UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
SUPPLEMENTARY EXAMINATION 2015/2016

TITLE OF PAPER : STATISTICS FOR ECONOMISTS

COURSE CODE : ECON 209

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :

1. QUESTION ONE (1) IN SECTION A IS COMPULSORY AND IT CARRIES 40 MARKS
2. ANSWER ANY OTHER THREE (3) QUESTIONS IN SECTION B. ALL QUESTIONS IN SECTION B CARRY 20 MARKS EACH.
3. ONLY SCIENTIFIC NON-PROGRAMMABLE CALCULATORS ARE ALLOWED.
4. ROUND UP YOUR FINAL ANSWERS TO TWO (2) DECIMAL PLACES.
5. THE REQUIRED PROBABILITY TABLES ARE ATTACHED AT THE BACK OF QUESTION PAPER.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

SECTION A

QUESTION 1 (COMPULSORY QUESTION) [40 MARKS]

a) Describe an unbiased estimator [5 Marks]

b) An agricultural experimenter, investigating the effect of the amount of nitrogen (x) applied in $100kg$ per hectare on the yield of maize meal (y) measured in tonnes per hectare, collected the data on the following table:

y	22	38	57	68	19	41	54	65
x	1	2	3	4	1	2	3	4

i. Fit the least squares line for the data. [10 Marks]

ii. Is there sufficient evidence to indicate that the yield of maize-meal is linearly related to the amount of nitrogen applied? Use the 95% confidence to come up with your conclusion. [5 Marks]

iii. Find a 95% confidence interval estimate for the slope of the amount of nitrogen usage (β), to confirm the result you obtained in part (b) above. [5 Marks]

iv. Predict the expected yield of maize-meal with 95% confidence if $250kg$ of nitrogen per hectare are applied. [5 Marks]

v. Calculate the correlation coefficient (r) of maize-meal yield and the amount of nitrogen used, and interpret what it means. [5 Marks]

vi. Of what value is this linear model when compared to $\bar{y}$ in predicting maize-meal yield? [5 Marks]

SECTION B

ANSWER ANY 3 QUESTIONS FROM THIS SECTION.

QUESTION 2 [20 Marks]

a) Describe a random variable, then differentiate between a discrete and continuous random variable. [6 Marks]

b) State 3 characteristics that define a binomial experiment. [3 Marks]

c) In an experiment of rolling a fair die 1000 times:

i. Define the sample space for this experiment. [2 Marks]

ii. Find the expected number of times that the number 5 comes up. [4 Marks]

iii. Find the variance of this experiment. [3 Marks]

iv. What is the standard deviation? [2 Marks]

QUESTION 3

a) List the five elements that are involved in a statistical hypothesis test. [5 Marks]

b) An experiment to determine whether a coin tossed 2000 times is fair or not is conducted. Using $= 0.05$, set up and conduct a statistical test that will provide a range of values for you to conclude that the coin is fair. [13 Marks]

c) If we use a stricter value of α, say $\alpha = 0.01$, without doing any calculations, state whether the range of values within which a fair coin will lie increases or decreases. [2 Marks]

QUESTION 4

a) State two (2) properties of the function $f(x)$, which is a probability distribution function (pdf) of a continuous random variable. [4 Marks]

b) Suppose that X is a continuous random variable whose probability distribution function is given by the following: $f(x) = \begin{cases} bx^2, & 0 < x < 3 \\ 0, & otherwise \end{cases}$

i. What is the value of b (constant) that will make $f(x)$ a valid pdf? [8 Marks]

ii. Find $P(1 < x < 2)$ [5 Marks]

iii. Find $P(x > 4)$ [3 Marks]

QUESTION 5

In a 100 *metre* race, four (4) equally qualified athletes, Mbuso, Sipho, and Dan, and John compete and the order of their finish in the race is recorded.

a) How many simple events are possible in this experiments? [3 Marks]

b) Since all the athletes are equally qualified, what is the probability that should be assigned to each simple event? [3 Marks]

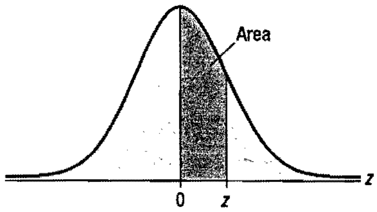
c) What is the probability that Dan comes first in the race? [5 Marks]

d) What is the probability that Dan wins and Sipho comes second? [5 Marks]

e) What is the probability that John is last in the race? [4 Marks]

C Standard Normal Distribution

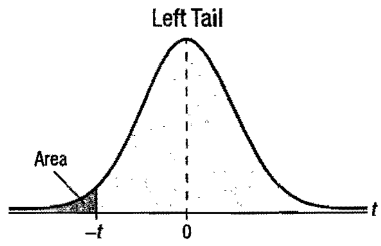
Numerical entries represent the probability that a standard normal random variable is between 0 and z where $z = \frac{x - \mu}{\sigma}$.

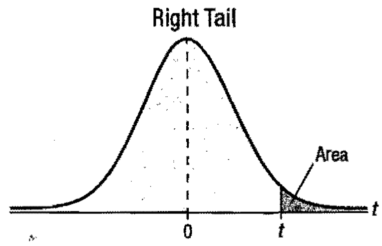


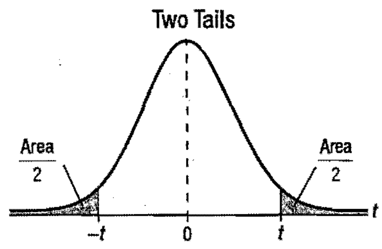
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

D Critical Values of t

	Area in One Tail				
	0.100	0.050	0.025	0.010	0.005
	Area in Two Tails				
df	0.200	0.100	0.050	0.020	0.010
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
200	1.286	1.653	1.972	2.345	2.601
300	1.284	1.650	1.968	2.339	2.592
400	1.284	1.649	1.966	2.336	2.588
500	1.283	1.648	1.965	2.334	2.586
750	1.283	1.647	1.963	2.331	2.582
1000	1.282	1.646	1.962	2.330	2.581
∞	1.282	1.645	1.960	2.326	2.576







APPENDIX

Useful Formulae

1. $S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$

2. $S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n} = \sum_{i=1}^{n}(x_i - \overline{x})^2$

3. $S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n} = \sum_{i=1}^{n}(y_i - \overline{y})^2$

4. $\widehat{\beta}_1 = \frac{S_{xy}}{S_{xx}},$

5. $\widehat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \widehat{\beta}_1 \frac{\sum_{i=1}^{n} x_i}{n} = \overline{y} - \widehat{\beta}_1 \overline{x}$

6. $SE(\widehat{\beta}_1) = \sqrt{\frac{MSE}{S_{xx}}}$

7. $MSE = \frac{SSE}{n-2}$

8. $TSS = S_{yy}$

9. $SSR = \frac{(S_{xy})^2}{S_{xx}}$

10. $SE(\widehat{y}) = \sqrt{MSE\left(\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{S_{xx}}\right)}$

Binomial Probability Distribution Function

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

Normal Probability Distribution Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty$$