UNIVERSITY OF SWAZILAND

Final Examination 2006

Title of Paper	:	Quantitative Techniques
Program	:	Dip.Comm./B.Ed. Comm.
Course Number	:	MS202/IDE-MS202
Time Allowed	:	Three (3) Hours
Instructions	:	

1. This paper consists of seven (7) questions on FIVE (5) pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements : None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1. (a) The demand functions for two products are

$$p = 20 - x, \qquad q = 19 - y$$

where p and q are the respective prices (in thousands of Emalangeni) and x and y are the respective amounts (in thousands of units) of each product sold. If the joint cost function is given by

$$C(x, y) = 2x^2 + xy + 3y^2 + 50,$$

determine the quantities and prices that maximize profit (Verify they give maximum profit). What is the maximum profit? [12 marks]

(b) A lounge suite, priced at E8000, is purchased on hire purchase with a deposit of E1000. If equal payments are to be made over 24 months and interest is charged at 24%, calculate the monthly installment.

[8 marks]

Question 2. (a) Aircrafts arrive at an airport with a single runway in a pattern which can be characterized by the Poisson distribution. The average rate of arrivals is 30 per hour, and the landing times are exponentially distributed with a mean of 1.5 minutes.

(i) The management is thinking of constructing an additional runway but only if there are more than two aircrafts waiting to land. Should management build another runway?

(ii) What is the probability that there is no plane arriving in a given hour?

[6 marks]

(b) A supermarket chain takes out a loan of E150,000 for the purchase of a new store, due in 10 years' time. The financier charges interest at 10%, compounded semiannually. Meanwhile, the company makes semiannual payments to a sinking fund that pays 12% interest, also compounded semiannually. Compute the total semiannual cost of the debt (Interest + Sinking fund). [8 marks]

(c) Use the method of Lagrange multipliers to optimize $z = 80x - 2x^2 - xy - 3y^2 + 100y$ subject to $x + y = 12$. [6 marks]

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Question 3. (a) Use Gaussian elimination to solve

$$\begin{aligned} -x + 3y + 2z &= 7 \\ 4x + 5y - 2z &= 12 \\ x + 4y + 3z &= 12 \end{aligned}$$

[10 marks]

(b) For the function, $z = 3x^3 - 5y^2 - 225x + 70y + 23$, determine all local extrema and classify them accordingly.

[10 marks]

Question 4. (a) Transform the following determinant to triangular form, and evaluate:

$$\begin{vmatrix} 2 & 0 & 3 \\ 4 & 1 & 2 \\ -6 & 3 & 5 \end{vmatrix}$$

[10 marks]

(b) Consider a three-industry economy, the information relating to the flow of goods for a particular output level is presented in the table below:

	Ind. A	*Ind. B*	*Ind. C*	*Consumers*	*Output Level*
Industry A	100	150	140	110	500
Industry B	400	100	280	220	1,000
Industry C	150	300	70	180	700

(i) Obtain the technological coefficient matrix for this economy.

(ii) Find the total export levels that correspond to consumer demands of 215 units for Industry A, 310 units for Industry B and 210 units for Industry C.

[10 marks]

Question 5. (a) Three cosmetics companies are competing for market shares for their hair products. The pattern of hair product users over a year for the various companies is given by the following statements:

Company 1 will retain 80% of its customers and lose 10% to company 2 and 10% to company 3;

Company 2 will lose 15% of its customers to company 1, 5% to company 3 and retain 80% of its customers;

Company 3 will lose 5% to each companies 1 and 2 and remain popular to 90% of its customers.

(i) Construct a transition probability matrix for the above market..

[3 marks]

(ii) Assuming that the initial shares for the three companies are 25%, 40% and 35% respectively, what is the market share for Company 2 in two year's time? [4 marks]

(iii) What are the equilibrium market shares? [5 marks]

(b) The Swazi Pump Company sells 8640 industrial pumps each year. Each of the pumps has a built-in filter that is purchased from a supplier. It costs the company R20 to place an order for these filters and R1.50 per unit to hold them in inventory for a year.

(i) What is the economic order quantity for this product?

(ii) What is the total annual cost associated with the EOQ, given that each filter costs R110? [8 marks]

Question 6. (a) A stereo mail order warehouse has 81 m^2 available for storage of loudspeakers. The jumbo speakers require 4 m^2 of space while the mid-size and economy speakers require 3 m^2 and 4 m^2, respectively. The demand for the jumbo speakers is at most 20 per month. The wholesale has R10 000 to invest in loudspeakers this month. Assume that the jumbo speakers contribute R105 to profit, the mid-size contribute R50, and the economy contribute R28. The management desires to maximize profits. Formulate the model to be used to find the combination of various types of loudspeakers which should be produced in order to meet the goal.

[10 marks]

(b) Solve the following linear programming problem graphically

$$\begin{aligned} \text{Maximize:} \quad & P = 4x + 2y \\ \text{Subject to:} \quad & x + 2y \leq 14 \\ & 3x + 2y \leq 20 \\ & x, y \geq 0 \end{aligned}$$

[10 marks]

Question 7. Consider the LP problem

$$\begin{aligned} \text{Maximize:} \quad & z = 15x_1 + 20x_2 + 24x_3 \\ \text{Subject to:} \quad & 3x_1 + x_2 + 3x_3 \leq 120 \\ & x_1 + 5x_2 + 2x_3 \leq 60 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(a) Solve the primal problem using the simplex method.

[12 marks]

(b) Write down the dual problem, and deduce its solution from that of the primal problem.

[8 marks]

********** *END OF EXAM* **********

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Some useful formulae.

Hire Purchase Instalment: $p = \dfrac{B\left(1+\frac{n}{m}i\right)}{n+\frac{n(n-1)}{2m}i}$

Growing Investment: $S = \left(P+\dfrac{p}{i}\right)(1+i)^n - \dfrac{p}{i}$

Future Value of an Annuity: $S = p \cdot \dfrac{(1+i)^n - 1}{i}$

Present Value of an Annuity: $P = p \cdot \dfrac{1-(1+i)^{-n}}{i}$

For an EOQ model without shortages: $Q_0 = \sqrt{\frac{2dK}{h}}$

For a queueing model with one service channel and arrival and service rates λ, μ respectively:

$\rho = \frac{\lambda}{\mu}$; $p_0 = 1 - \frac{\lambda}{\mu}$ and $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$

$L_s = \frac{\lambda}{\mu-\lambda}$ $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$ $W_s = \frac{L_s}{\lambda}$ $W_q = \frac{L_q}{\lambda}$

(END)