University of Swaziland



FINAL EXAMINATION, MAY 2017

BA in Social Sciences I

Title of Paper

: Elementary Quantitative Techniques II

Course Number

: MAT102

Time Allowed

: Two (2) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.

- 2. Section A is **COMPULSORY** and is worth 50%. Answer ALL questions in this section.
- 3. Section B consists of FOUR questions, each worth 25%. Answer ANY TWO (2) questions in this section.
- 4. Show all your working.
- 5. Start each new major question (A1, B4 B7) on a new page and clearly indicate the question number at the top of the page.
- 6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A

Answer ALL Questions in this section

QUESTION A1

a . Evaluate

i.
$$\lim_{x \to 3} (2 + 4x^2 - x^3)$$
 [2]

$$\lim_{x \to -2} \left(\frac{3x}{5 - 3x} \right)$$
[2]

iii.
$$\lim_{x \to 4} \left(\frac{x^2 - 4x}{x - 4} \right)$$
 [3]

iv.
$$\lim_{x \to \infty} \left(\frac{x+7}{x^2 - 2} \right)$$
 [4]

QUESTION A2

a. State the *limit definition* of the derivative of the function f(x).

[2]

b. Use the limit definition to find $\frac{df}{dx}$ given

$$f(x) = 5 - x^2$$

[7]

c. Find y' if

i.
$$y = 4 - 2x + \frac{2}{7}x^7$$
 [2]

ii.
$$y = 3X^{\frac{2}{3}} + X^{-\frac{4}{3}}$$
 [3]

iii.
$$y = 1 - \frac{2}{x^2}$$
 [3]

$$iv. \quad y = e^{2-5x}$$
 [2]

$$\mathbf{v.} \quad y = \ln(3x) \tag{3}$$

[3]

QUESTION A3

- a. State the Fundamental Theorem of Calculus.
- b. Integrate

i.
$$\int_0^1 \left(3 + 4x - 15x^2\right) dx$$
 [5]

ii.
$$\int \left(7X^{\frac{1}{2}} + \frac{3}{X}\right) dX$$
 [3]

iii.
$$\int \left(\frac{5}{x^5} - \frac{3}{x^4}\right) dx$$
 [3]

iv.
$$\int e^{\frac{1}{2}x} dx$$
 [3]

Section B

Answer and 2 Questions in this section

QUESTION B4

a. Find the value of the limit of

$$\lim_{x \to -3} \frac{x+3}{x^2 + 4x + 3}$$

[7]

b. Find the derivative

i.
$$y'$$
 for $y = (1 - 4x^2)^{14}$

ii.
$$y'$$
 for $y = (x+3)e^x$

$$[4]$$

iii.
$$y'$$
 for $y = \frac{3x-2}{7x+4}$

iv.
$$y'''$$
 for $y = 3\sqrt{x} + \frac{1}{x}$

[5]

QUESTION B5

a. Consider the function

$$y = 2 + 27x - x^3$$

i. Find the *stationary point(s)* and classify them as relative maxima or minima.

[10]

ii. Find the y-intercept.

[2]

iii. Make a neat sketch of the graph of y.

4

b. The total costs C of a company (under certain conditions) are given by

$$C(x) = x^2 + \frac{16000}{x}$$

where x is the number of units produced per month. Find the number of units that minimises the monthly average costs. Hence find the minimum cost.

[9]

QUESTION B6

a. By first making the substitution $u = x^2 + 1$, evaluate the integral

$$\int 6x \left(x^2 + 1\right)^5 dx.$$

[10]

b. Use the method of partial fractions to integrate

$$\int \frac{5x-6}{(x-1)(x-2)} \, dx.$$

[15]

QUESTION B7

a. Use the method of tabular integration to evaluate

$$\int 20x^2c^x\,dx.$$

[12]

b. i. Evaluate

$$\int_0^1 \frac{2t^4 - 3t^3 + 5}{t^3} \, dt.$$

[7]

ii. Find the area of the shaded region in the figure below

[6]

