

UNIVERSITY OF ESWATINI
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
RE-SIT EXAMINATION

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TITLE OF PAPER: **MATHEMATICS FOR ECONOMISTS II**
COURSE CODE: **ECO206 / IDE – ECO206**
TIME ALLOWED: **2 HOURS**
INSTRUCTIONS: **ANSWER ANY FOUR QUESTIONS [20 MARKS EACH]**

REQUIREMENTS

- SCIENTIFIC CALCULATOR
- STATISTICAL TABLES

DO NOT OPEN THIS PAPER UNTIL YOU HAVE BEEN INSTRUCTED TO DO SO

Question 1

(a) Heritage Properties is a national company specializing in rental office accommodation. An analysis of lease records in Durban branch has established that, on average, five lease agreements are signed in the Durban Metropolitan Area.

- i. What is the probability that, on a given day, the Durban branch will sign only three lease agreements for office space? [2]
- ii. What is the probability that on any given day, the Durban branch will sign at most two lease agreements for office space? [2]
- iii. What is the probability that the Durban Branch will sign more than four lease agreements for office space on any given day? [3]
- iv. What is the probability that the Durban office will sign will sign more than four lease agreements for office space in any two-day period? [3]

(b) A courier service company has found that their delivery time of parcels to clients is normally distributed with a mean of 45 minutes ($\mu = 45$) and a standard deviation of eight minutes ($\sigma = 8$).

(b) What is the probability that a randomly selected parcel:

- (i) Will take between 45 and 51 minutes to deliver to the client? [5]
- (ii) Will take less than 48 minutes to deliver? [5]

Question 2

(a) Assume that the purchase value of transactions, x , at a national clothing store such as Edgars, is normally distributed with a mean of R244 and a standard deviation of R68.

- (i) What is the minimum purchase value of transactions for the highest-spending 15% of clothing store customers? [5]

(ii) What purchase value of transactions separates the lowest-spending 20% of clothing store customers from the remaining customers?

[5]

(b) A human resources director at the Chamber of Mines wishes to estimate the true mean employment period of all coalminers. From a random sample of 144 coalminers' records, the sample mean employment period was found to be 88.4 months. The population standard deviation is assumed to be 21.5 months and normally distributed.

Find the 95 % confidence interval estimate for the actual mean employment period (in months) for all miners employed in coal mines. [10]

Question 3

(a) Define the following terms, giving relevant examples [2 marks Each]

- Population
- Census
- Sample
- Probability sampling methods
- Non probability sampling methods
- Statistic
- Parameter
- Point estimate
- Interval estimate
- Confidence interval

Question 4

An international study on executive working hours reported that company CEOs (chief executive officers) worked *more than* 60 hours per week on average. The South African Institute of Management (SAIM) wanted to test whether this norm applied to South African CEOs as well.

A random sample of 90 CEOs from South African companies was drawn, and each executive was asked to record the number of hours worked during a given week. The sample mean number

of hours worked per week was found to be 61.3 hours. Assume a normal distribution for weekly hours worked and a population standard deviation of 8.8 hours.

Do South African CEOs work *more than* 60 hours per week, on average? Test this claim

at the 5% level of significance. [20]

Question 5

A mobile phone service provider, Cell D Mobile, claims that it has 15% of the prepaid mobile phone market. A competitor, who commissioned a market research company to conduct a survey amongst prepaid mobile phone users, challenged this claim. The market research company randomly sampled 360 prepaid mobile users and found that 42 users subscribe to Cell D Mobile as a service provider.

(a) Test, at the 1% level of significance, Cell D Mobile's claim that they have a 15% share of the prepaid mobile phone market. [10]

(b) Test at the 10% level of significance, whether Cell D Mobile's share of the prepaid mobile phone market is significantly *less than* 15%. Use the same market research data as for (a) above.

[10]

STATISTICAL TABLES

TABLE A.1

Cumulative Standardized Normal Distribution

$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z or particular importance:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5783	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7589	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7818	0.7910	0.7939	0.7967	0.7995	0.8023	0.8050	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8645	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8979	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9182	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9463	0.9463	0.9474	0.9484	0.9495	0.9515	0.9525	0.9535	0.9545	0.9555
1.7	0.9556	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9836	0.9844	0.9853	0.9862	0.9871	0.9875	0.9884	0.9890
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9929	0.9932	0.9934	0.9936	0.9938	0.9940
2.5	0.9938	0.9940	0.9941	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.9954
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9964	0.9966
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0.9975
2.8	0.9974	0.9975	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9986	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

