UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS
MAIN EXAMINATION 2015/2016

TITLE OF PAPER : STATISTICS FOR ECONOMISTS

COURSE CODE : ECON 209

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :

1. QUESTION ONE (1) IN SECTION A IS COMPULSORY AND IT CARRIES 40 MARKS
2. ANSWER ANY OTHER THREE (3) QUESTIONS IN SECTION B. ALL QUESTIONS IN SECTION B CARRY 20 MARKS EACH.
3. ONLY SCIENTIFIC NON-PROGRAMMABLE CALCULATORS ARE ALLOWED.
4. ROUND UP YOUR FINAL ANSWERS TO TWO (2) DECIMAL PLACES.
5. THE REQUIRED PROBABILITY TABLES ARE ATTACHED AT THE BACK OF QUESTION PAPER.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR

SECTION A

QUESTION 1 (COMPULSORY QUESTION) [40 MARKS]

a) Write short explanatory notes on properties of ordinary least squares (OLS) estimators. [5 Marks]

b) A horticulturist devised a scale to measure the freshness of roses that were packaged and stored for varying periods of time before transplanting. The freshness measurement (y) and the length of time in days that the rose is packaged and stored before transplanting (x) are given on the following table:

y	15.3	13.6	9.8	5.5	1.8	16.8	13.8	8.7	4.7	1.0
x	5	10	15	20	25	5	10	15	20	25

i. Fit a least squares line to the data. [10 Marks]

ii. Is there sufficient evidence to indicate that freshness is linearly related to storage time? Use $\alpha = 0.05$ to come up with a decision. [5 Marks]

iii. Find a 95% confidence interval estimate for the slope of storage time (β), to confirm the result you obtained in part (b) above. [5 Marks]

iv. Estimate the expected freshness measurement for a storage time of 14-days with a 95% confidence interval. [5 Marks]

v. Calculate the correlation coefficient (r) of freshness of roses and the storage time of the roses and interpret what it means. [5 Marks]

vi. Of what value is this linear model when compared to $\bar{y}$ in predicting freshness? [5 Marks]

SECTION B

ANSWER THREE (3) QUESTIONS FROM THIS SECTION.

QUESTION 2 **[20 Marks]**

a) Define three (3) axioms of probability [6 Marks]

b) With the aid of an example, write short explanatory notes on the following:

i. Conditional Probability [3 Marks]

ii. Mutually exclusive events [3 Marks]

c) Two fair coins are tossed, and the outcome is recorded. The following are the events of interest:

A: At least one head is observed

B: At least one tail is observed

Define the following events: A, B, $A \cap B$, $A \cup B$, and A^c [8 Marks]

QUESTION 3 **[20 Marks]**

a) Differentiate between a parameter and a statistic [5 Marks]

b) John flips a fair coin three (3) times. Let the following events observed be defined as follows:

A : Tails on the first (1^{st}) flip

B : Tails on the second (2^{nd}) flip

C : Exactly two heads are observed

i. Define each event and state it's equivalent probability [6 Marks]

ii. Are events A and B independent? [5 Marks]

iii. Are events B and C independent? [4 Marks]

QUESTION 4

a) Define four (4) characteristics of a binomial experiment. [4 Marks]

b) Consider a binomial random variable with the number of trials, $n = 9$ and the probability of success, $p = 0.3$. Let x be the number of successes in the sample. Find the probability that:

i. x is less than two (2) [3 Marks]

ii. x is between two(2) and four(4) i.e $P(2 \leq x \leq 4)$ [3 Marks]

iii. x is greater than two(2) i.e $P(x > 2)$ [3 Marks]

c) The number of people (x) entering a the intensive care unit of a hospital on any given day is considered to exhibit a Poisson probability distribution with mean equals five (5) persons per day. What is the probability that the number of people entering the intensive care unit on a particular day is:

i. less than or equal to two i.e $[P(x \leq 2)]$ [4 Marks]

ii. More than 4 i.e $[P(x > 2)]$ [3 Marks]

QUESTION 5

a) Briefly define what is a **p-value**. [4 Marks]

b) A random sample of $n = 35$ observations from a certain population produced a mean $(\bar{x} = 2.4)$ and a standard deviation $(s = 0.29)$. Suppose the research objective is to show that the population mean (μ) exceeds 2.3

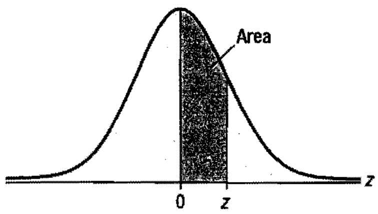
i. State the null and alternative hypothesis [2 Marks]

ii. Do the data provide sufficient evidence to suggest the population mean is greater than 2.3 i.e $(\mu > 2.3)$. [7 Marks]

c) Calculate the p-value for the test statistic above and use it to draw a conclusion at the 5% significance level. [7 Marks]

C Standard Normal Distribution

Numerical entries represent the probability that a standard normal random variable is between 0 and z where $z = \frac{x - \mu}{\sigma}$.

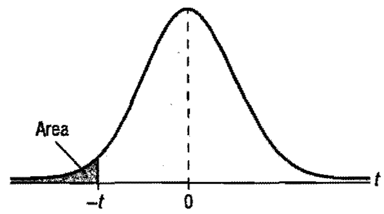


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

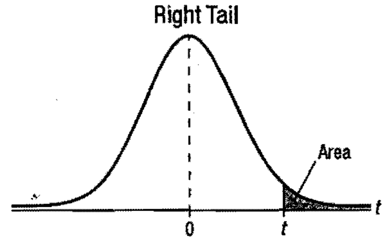
D Critical Values of *t*

	Area in One Tail				
	0.100	0.050	0.025	0.010	0.005
	Area in Two Tails				
df	0.200	0.100	0.050	0.020	0.010
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.318	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.269	1.658	1.980	2.358	2.617
200	1.286	1.653	1.972	2.345	2.601
300	1.284	1.650	1.968	2.339	2.592
400	1.284	1.649	1.966	2.338	2.568
500	1.283	1.648	1.965	2.334	2.586
750	1.283	1.647	1.963	2.331	2.562
1000	1.282	1.646	1.962	2.330	2.581
∞	1.282	1.645	1.960	2.326	2.576

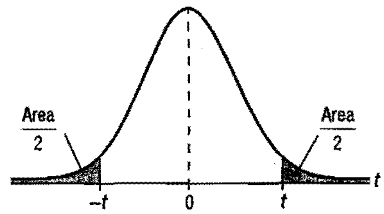
Left Tail



Right Tail



Two Tails



APPENDIX

Useful Formulae

1. $S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$

2. $S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n} = \sum_{i=1}^{n}(x_i - \bar{x})^2$

3. $S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n} = \sum_{i=1}^{n}(y_i - \bar{y})^2$

4. $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}},$

5. $\hat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \hat{\beta}_1 \frac{\sum_{i=1}^{n} x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x}$

6. $SE(\hat{\beta}_1) = \sqrt{\frac{MSE}{S_{xx}}}$

7. $MSE = \frac{SSE}{n-2}$

8. $TSS = S_{yy}$

9. $SSR = \frac{(S_{xy})^2}{S_{xx}}$

10. $SE(\hat{y}) = \sqrt{MSE\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$

Binomial Probability Distribution Function

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

Normal Probability Distribution Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \qquad -\infty < x < \infty$$