

**UNIVERSITY OF SWAZILAND  
FACULTY OF SOCIAL SCIENCE  
DEPARTMENT OF ECONOMICS**

**SUPPLEMENTARY EXAMINATION PAPER: JULY 2017**

**TITLE OF PAPER: MATHEMATICS FOR ECONOMISTS I  
COURSE CODE: ECO 205  
TIME ALLOWED: TWO (2) HOURS**

**INSTRUCTIONS:**

1. Answer question1 (**COMPULSORY**), and any other two questions
2. Show all relevant workings to your answer
3. Question 1 carries 50 marks and the rest of the questions are graded out of 25 marks

**SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR**

**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY  
THE INVIGILATOR**

**Question 1 (COMPULSORY)**

a) Write short explanatory notes ( not more than a quarter of a page each) on the following:

- (a) Properties of the inverse
- (b) Behavioural equations and identities
- (c) Symmetric and idempotent matrix
- (d) Assumptions of the input-output model
- (e) Autonomous and induced expenditures

[ 5 marks each]

b) The demand and supply functions for a good are given by:

$$\text{Demand function: } P_d = 100 - 0.5 Q_d$$

$$\text{Supply function: } P_s = 10 + 0.5 Q_s$$

- i) Analyse the effect of the introduction of a price ceiling of E40 in this market [15 marks]
- ii) Calculate the profit made by black marketeers if a black market operated in this market. [10 marks]

**ANSWER ANY TWO QUESTIONS FROM THE FOLLOWING:**

**Question 2**

The following is the general macroeconomic model for a closed economy with no external sector:

$$\begin{aligned} Y &= C + I + G \\ C &= a + bY^d && \text{(Consumption)} \\ I &= \gamma + \epsilon i && \text{(Investment)} \\ Y^d &= Y - T && \text{(Disposable income)} \\ T &= \beta + tY && \text{(Taxation)} \\ M_d &= \sigma + hY + ki && \text{(Demand for Money)} \\ M_s &= M_0 \end{aligned}$$

Where the values of the parameters are:  $a = 150$ ,  $b = 0.6$ ,  $\gamma = 1150$ ,  $\epsilon = -58$ ,  $\sigma = 1850$ ,  
 $H = 0.7$ ,  $k = -77$ ,  $\beta = 30$ ,  $t = 0.3$ ,  $G = 1160$ ,  $I = 815.95$ ,  $M_0 = 3950$

- i. Algebraically determine the expressions for IS and LM curve. [15 marks]
- ii. Determine the equilibrium value of income and interest rate where commodity and money markets together are in equilibrium. [7 marks]
- iii. Determine the value of government expenditure multiplier. [3 marks]

**Question 3**

- a) Algebraically derive the relationship between marginal revenue (MR) and price elasticity of demand. State all relevant assumptions. (Hint: show that  $MR = P(1 + 1/\epsilon_p)$ ). [5 marks]
- b) A utility function is given by the equation  $U = 8x^{1/2}Y^{1/2}$ 
  - i) Graph the indifference curves for  $U = 40$ ,  $U = 48$ , and  $U = 56$  [9 marks]
  - ii) Derive expressions for the marginal utility of good X, and Y [5 marks]
  - iii) Deduce an expression for the slope of the indifference curve,  $\frac{\partial Y}{\partial X}$  [6 marks]

#### Question 4

A publisher of books has the following production function:

$$40Q = 1800 - 4(x - 12)^2 - 3(y - 20)^2$$

where  $Q$  is the number of books published over the year, and  $x$  and  $y$  respectively are the amounts of labour and materials used. The price of the book is E500, and the costs per unit of inputs (under pure competition) are E200 for labour and E150 for materials.

- a) Using the method of Lagrange, determine the optimum quantities of the two inputs. [18 marks]
- b) What is the effect on the solution if the price of the materials is raised to E225? (assume that the second order conditions are satisfied) [7 marks]

#### Question 5

- (a) Apply Cramer's rule and matrix inversion to show that the solution to the system of equations is identical using either methods:

$$Y = C + I + G$$

$$C = 100 + 0.6 Y$$

$$I = 50 + 0.2 Y$$

$$G = 500$$

[12 marks]

- (b) A large energy company produces electricity, natural gas, and oil. The production of a Lilangeni's worth of electricity requires inputs of E0.30 from electricity, E0.10 from natural gas, and E0.20 from oil. Production of a Lilangeni's worth of natural gas requires inputs of E0.30 from electricity, E0.10 from natural gas, and E0.20 from oil. Production of a Lilangeni's worth of oil requires inputs of E0.10 from each sector. Find the output for each sector that is needed to satisfy a final demand of E25 million for electricity, E15 million for natural gas, and E20 million for oil.

[13 marks]