UNIVERSITY OF SWAZILAND

FACULTY OF SOCIAL SCIENCE

DEPARTMENT OF ECONOMICS

FINAL EXAMINATION PAPER: MAY, 2010

TITLE OF PAPER: QUANTITATIVE METHODS

COURSE CODE: ECON 205

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

1. Answer **FOUR** Questions. Two from Section A and two from Section B.

2. Show all relevant workings to your answer

3. All Questions carry a total of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR.

SECTION A

Question 1

Complete the following statements by filling in the blanks.

a)

i) __________________ __________________ is the technique used to solve problems in which the objective is to maximize or minimize a linear function subject to a set of linear inequality constraints.

ii) A_________________ matrix is one that is unaltered by transposition.

iii) A ______________________ equation defines the manner in which a variable changes in response to changes in other variables.

iv) A _______________ equation expresses the value of y in any period, t, as a function of its own value in the previous period, t-1.

v) The constraints of a maximum in standard form are changed from an inequality to an equation by introducing__________________ _____________.

vi) The ____________ ____________________ ___________________ principle states that the optimal solution of a minimum linear programming problem, if it exists, has the same value as the optimal solution of the maximum problem, which is its dual.

vii) A function of the form $f(x) = a^x, a > 0, a \neq 1,$ is called an __________________ function

viii) A ___________ ____________ is a sequence of terms in which each term is obtained from the preceding term by multiplying by a constant known as the common ratio.

ix) If $AB = I$, the identity matrix, then B is called the ____________ of A.

x) ___________________________ is a measure of the net benefit to a consumer from purchasing a commodity at a certain price that is below what the consumer would have been willing to pay.

[10 marks]

b) Consider the following macroeconomic model

$C = 97 + 0.7Y$

$I = 180 - 125r$

$M_s = 255$

$L_1 = 0.2Y$ (transactionary demand for money)

$L_2 = 220 - 175r$ (speculative demand for money)

i. determine the nature of the function Y = f(x) for the product market. [3 marks]

ii. Determine the function Y = f(r) which equates the demand for money (M_d) with the supply for money (M_s) [3 marks]

iii. Find the equilibrium levels of income, interest rate, consumption, investment, transactions and precautionary demand. [9 marks]

Question 2

a) A man receives a salary of E8500 per annum which increases annually by 5% of the current salary. What salary will he receive after seven years? What will be his total income during the first five years? [8 marks]

b) Given the following national income model

$$Y = C + I$$

$$C = C_0 + bY^d$$

$$T = T_0 + tY$$

$$Y^d = Y - T$$

$$where I = I_0 = 30, C_0 = 85, b = 0.75, T = 0.2 and T_0 = 20$$

(i) find the reduced form of the model [10 marks]

(ii) find the numerical value of income at equilibrium [3 marks]

(iii) what is the effect on the multiplier if a proportional income tax (t) is incorporated into the model [4 marks]

Question 3

(a) Find the inverse of the matrix A = $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ [4 marks]

(b) Use the Guass Jordan method to solve the following system of equations;

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + 2x_2 - x_3 = 4$$
$$3x_1 + x_2 + 2x_3 = 11$$

[11 marks]

(c) Determine the total demand for X for industries 1, 2, and 3, given the matrix of technical coefficients and the final demand vector D, i.e.

$$A = \begin{pmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{pmatrix} \qquad B = \begin{pmatrix} 20 \\ 10 \\ 30 \end{pmatrix}$$

[10 marks]

Question 4

a) Algebraically derive the relationship between marginal revenue (MR) and price elasticity of demand. State all relevant assumptions. (Hint: show that MR= $(1+1/\varepsilon_p)$)
[5 marks]

b) The Cobb- Douglas production function for a new product is given by:

$$N(x,y) = 16x^{0.25}y^{0.75}$$

Where x is the number of units of labor and y is the number of units of capital required to produce N(x,y) units of the product. Each unit of labor costs E50 and each unit of capital costs E100. If E500,000 has been budgeted for the production of this product, how should this amount be allocated between labor and capital in order to maximize production? What is the maximum number of units that can be produced? [10 marks]

