTIONS.
    b. **SECTION B: 60 MARKS**
       Answer ANY THREE questions.
       Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

QUESTION 1

a. State the remainder theorem. [2]

b. By using the remainder theorem which of the following values

i. $x = \frac{1}{3}$, [1]

ii. $x = 2$, [1]

are roots of the polynomial

$$P(x) = -3x^4 + 10x^3 + 2x - 1.$$

c. Using the long division method find the quotient and remainder when

$$P(x) = x^4 - 3x^3 - 4x + 2$$

is divided by $D(x) = x^2 + 3$. [3]

d. Solve

i. $x^{\frac{4}{3}} = 16.$ [3]

ii. $\log(x+1) - \log(2x-1) = \log 4 + \log \dfrac{1}{6}.$ [3]

iii. $4^{x-2} = 3^{2x+1}.$ [3]

iv. $x - \sqrt[3]{-\dfrac{1}{27}} = 0.$ [3]

e. Expand $(x + 2y)^4$ using the binomial theorem. [3]

f. Without using a calculator, find the exact value of $\sin 1305°$. [3]

g. Find the equation of a straight line passing through $(-1, 2)$ and having $y-$ intercept of 4 units. [3]

h. Calculate $AB^T + A$ if the matrices A and B are given by

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & -4 & 4 \end{bmatrix} \text{ and } \quad B = \begin{bmatrix} 0 & 2 & -3 \\ -6 & 5 & 1 \\ 2 & 1 & 0 \end{bmatrix}.$$

[4]

i. A new car costs E 9 000. Assume that it depreciates 21% the first year, 18% the second year, 15% the third year, and continues in the same manner for 5 years. If all depreciations apply to the original cost, what is the value of the car in 5 years? [4]

j. If $\cos\theta = -\frac{\sqrt{3}}{2}$; find the value of $\sin\theta$ and $\tan\theta$ when θ lies in the third quadrant. [4]

k. Given the complex number $Z_1 = 1 + 2i, Z_2 = 1 - i$ and $Z_3 = 3 - 4i$, express $\frac{\overline{Z_1}Z_2}{Z_3}$ in the form $a + ib$. [4]

SECTION B: ANSWER ANY 3 QUESTIONS

QUESTION 2

Given the following polynomial

$$P(x) = 3x^4 + 5x^3 - 10x^2 - 20x - 8 = 0$$

a. List all the possible roots of $P(x)$. [3]

b. Find the number of positive real zeros (roots) of $P(x)$. [3]

c. Find the number of negative real zeros (roots) of $P(x)$. [3]

d. Use the remainder theorem and synthetic division (ONLY) to find the roots of $P(x)$. [11]

QUESTION 3

a. Prove the following trigonometric identity $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$. [7]

b. Solve the following equations

i. $2\cos^2 x = 1 - \sin x, \quad 0° \leq x \leq 360°$. [7]

ii. $z^2 + 2iz - 4 = 0$. [6]

QUESTION 4

a. Use Cramer's rule to solve the following system of equations

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & 1 \\ x & - & y & - & z & = & 0 \\ 2x & + & y & + & z & = & 3. \end{array}$$

[10]

b. Find the first three terms of an arithmetic progress whose 9^{th} term is 16 and 40^{th} term is 47. [5]

c. Convert $3.38181818\cdots$ into and equivalent fraction. [5]

QUESTION 5

a. Given the following expression

$$\left(x^2 - \frac{1}{2x}\right)^{15},$$

Find

i. eight term [4]

ii. constant term [4]

iii. term involving x^6. [4]

b. Find the equation of a straight line passing through the intersection of $3x - y = 9$ and $x + 2y = -4$, parallel to $3 = 4y + 8x$. [8]

QUESTION 6

a. Find the center and radius of a circle defined by the equation

$$6x^2 + 12x - 4 + 6y^2 - 18y = 0.$$

[5]

b. Give the binomial expansion for $\sqrt[4]{1-3x}$ up to and including x^3 (where x is small). Use this expansion to find $\sqrt[4]{0.97}$. [5]

c. Prove by mathematical induction that the formula

$$3 + 3^2 + 3^3 + \cdots + 3^n = \frac{3(3^n - 1)}{2}$$

is valid for all positive integers. [10]

END OF EXAMINATION