TABLE A.2
Distribution: Critical Values of *t*

		Significance Level									
Degrees of freedom	Two-tailed test; One-tailed test:	.10%	.05%	.02%	.01%	.005%	.001%	.0005%	.0001%	.00005%	.00001%
1	1.6145	1.9930	2.1571	2.2658	2.3016	2.3539	2.3677	2.3888	2.4054	2.4118	2.4576
2	2.1851	1.9800	19.16	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44
3	3.0113	9.955	9.28	9.12	9.01	8.94	8.85	8.81	8.79	8.74	8.69
4	4.771	6.54	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.84
5	6.61	5.79	5.41	5.19	4.95	4.74	4.68	4.62	4.57	4.52	4.56
6	7.59	5.14	4.76	4.53	4.39	4.28	4.15	4.10	4.06	4.00	3.96
7	7.559	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57
8	8.52	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.20
9	9.512	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07
10	10.496	4.10	3.71	3.48	3.35	3.22	3.14	3.07	3.02	2.98	2.91
11	11.484	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79
12	12.475	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.64
13	13.467	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60
14	14.460	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.55
15	15.454	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.42
16	16.449	3.65	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42
17	17.445	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38
18	18.441	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34
19	19.438	3.52	3.13	2.90	2.74	2.63	2.54	2.42	2.38	2.31	2.26
20	20.435	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.26
21	21.432	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.20
22	22.430	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23
23	23.428	3.42	3.03	2.80	2.64	2.55	2.46	2.40	2.37	2.32	2.25
24	24.426	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21
25	25.424	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.21
26	26.422	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15
27	27.421	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13
28	28.420	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.23	2.18	2.11
29	29.418	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10
30	30.417	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09
31	31.416	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15	2.08
32	32.415	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.18	2.12	2.07
33	33.414	3.29	2.89	2.66	2.50	2.39	2.30	2.23	2.17	2.11	2.05
34	34.413	3.28	2.88	2.64	2.48	2.37	2.28	2.20	2.14	2.08	2.02
35	35.412	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.07
36	36.411	3.26	2.86	2.63	2.46	2.34	2.25	2.18	2.12	2.06	2.02
37	37.410	3.25	2.85	2.62	2.45	2.33	2.24	2.17	2.11	2.05	2.00
38	38.409	3.24	2.84	2.61	2.44	2.32	2.23	2.16	2.10	2.04	1.99
39	39.408	3.23	2.83	2.60	2.43	2.31	2.22	2.15	2.09	2.03	1.98
40	40.408	3.23	2.82	2.59	2.42	2.30	2.21	2.13	2.07	2.01	1.97
41	41.407	3.22	2.81	2.58	2.41	2.29	2.20	2.13	2.07	2.01	1.96
42	42.406	3.21	2.80	2.57	2.40	2.28	2.19	2.12	2.06	2.00	1.95
43	43.405	3.20	2.79	2.56	2.39	2.27	2.18	2.10	2.04	1.98	1.92
44	44.404	3.19	2.78	2.55	2.38	2.26	2.17	2.09	2.03	1.97	1.91
45	45.403	3.18	2.77	2.54	2.37	2.25	2.16	2.07	2.00	1.94	1.88
46	46.402	3.17	2.76	2.53	2.36	2.24	2.15	2.06	1.99	1.93	1.87
47	47.401	3.16	2.75	2.52	2.35	2.23	2.14	2.05	1.98	1.92	1.86
48	48.400	3.15	2.74	2.51	2.34	2.22	2.13	2.04	1.97	1.91	1.85
49	49.400	3.14	2.73	2.50	2.33	2.21	2.12	2.03	1.96	1.90	1.84
50	50.400	3.13	2.72	2.49	2.32	2.20	2.11	2.02	1.95	1.89	1.83
51	51.400	3.12	2.71	2.48	2.31	2.19	2.10	2.01	1.94	1.88	1.82
52	52.400	3.11	2.70	2.47	2.29	2.17	2.08	1.99	1.92	1.86	1.80
53	53.400	3.10	2.69	2.46	2.28	2.16	2.07	1.98	1.91	1.85	1.79
54	54.400	3.09	2.68	2.45	2.27	2.15	2.06	1.97	1.90	1.84	1.78
55	55.400	3.08	2.67	2.44	2.26	2.14	2.05	1.96	1.89	1.83	1.77
56	56.400	3.07	2.66	2.43	2.25	2.13	2.04	1.95	1.88	1.82	1.76
57	57.400	3.06	2.65	2.42	2.24	2.12	2.03	1.94	1.87	1.81	1.75
58	58.400	3.05	2.64	2.41	2.23	2.11	2.02	1.93	1.86	1.80	1.74
59	59.400	3.04	2.63	2.40	2.22	2.10	2.01	1.92	1.85	1.79	1.73
60	60.400	3.03	2.62	2.39	2.21	2.09	2.00	1.91	1.84	1.78	1.72
61	61.400	3.02	2.61	2.38	2.19	2.07	1.98	1.89	1.82	1.76	1.70
62	62.400	3.01	2.60	2.37	2.18	2.06	1.97	1.88	1.81	1.75	1.69
63	63.400	3.00	2.59	2.36	2.17	2.05	1.96	1.87	1.80	1.74	1.68
64	64.400	2.99	2.58	2.35	2.16	2.04	1.95	1.86	1.79	1.73	1.67
65	65.400	2.98	2.57	2.34	2.15	2.03	1.94	1.85	1.78	1.72	1.66
66	66.400	2.97	2.56	2.33	2.14	2.02	1.93	1.84	1.77	1.71	1.65
67	67.400	2.96	2.55	2.32	2.13	2.01	1.92	1.83	1.76	1.70	1.64
68	68.400	2.95	2.54	2.31	2.12	2.00	1.91	1.82	1.75	1.69	1.63
69	69.400	2.94	2.53	2.30	2.11	1.99	1.80	1.71	1.65	1.59	1.53
70	70.400	2.93	2.52	2.29	2.09	1.97	1.78	1.69	1.63	1.57	1.51
71	71.400	2.92	2.51	2.28	2.08	1.96	1.77	1.68	1.62	1.56	1.50
72	72.400	2.91	2.50	2.27	2.07	1.95	1.76	1.67	1.61	1.55	1.49
73	73.400	2.90	2.49	2.26	2.06	1.94	1.75	1.66	1.60	1.54	1.48
74	74.400	2.89	2.48	2.25	2.05	1.93	1.74	1.65	1.59	1.53	1.47
75	75.400	2.88	2.47	2.24	2.04	1.92	1.73	1.64	1.58	1.52	1.46
76	76.400	2.87	2.46	2.23	2.03	1.91	1.72	1.63	1.57	1.51	1.45
77	77.400	2.86	2.45	2.22	2.02	1.89	1.70	1.61	1.55	1.49	1.43
78	78.400	2.85	2.44	2.21	2.01	1.87	1.68	1.59	1.53	1.47	1.41
79	79.400	2.84	2.43	2.20	2.00	1.85	1.66	1.57	1.51	1.45	1.39
80	80.400	2.83	2.42	2.19	1.99	1.83	1.64	1.55	1.49	1.43	1.37
81	81.400	2.82	2.41	2.18	1.98	1.82	1.63	1.54	1.48	1.42	1.36
82	82.400	2.81	2.40	2.17	1.97	1.81	1.62	1.53	1.47	1.41	1.35
83	83.400	2.80	2.39	2.16	1.96	1.79	1.59	1.50	1.44	1.38	1.32
84	84.400	2.79	2.38	2.15	1.95	1.78	1.58	1.49	1.43	1.37	1.31
85	85.400	2.78	2.37	2.14	1.94	1.77	1.57	1.48	1.42	1.36	1.30
86	86.400	2.77	2.36	2.13	1.93	1.76	1.56	1.47	1.41	1.35	1.29
87	87.400	2.76	2.35	2.12	1.92	1.75	1.55	1.46	1.40	1.34	1.28
88	88.400	2.75	2.34	2.11	1.91	1.74	1.54	1.45	1.39	1.33	1.27
89	89.400	2.74	2.33	2.10	1.90	1.73	1.53	1.44	1.38	1.32	1.26
90	90.400	2.73	2.32	2.09	1.89	1.72	1.52	1.43	1.37	1.31	1.25
91	91.400	2.72	2.31	2.08	1.88	1.71	1.51	1.42	1.36	1.30	1.24
92	92.400	2.71	2.30	2.07	1.87	1.69	1.49	1.40	1.34	1.28	1.22
93	93.400	2.70	2.29	2.06	1.86	1.68	1.48	1.39	1.33	1.27	1.21
94	94.4										

