UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

B.A.S.S. I / D.COM I

TITLE OF PAPER   :   INTRODUCTORY MATHEMATICS FOR BUSINESS

COURSE NUMBER   :   MS 101 AND IDE MS100-1

TIME ALLOWED   :   THREE (3) HOURS

INSTRUCTIONS   :   1.   THIS PAPER CONSISTS OF
                                        SEVEN QUESTIONS.
                                2.   ANSWER ANY FIVE QUESTIONS
                                3.   USEFUL FORMULAE ARE PROVIDED
                                        AT THE END OF THE QUESTION PAPER.

SPECIAL REQUIREMENTS   :   NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL
PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. Solve the following equations for the unknown variables

    (a) $\cos^2 x + \cos x = \sin^2 x$ $(0^o \leq x \leq 360^o)$ [5 marks]

    (b) $x^3 - 2x^2 - 13x - 10 = 0$ [5 marks]

    (c) $27^{4x} = 9^{x+1}$ [5 marks]

    (d) $\log_3(2x - 1) = \log_3(4x - 3) - \log_3 x$ [5 marks]

QUESTION 2

2. (a) Find the sum of the following series

$$2 + 5 + 8 + \ldots\ldots + 1001$$

                                [5 marks]

  (b) Find a fraction representation of $0.568888\ldots$ [5 marks]

  (c) Find the 20th term of a geometric progression whose 2nd
term is 12 and 8th term is 4.   [5 marks]

  (d) The expenses of a company are E200 000 a year. It is
decided that each year they shall be reduced by 5% of
those for the preceding year. What will be the expenses
during the fourth year, the first reduction taking place
at the end of the first year?   [5 marks]

QUESTION 3

3. (a) Solve the following trigonometric equation giving all solutions for x between 0° and 360°

$$\cos 2x + \cos x = 0$$

[5 marks]

(b) Prove the following trigonometric identities

  i. $\dfrac{\sec x - \cos x}{\tan x} = \sin x$ [4 marks]

  ii. $\csc 2x - \cot 2x = \tan x$ [4 marks]

(c) Without using calculators, find $\cos 105^o$.   [3 marks]

(d) Given that $\sec \theta = -\frac{3}{2}$ and that θ is in the 3rd quadrant, find $\sin \theta$ and $\cos 2\theta$.   [4 marks]

QUESTION 4

4. (a) Use the general formula for the rth term to find the 22nd term in the binomial expansion of   [3 marks]

$$\left(x + \frac{1}{x} \right)^{25}$$

(b) Write down the first four terms of the binomial expansion of   [3 marks]

$$\frac{1}{1-x}$$

(c) Find the constant term (i.e term without x) in the binomial expansion of

$$\left(x^3 + \frac{1}{2x^2} \right)^{20}$$

[7 marks]

(d) Use Mathematical Induction to prove that the following formula is valid for all positive integers.

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

[7 marks]

5. (a) Assuming an inflation rate of 5% compounded annually, how
long will it take for prices to double? [5 marks]

(b) Sphiwe wants to open a Saloon. How much must she deposit now
at 8% interest compounded quarterly to have E20 000 at the end
of 4 years. [5 marks]

(c) Find the interest rate needed for E6 000 to grow to E8 000 in
3 years if the interest is compounded monthly. [5 marks]

(d) How long will it take for E7200 to grow to E39 600 at an interest
rate of 4.5% if the interest is compounded continuously?

[5 marks]

QUESTION 6

6. (a) Find x, y, z and p [5 marks]

$$\begin{pmatrix} x & 3 \\ -2 & 2x \end{pmatrix} + \begin{pmatrix} 2 & -y \\ z & p \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

(b) Calculate $A^T B$ if the matrices A and B are given by [5 marks]

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ -1 & 4 \\ 2 & 6 \\ 3 & 2 \end{pmatrix}$$

(c) Solve the following system of equations using Cramer's rule. [10 marks]

$$2x + 3y + z = 2$$
$$-x + 2y + 3z = -1$$
$$-3x - 3y + z = 0$$

7. (a) Find the equation of a circle passing through the point (-2,-4) and with centre located at (2,-7). [4 marks]

    (b) Find the centre and radius of a circle whose equation is

    $$x^2 + y^2 + 3x - 5y - \frac{1}{2} = 0$$

    [4 marks]

    (c) Find the equation for the line with slope of -2 which passes through the intersection of $-3y + x = 4$ and $5y + 3 = -2x$ [4 marks]

    (d) Evaluate the complex number $\sqrt{8 - 6i}$ and express your answer in the form $a + ib$ where both a and b are real number. [4 marks]

    (e) Express the complex number $-1 - \sqrt{3}i$ in mod-arg form. [4 marks]

END OF EXAMINATION

Useful Formulas

The general rth term of a Binomial expansion of

$$(a+b)^n$$

is given by

$$C(n, r-1)a^{n-r+1}b^{r-1}$$

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
3. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
4. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
5. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
6. $2\cos A \cos B = \cos(A+B) + \cos(A-B)$
7. $\sin 2A = 2\sin A \cos A$
8. $\cos 2A = \cos^2 A - \sin^2 A$
9. $\cos 2A = 2\cos^2 A - 1$
10. $\cos 2A = 1 - 2\sin^2 A$

Degrees	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	