(c) Biella Textiles Pty (Ltd) produces a unique fabric whose marginal revenue function is given by :

$$MR = 200 - 3Q$$

and the corresponding marginal cost associated with the production of the fabric is :

$$MC = 2Q$$

Compute the value of profits if output is 30 units and hence find the maximum value of profit. [10 marks]

SECTION B

Question 5

a) Part one of the examinations of the Institute of Chartered Secretaries and Administrators consists of two modules. The subjects of module one are: Communication, and General Principles of Law. The subjects of module two are : Principles of Economics and Statistics. A student attempts part one of the examinations of the Institute of Chartered Secretaries and Administrators, i.e. he takes module one and two. He considers that his chances of passing Communication are 0.7, General Principles of Law is 0.6, Principles of Economics 0.8 and Statistics is 0.9. Assuming that the probability of his passing one subject is independent of the probability of his passing the other three subjects, find the probability :

i. That he passes part one [5 marks]
ii. That he fails all four examinations [5 marks]
iii. That he passes just one module, i.e passes either module one or module two [8 marks]

b) Another student attempts module three. If the probability of passing this module is constant and equal to 0.6, find the probability that the student passes module three at the third attempt. [4 marks]

c) State clearly what is meant by two events being statistically independent.

[3 marks]

Question 6

a) Using the concept of the dual use the simplex method to solve the maximization problem that derives from the following:

$$\text{Minimize } C = 40X_1 + 12x_2 + 40X_3$$
$$\text{Subject to} \quad 2X_1 + X_2 + 5X_3 \geq 20$$
$$4X_1 + X_2 + X_3 \geq 30$$
$$X_1, X_2, X_3 \geq 0$$

[20 marks]

b) A farmer uses two feedstuffs Y and X to feed his animals. To obtain meat of good quality he has to ensure that he obtains each day 240 kilos of fat, 150 kilos of protein and 30 kilos of vitamins. Feed Y contains by weight 30% fat, 10% protein, and 6% vitamin. Feed X contains by weight 20% fat, 25% protein and 2% vitamins. The cost per kilo of feed Y is 30 cents and feed X is 15 cents. Formulate a linear programming problem that depicts the minimum cost quantities of feed Y and X to purchase each day to meet the needs of the animals. (do not solve).

[5 marks]

Question 7

A sales report, as indicated in the data below, shows the number of dresses sold, number of hours worked, and months of experience, for 10 randomly selected part-time sales personnel in the women's dress department of a large store.

Y Dresses Sold	X_1 Hours worked	X_2 Months of experience
2	6	0
4	4	2
16	16	4
10	10	6
12	12	8
8	8	10
14	12	12
18	16	14
16	14	16
3	4	3

a) Fit an equation of the form $Y = b_0 + b_1X_1 + b_2X_2$ and plot the data. [20 marks]

b) Predict the sales by a part-time salesperson who works 5 hours and who has 2.5 months of experience [5 marks]

Question 8

a) Find K(t), the time path of capital, given the following net investment flow function and the corresponding initial conditions:

$I(t) = 900\sqrt{t}$ $t = 0$ and $K(0) = 1300$ [5 marks]

b) Consider the following macroeconomic model.

$$C_t = 70 + \tfrac{2}{3} Y_{t-1}$$
$$Y_t = C_t + I_t$$

(i) suppose that I_t is kept constant at the value I_0. Obtain a first order difference equation for income Y_t and solve it. Show that the system will reach long-term equilibrium in which Y_t, C_t are also constant and find their values when $I_0 = 20$.

[10 marks]

(ii) Suppose that I_t has been fixed at 20 for many years, and then suddenly rises to a new fixed value of 30. Find the new equilibrium value for Y_t. How many years will it take for Y_t to complete two-thirds of the rise to its new equilibrium value. [10 marks]