TABLE A.3 (continued)
 F Distribution: Critical Values of F (5% significance level)

v_1		F Distribution: Critical Values of F (5% significance level)																	
v_2		1	2	3	4	5	6	7	8	9	10	12	14	16	18	20			
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68									
2	19.46	18.66	19.47	19.48	19.48	19.48	19.49	19.49	19.49	19.49									
3	8.63	8.62	8.60	8.59	8.58	8.56	8.55	8.54	8.54	8.54									
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65									
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39									
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69									
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25									
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95									
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73									
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56									
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43									
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32									
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23									
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16									
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10									
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04									
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99									
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95									
19	2.11	2.07	2.05	2.03	2.03	2.00	1.98	1.96	1.94	1.92									
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88									
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84									
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82									
23	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79	1.79									
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77									
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75									
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73									
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71									
28	1.91	1.87	1.84	1.82	1.80	1.78	1.75	1.73	1.70	1.69									
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67									
30	1.83	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66									
35	1.82	1.79	1.76	1.74	1.72	1.69	1.66	1.63	1.61	1.60									
40	1.78	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55	1.55									
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48									
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.44	1.44									
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.42									
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.40	1.40									
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36									
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34									
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.35	1.32									
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29									
200	1.56	1.52	1.48	1.46	1.43	1.40	1.37	1.34	1.31	1.28									
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.28	1.25									
300	1.54	1.50	1.46	1.43	1.40	1.37	1.34	1.31	1.28	1.25									
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.25	1.22									
500	1.53	1.48	1.45	1.42	1.38	1.35	1.32	1.28	1.25	1.21									
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20									
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20									
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19									

TABLE A.3 (continued)

F Distribution: Critical Values of F (5% significance level)

TABLE A.3 {continued}

Table 3 (continued)

ν_1	25	30	35	40	50	60	75	100	150	200
ν_2	1.635983	6.026655	67255.57	638678	G102.32	G111.05	6323.56	6334.11	G54.68	G39.97
2	99.46	99.47	99.47	99.48	99.48	99.48	99.49	99.49	99.49	99.49
4	26.58	26.58	26.41	26.35	26.32	26.28	26.24	26.20	26.18	26.16
6	13.91	13.84	13.79	13.75	13.69	13.65	13.61	13.58	13.52	13.52
8	5.26	5.20	5.15	5.12	5.07	5.03	5.00	4.96	4.93	4.91
9	4.71	4.65	4.60	4.57	4.52	4.48	4.45	4.41	4.38	4.36
10	4.51	4.25	4.20	4.17	4.12	4.08	4.05	4.01	3.98	3.96
11	4.01	3.94	3.89	3.85	3.81	3.78	3.74	3.71	3.67	3.66
12	3.76	3.70	3.65	3.62	3.57	3.54	3.50	3.47	3.43	3.41
13	3.57	3.51	3.46	3.43	3.38	3.34	3.31	3.27	3.24	3.22
14	3.41	3.35	3.30	3.27	3.22	3.18	3.15	3.11	3.06	3.05
15	3.28	3.21	3.17	3.13	3.08	3.05	3.01	2.98	2.94	2.92
16	3.16	3.10	3.05	3.02	2.97	2.93	2.90	2.86	2.83	2.81
17	3.07	3.00	2.96	2.92	2.87	2.83	2.80	2.76	2.73	2.71
18	2.98	2.92	2.87	2.84	2.78	2.75	2.71	2.68	2.64	2.62
19	2.91	2.84	2.80	2.76	2.71	2.67	2.64	2.60	2.57	2.55
20	2.84	2.78	2.73	2.69	2.64	2.61	2.57	2.54	2.50	2.48
21	2.79	2.72	2.67	2.64	2.58	2.55	2.51	2.48	2.44	2.42
22	2.73	2.67	2.62	2.58	2.53	2.50	2.46	2.42	2.38	2.36
23	2.62	2.57	2.57	2.54	2.58	2.45	2.41	2.37	2.34	2.32
24	2.64	2.58	2.53	2.49	2.44	2.40	2.37	2.33	2.29	2.27
25	2.60	2.54	2.49	2.45	2.40	2.36	2.33	2.29	2.25	2.23
26	2.57	2.50	2.45	2.42	2.36	2.33	2.29	2.25	2.21	2.19
27	2.54	2.47	2.42	2.38	2.33	2.29	2.26	2.22	2.18	2.16
28	2.51	2.44	2.39	2.35	2.30	2.26	2.23	2.19	2.15	2.13
29	2.48	2.41	2.36	2.33	2.27	2.23	2.20	2.16	2.12	2.10
30	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.13	2.09	2.07
35	2.35	2.23	2.23	2.19	2.14	2.10	2.06	2.02	1.98	1.96
40	2.27	2.20	2.15	2.11	2.06	2.01	1.98	1.94	1.90	1.87
50	2.17	2.10	2.05	2.01	1.96	1.91	1.87	1.83	1.78	1.76
60	2.10	2.03	1.98	1.94	1.88	1.84	1.79	1.75	1.70	1.68
70	2.05	1.98	1.95	1.89	1.83	1.78	1.74	1.70	1.65	1.62
80	2.01	1.94	1.89	1.85	1.76	1.72	1.67	1.62	1.57	1.58
90	1.99	1.92	1.86	1.82	1.76	1.72	1.67	1.62	1.56	1.52
100	1.97	1.89	1.84	1.80	1.74	1.69	1.65	1.60	1.55	1.52
120	1.93	1.86	1.81	1.76	1.70	1.65	1.61	1.56	1.51	1.48
150	1.90	1.83	1.77	1.73	1.66	1.62	1.57	1.52	1.46	1.43
200	1.87	1.79	1.74	1.69	1.63	1.58	1.53	1.48	1.42	1.39
250	1.85	1.77	1.72	1.67	1.61	1.56	1.51	1.46	1.40	1.36
300	1.84	1.76	1.70	1.66	1.59	1.55	1.50	1.44	1.38	1.35
400	1.82	1.75	1.69	1.64	1.58	1.53	1.48	1.42	1.36	1.32
500	1.81	1.74	1.68	1.63	1.57	1.52	1.47	1.41	1.34	1.31
600	1.80	1.73	1.67	1.63	1.56	1.51	1.46	1.40	1.34	1.30
700	1.79	1.72	1.66	1.62	1.55	1.50	1.45	1.39	1.33	1.29
1000	1.79	1.72	1.66	1.61	1.54	1.50	1.44	1.38	1.32	1.28

F Distribution: Critical Values of F (0.1% significance level)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
y_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
y_2	1	4.05e-05	5.00e-05	5.60e-05	5.62e-05	5.76e-05	5.86e-05	5.93e-05	5.98e-05	6.00e-05	6.06e-05	6.11e-05	6.14e-05	6.17e-05	6.19e-05	6.21e-05			
	2	9.98e-09	9.99e-09	9.99e-09	9.99e-09														
	3	1.67e-05	1.68e-05	1.69e-05	1.70e-05	1.71e-05	1.72e-05	1.73e-05	1.74e-05	1.75e-05	1.76e-05	1.77e-05	1.78e-05	1.79e-05	1.80e-05	1.81e-05	1.82e-05	1.83e-05	1.84e-05
	4	7.44e-13	6.12e-13	5.61e-13	5.13e-13	4.73e-13	4.41e-13	4.13e-13	3.87e-13	3.64e-13	3.43e-13	3.24e-13	3.06e-13	2.89e-13	2.74e-13	2.61e-13	2.50e-13	2.41e-13	2.34e-13
	5	47.18e-12	33.20e-12	31.09e-12	30.75e-12	28.83e-12	28.16e-12	27.65e-12	27.24e-12	26.92e-12	26.42e-12	26.06e-12	25.78e-12	25.57e-12	25.39e-12				
	6	35.51e-09	27.00e-09	23.70e-09	21.92e-09	20.80e-09	20.03e-09	19.46e-09	19.03e-09	18.69e-09	18.41e-09	17.99e-09	17.68e-09	17.45e-09	17.27e-09	17.12e-09			
	7	2.95e-16	2.16e-16	1.69e-16	1.26e-16	1.02e-16	8.20e-17	6.52e-17	5.12e-17	4.02e-17	3.15e-17	2.48e-17	1.95e-17	1.54e-17	1.24e-17	1.02e-17	8.50e-18	7.12e-18	6.04e-18
	8	2.54e-19	1.89e-19	1.35e-19	1.03e-19	7.85e-20	5.93e-20	4.47e-20	3.25e-20	2.27e-20	1.55e-20	1.11e-20	7.77e-21	5.54e-21	4.07e-21	2.87e-21	1.97e-21	1.36e-21	9.64e-22
	9	2.26e-16	1.63e-16	1.39e-16	1.20e-16	1.15e-16	1.12e-16	1.10e-16	1.07e-16	1.04e-16	1.01e-16	9.87e-17	9.58e-17	9.30e-17	9.05e-17	8.82e-17	8.61e-17	8.42e-17	8.24e-17
	10	2.10e-14	1.49e-14	1.25e-14	1.11e-14	1.03e-14	9.58e-15	9.05e-15	8.66e-15	8.35e-15	8.07e-15	7.82e-15	7.63e-15	7.47e-15	7.34e-15	7.24e-15	7.11e-15	7.01e-15	6.90e-15
	11	1.93e-13	1.31e-13	1.16e-13	1.05e-13	9.58e-14	8.90e-14	8.38e-14	8.00e-14	7.71e-14	7.48e-14	7.25e-14	7.00e-14	6.79e-14	6.55e-14	6.31e-14	6.06e-14	5.83e-14	5.63e-14
	12	1.86e-12	1.24e-12	1.09e-12	1.01e-12	9.20e-13	8.07e-13	7.36e-13	6.79e-13	6.34e-13	5.98e-13	5.65e-13	5.36e-13	5.09e-13	4.84e-13	4.63e-13	4.43e-13	4.24e-13	4.06e-13
	13	1.82e-12	1.21e-12	1.02e-12	9.31e-13	7.85e-13	7.36e-13	6.74e-13	6.20e-13	5.70e-13	5.24e-13	4.80e-13	4.40e-13	4.04e-13	3.71e-13	3.40e-13	3.11e-13	2.82e-13	2.54e-13
	14	1.71e-12	1.17e-12	9.73e-13	8.62e-13	7.42e-13	6.74e-13	6.08e-13	5.58e-13	5.13e-13	4.70e-13	4.30e-13	3.92e-13	3.54e-13	3.20e-13	2.87e-13	2.56e-13	2.26e-13	1.95e-13
	15	1.65e-12	1.13e-12	9.34e-13	8.25e-13	7.25e-13	6.57e-13	5.98e-13	5.52e-13	5.13e-13	4.75e-13	4.37e-13	3.98e-13	3.60e-13	3.29e-13	2.96e-13	2.64e-13	2.32e-13	1.99e-13
	16	1.61e-12	1.09e-12	9.01e-13	7.94e-13	7.27e-13	6.60e-13	6.04e-13	5.58e-13	5.18e-13	4.83e-13	4.48e-13	4.13e-13	3.80e-13	3.47e-13	3.14e-13	2.81e-13	2.48e-13	2.14e-13
	17	1.57e-12	1.06e-12	9.66e-13	7.68e-13	7.08e-13	6.56e-13	6.02e-13	5.56e-13	5.19e-13	4.84e-13	4.50e-13	4.16e-13	3.83e-13	3.50e-13	3.17e-13	2.84e-13	2.51e-13	2.18e-13
	18	1.53e-12	1.03e-12	9.08e-13	8.49e-13	7.46e-13	6.81e-13	6.35e-13	5.92e-13	5.50e-13	5.17e-13	4.84e-13	4.51e-13	4.18e-13	3.85e-13	3.52e-13	3.19e-13	2.86e-13	2.53e-13
	19	1.50e-12	9.98e-13	1.01e-12	8.28e-13	7.27e-13	6.62e-13	6.18e-13	5.85e-13	5.50e-13	5.15e-13	4.81e-13	4.47e-13	4.13e-13	3.80e-13	3.47e-13	3.14e-13	2.81e-13	2.48e-13
	20	1.48e-12	9.59e-13	8.95e-13	8.10e-13	7.10e-13	6.60e-13	6.02e-13	5.69e-13	5.34e-13	5.01e-13	4.69e-13	4.36e-13	4.03e-13	3.70e-13	3.37e-13	3.04e-13	2.71e-13	2.38e-13
	21	1.45e-12	9.77e-13	9.64e-13	9.55e-13	9.35e-13	9.15e-13	8.95e-13	8.75e-13	8.55e-13	8.31e-13	8.07e-13	7.83e-13	7.59e-13	7.35e-13	7.11e-13	6.87e-13	6.63e-13	6.39e-13
	22	1.43e-12	9.61e-13	7.80e-13	6.19e-13	5.76e-13	5.36e-13	4.96e-13	4.56e-13	4.16e-13	3.76e-13	3.36e-13	2.96e-13	2.56e-13	2.16e-13	1.76e-13	1.36e-13	9.64e-14	5.63e-14
	23	1.40e-12	9.47e-13	7.57e-13	6.70e-13	6.08e-13	5.53e-13	5.03e-13	4.53e-13	4.03e-13	3.53e-13	3.03e-13	2.53e-13	2.03e-13	1.53e-13	1.03e-13	5.33e-14	1.03e-13	1.53e-13
	24	1.39e-12	7.94e-13	7.24e-13	7.55e-13	6.59e-13	5.98e-13	5.35e-13	4.72e-13	4.19e-13	3.66e-13	3.13e-13	2.60e-13	2.07e-13	1.54e-13	1.01e-13	5.11e-14	1.01e-13	1.51e-13
	25	1.38e-12	7.55e-13	7.24e-13	7.45e-13	6.49e-13	5.89e-13	5.26e-13	4.63e-13	4.10e-13	3.57e-13	3.04e-13	2.51e-13	1.98e-13	1.45e-13	8.24e-14	1.45e-13	1.95e-13	2.45e-13
	26	1.37e-12	7.34e-13	7.36e-13	6.41e-13	5.80e-13	5.38e-13	5.07e-13	4.63e-13	4.20e-13	3.77e-13	3.34e-13	2.91e-13	2.48e-13	1.95e-13	1.42e-13	8.14e-14	1.42e-13	1.92e-13
	27	1.36e-12	7.27e-13	7.33e-13	6.51e-13	5.90e-13	5.47e-13	5.14e-13	4.71e-13	4.28e-13	3.85e-13	3.42e-13	2.99e-13	2.56e-13	2.13e-13	1.60e-13	1.07e-13	1.60e-13	2.07e-13
	28	1.35e-12	7.19e-13	7.25e-13	6.56e-13	5.94e-13	5.42e-13	5.09e-13	4.66e-13	4.23e-13	3.80e-13	3.37e-13	2.94e-13	2.51e-13	2.08e-13	1.55e-13	1.02e-13	1.55e-13	1.92e-13
	29	1.34e-12	7.13e-13	7.12e-13	6.59e-13	5.98e-13	5.45e-13	5.12e-13	4.69e-13	4.26e-13	3.83e-13	3.39e-13	2.96e-13	2.53e-13	2.10e-13	1.57e-13	1.04e-13	1.57e-13	1.94e-13
	30	1.32e-12	7.07e-13	7.05e-13	6.12e-13	5.53e-13	5.12e-13	4.82e-13	4.39e-13	3.96e-13	3.53e-13	3.10e-13	2.67e-13	2.24e-13	1.81e-13	1.38e-13	8.54e-14	1.38e-13	1.85e-13
	31	1.30e-12	6.98e-13	6.97e-13	6.41e-13	5.48e-13	5.10e-13	4.76e-13	4.33e-13	3.90e-13	3.47e-13	3.04e-13	2.61e-13	2.18e-13	1.75e-13	1.32e-13	7.95e-14	1.32e-13	1.82e-13
	32	1.29e-12	6.92e-13	6.91e-13	6.35e-13	5.38e-13	5.00e-13	4.67e-13	4.24e-13	3.81e-13	3.38e-13	2.95e-13	2.52e-13	2.09e-13	1.66e-13	1.23e-13	7.75e-14	1.23e-13	1.73e-13
	33	1.28e-12	6.86e-13	6.85e-13	6.29e-13	5.31e-13	4.93e-13	4.50e-13	4.07e-13	3.64e-13	3.21e-13	2.78e-13	2.35e-13	1.92e-13	1.49e-13	1.06e-13	6.95e-14	1.06e-13	1.56e-13
	34	1.27e-12	6.81e-13	6.80e-13	6.23e-13	5.24e-13	4.86e-13	4.43e-13	4.00e-13	3.57e-13	3.14e-13	2.71e-13	2.28e-13	1.85e-13	1.42e-13	9.95e-14	9.95e-13	1.49e-13	1.89e-13
	35	1.26e-12	6.77e-13	6.78e-13	6.15e-13	5.19e-13	4.83e-13	4.40e-13	3.97e-13	3.54e-13	3.11e-13	2.68e-13	2.25e-13	1.82e-13	1.39e-13	9.65e-14	9.65e-13	1.37e-13	1.77e-13
	36	1.25e-12	6.73e-13	6.75e-13	6.09e-13	5.05e-13	4.72e-13	4.29e-13	3.86e-13	3.43e-13	3.00e-13	2.57e-13	2.14e-13	1.71e-13	1.28e-13	8.55e-14	8.55e-13	1.26e-13	1.66e-13
	37	1.24e-12	6.69e-13	6.71e-13	6.03e-13	4.96e-13	4.63e-13	4.20e-13	3.77e-13	3.34e-13	2.91e-13	2.48e-13	2.05e-13	1.62e-13	1.19e-13	7.85e-14	7.85e-13	1.18e-13	1.58e-13
	38	1.23e-12	6.65e-13	6.65e-13	5.97e-13	4.87e-13	4.54e-13	4.11e-13	3.68e-13	3.25e-13	2.82e-13	2.39e-13	1.96e-13	1.53e-13	1.10e-13	7.25e-14	7.25e-13	1.15e-13	1.55e-13
	39	1.22e-12	6.61e-13	6.61e-13	5.91e-13	4.81e-13	4.48e-13	4.05e-13	3.62e-13	3.19e-13	2.76e-13	2.33e-13	1.90e-13	1.47e-13	1.04e-13	6.65e-14	6.65e-13	1.03e-13	1.43e-13
	40	1.21e-12	6.57e-13	6.57e-13	5.85e-13	4.73e-13	4.40e-13	3.97e-13	3.55e-13	3.12e-13	2.69e-13	2.26e-13	1.83e-13	1.40e-13	9.75e-14	5.15e-13	5.15e-13	9.75e-13	1.35e-13
	41	1.20e-12	6.53e-13	6.53e-13	5.79e-13	4.68e-13	4.35e-13	3.92e-13	3.49e-13	3.06e-13	2.63e-13	2.20e-13	1.77e-13	1.34e-13	8.85e-14	4.15e-13	4.15e-13	8.85e-13	1.25e-13
	42	1.19e-12	6.49e-13	6.49e-13	5.73e-13	4.63e-13	4.30e-13	3.87e-13	3.44e-13	3.01e-13	2.58e-13	2.15e-13	1.72e-13	1.29e-13	7.95e-14	3.25e-13	3.25e-13	7.95e-13	1.15e-13
	43	1.18e-12	6.45e-13	6.45e-13	5.67e-13	4.58e-13	4.25e-13	3.82e-13	3.39e-13	2.96e-13	2.53e-13	2.10e-13	1.67e-13	1.24e-13	7.05e-14	2.35e-13	2.35e-13	7.05e-13	1.05e-13
	44	1.17e-12	6.41e-13	6.41e-13	5.61e-13	4.53e-13	4.20e-13	3.77e-13	3.35e-13	2.92e-13	2.49e-13	2.06e-13	1.63e-13	1.21e-13	6.15e-14	1.45e-13	1.45e-13	6.15e-13	9.55e-14
	45	1.16e-12	6.37e-13	6.37e-13	5.55e-13	4.48e-13	4.15e-13	3.72e-13	3.30e-13	2.87e-13	2.44e-13	2.01e-13	1.58e-13	1.15e-13	5.25e-14	8.55e-13	8.55e-13	5.25e-13	8.55e-14
	46	1.15e-12	6.33e-13	6.33e-13	5.49e-13	4.43e-13	4.09e-13	3.66e-13	3.23e-13	2.79e-13	2.36e-13	1.93e-13	1.50e-13	1.07e-13	4.35e-14	7.65e-13	7.65e-13	4.35e-13	7.65e-14
	47	1.14e-12	6.29e-13	6.29e-13	5.43e-13	4.38e-13	4.05e-13	3.62e-13	3.19e-13	2.76e-13	2.33e-13	1.90e-13	1.47e-13	1.04e-13	3.45e-14	6.75e-13	6.75e-13	3.45e-13	6.

TABLE A.3 (continued)

F Distribution: Critical Values of F (0.1% significance level)

v_1	25	30	35	40	50	60	75	100	150	200
1	6.24465	6.26465	6.28465	6.30465	6.31465	6.33465	6.34465	6.35465	6.36465	6.37465
2	9.9946	9.9947	9.9947	9.9947	9.9948	9.9948	9.9949	9.9949	9.9949	9.9949
3	12.84	12.51	12.51	12.51	12.51	12.51	12.51	12.51	12.51	12.51
4	4.45	4.53	4.53	4.53	4.53	4.53	4.53	4.53	4.53	4.53
5	25.08	24.87	24.72	24.60	24.44	24.33	24.22	24.12	24.01	23.95
6	16.85	16.67	16.54	16.44	16.31	16.21	16.12	16.03	15.93	15.89
7	12.69	12.53	12.41	12.33	12.20	12.12	12.04	11.95	11.87	11.82
8	10.26	10.11	10.00	9.92	9.83	9.73	9.65	9.57	9.49	9.45
9	8.69	8.55	8.46	8.37	8.26	8.19	8.11	8.04	7.95	7.95
10	7.60	7.47	7.37	7.30	7.19	7.12	7.05	6.98	6.91	6.87
11	6.81	6.68	6.59	6.52	6.42	6.35	6.28	6.21	6.14	6.10
12	6.22	6.09	6.00	5.93	5.83	5.76	5.70	5.63	5.56	5.52
13	5.75	5.63	5.54	5.47	5.30	5.24	5.17	5.10	5.07	5.07
14	5.38	5.25	5.17	5.10	5.00	4.94	4.87	4.81	4.74	4.71
15	5.07	4.95	4.86	4.80	4.76	4.64	4.57	4.51	4.44	4.41
16	4.82	4.70	4.61	4.54	4.45	4.39	4.32	4.26	4.19	4.16
17	4.60	4.48	4.40	4.33	4.24	4.18	4.11	4.05	3.98	3.95
18	4.42	4.30	4.22	4.15	4.06	3.99	3.92	3.87	3.80	3.77
19	4.26	4.14	4.06	3.99	3.90	3.84	3.78	3.71	3.65	3.61
20	4.12	4.00	3.92	3.85	3.77	3.70	3.64	3.58	3.51	3.48
21	4.00	3.88	3.80	3.74	3.64	3.58	3.52	3.46	3.39	3.36
22	3.89	3.78	3.70	3.63	3.54	3.48	3.41	3.35	3.28	3.25
23	3.79	3.68	3.60	3.53	3.44	3.38	3.32	3.25	3.19	3.16
24	3.71	3.59	3.51	3.45	3.36	3.29	3.23	3.17	3.10	3.07
25	3.63	3.52	3.45	3.37	3.28	3.22	3.15	3.09	3.03	2.99
26	3.56	3.44	3.36	3.30	3.21	3.15	3.08	3.02	2.95	2.92
27	3.49	3.38	3.30	3.23	3.14	3.08	3.02	2.96	2.89	2.86
28	3.43	3.32	3.24	3.16	3.09	3.02	2.96	2.90	2.83	2.80
29	3.38	3.27	3.18	3.12	3.03	2.97	2.91	2.84	2.78	2.74
30	3.33	3.22	3.13	3.07	2.98	2.92	2.86	2.79	2.73	2.69
35	3.13	3.02	2.93	2.87	2.78	2.72	2.66	2.59	2.52	2.49
40	2.98	2.87	2.79	2.73	2.64	2.57	2.51	2.44	2.38	2.34
50	2.79	2.68	2.60	2.53	2.44	2.38	2.31	2.25	2.18	2.14
60	2.67	2.55	2.47	2.41	2.32	2.25	2.19	2.12	2.05	2.01
70	2.58	2.47	2.39	2.32	2.23	2.16	2.10	2.03	1.95	1.92
80	2.52	2.41	2.32	2.26	2.16	2.10	2.03	1.96	1.89	1.85
90	2.47	2.36	2.27	2.19	2.05	1.98	1.91	1.83	1.79	1.75
100	2.43	2.32	2.24	2.17	2.08	2.01	1.94	1.87	1.79	1.75
120	2.37	2.26	2.18	2.11	2.02	1.95	1.88	1.81	1.73	1.68
150	2.32	2.21	2.12	2.06	1.96	1.89	1.82	1.74	1.66	1.62
200	2.26	2.15	2.07	2.00	1.90	1.83	1.76	1.68	1.60	1.55
250	2.23	2.12	2.05	1.97	1.87	1.80	1.72	1.65	1.56	1.51
300	2.21	2.10	2.01	1.94	1.85	1.78	1.70	1.62	1.53	1.48
400	2.18	2.07	1.98	1.92	1.82	1.75	1.67	1.59	1.50	1.45
500	2.17	2.05	1.97	1.90	1.83	1.73	1.65	1.57	1.48	1.43
600	2.16	2.04	1.96	1.89	1.79	1.72	1.64	1.56	1.46	1.41
750	2.15	2.03	1.95	1.88	1.78	1.71	1.63	1.55	1.46	1.40
1000	2.14	2.02	1.94	1.87	1.77	1.69	1.62	1.53	1.44	1.38

TABLE A.4

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

Degrees of freedom	Significance level
1	3.841
2	5.991
3	7.815
4	9.483
5	11.070
6	12.592
7	14.057
8	15.507
9	16.919
10	18.307