

**UNIVERSITY OF SWAZILAND
FACULTY OF SOCIAL SCIENCE
DEPARTMENT OF ECONOMICS**

MAIN EXAMINATION PAPER: MAY, 2008

TITLE OF PAPER: QUANTITATIVE METHODS

COURSE CODE: ECON 205

TIME ALLOWED: THREE (3) HOURS

INSTRUCTIONS:

1. Answer **FOUR** Questions. Two from Section A and two from Section B.
2. Show all relevant workings to your answer
3. All Questions carry a total of 25 marks

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR

**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY
THE INVIGILATOR.**

SECTION A

Question 1

Write explanatory notes on the following concepts

- (a) The meaning of equilibrium
- (b) Endogenous vs. exogenous variables
- (c) Matrix singularity
- (d) The objective of linear programming
- (e) Assumptions of the input-output model

[5 marks each]

Question 2

- (a) Estrada Engineering Components (Pty) Ltd faces the following demand curve:

$$Q_d = 60 - 3P$$

The firm's accountants believe that the supply curve is given by

$$Q_s = 2P - 16$$

- (i) Determine the equilibrium price and quantity in the market for the firm.
[4 marks]
- (ii) If the government now decides to impose a tax of E3 per unit on the quantity supplied and the producer adjusts the supply function to include the tax, calculate the new equilibrium price and quantity? Who pays the tax?
[6 marks]

- (b) Consider the following macroeconomic model

$$C = 97 + 0.7Y$$

$$I = 180 - 125r$$

$$M_s = 255$$

$$L_1 = 0.2Y \quad (\text{transactionary demand for money})$$

$$L_2 = 220 - 175r \quad (\text{speculative demand for money})$$

- (i) determine the nature of the function $y = f(x)$ for the product market
[3 marks]
- (ii) determine the function $y = f(r)$ which equates the demand for money (M_d) with the supply for money (M_s).
[3 marks]

- (c) Find the equilibrium levels of income, interest rate, consumption, investment, transactions and precautionary demand. [9 marks]

Question 3

- (a) Apply Cramer's rule and matrix inversion to show that the solution to the system of equations is identical using either methods:

$$\begin{aligned}Y &= C + I + G \\C &= 100 + 0.6 Y \\I &= 50 + 0.2Y \\G &= 500\end{aligned}$$

[12 marks]

- (b) A large energy company produces electricity, natural gas, and oil. The production of a Lilangeni's worth of electricity requires inputs of E0.30 from electricity, E0.10 from natural gas, and E0.20 from oil. Production of a Lilangeni's worth of natural gas requires inputs of E0.30 from electricity, E0.10 from natural gas, and E0.20 from oil. Production of a Lilangeni's worth of oil requires inputs of E0.10 from each sector. Find the output for each sector that is needed to satisfy a final demand of E25 million for electricity, E15 million for natural gas, and E20 million for oil.

[13 marks]

Question 4

(a) A rancher wants to construct two feeding pens of the same size along an existing fence. If the rancher has 720 metres of fencing material available, how long should x and y be in order to obtain the maximum total area? What is the maximum area? [12 marks]

(b) Given the demand function $P_d = 25 - Q^2$ and the supply function $P_s = 2Q + 1$. Assuming pure competition, find:

- | | | |
|-------|----------------------------------|-----------|
| (i) | the consumers surplus | [5 marks] |
| (ii) | the producers surplus | [5 marks] |
| (iii) | present your results graphically | [3 marks] |

SECTION B

Question 5

a) Distinguish between:

- (i) A simple and a Composite index [5 marks]
(ii) Consumer Price Index and Price deflator for GDP [5 marks]

b) Consider the following data:

Commodity/ Year	2003		2004	
	Price(E)	Quantity	Price(E)	Quantity
A	1	60	1.50	80
B	2	110	2.30	130
C	3	80	4.20	120
D	4	150	5	230

- (i) Define, calculate the Laspyres and Paasche index numbers for 2004 using 2003 as a base [10 marks]
(ii) Interpret the results and explain why they are different [5 marks]

Question 6

a) Suppose a guided missile has five distinct circuit sections through which a signal must pass if the missile is to operate properly. Each of the individual sections has 2 circuits through which a signal may pass, at least one of which must function if the signal is to traverse that section. If the probability that any single circuit will fail is 0.2, what is the probability that a signal will pass through:

- a) the first section [3 marks]
c) all 5 sections [3 marks]

b) If a list of 20 individuals who volunteered to donate blood when it is needed for a transfusion, has 15 individuals of type B blood and if three individuals are selected at random from the list, what is the probability that:

- i) all three will be of type B [3 marks]
ii) 2 will be of type B and one will not. [3 marks]

iii) at least one will be of type B [3 marks]

c) A device for testing radio tubes can always detect a bad tube, but 2% of the time it claims that a tube is bad when, in fact, it is a good tube. If 97% of manufactured tubes are good, what is the probability that if a new tube is selected at random and the testing device claims it is a bad tube, the tube is actually a good one.

[10 marks]

Question 7

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

- a) Determine the regression line of Y on X and interpret its coefficients [10 marks]
- b) Find the standard errors for the estimators [6marks]
- c) Find the Coefficient of Determination and the Correlation Coefficient and interpret the results. [9 marks]

Question 8

a) The fish pond at a school fund-raising affair has 25 packages worth £1 each, 5 packages worth £5 each, and 1 package worth £10. How much should you be willing to pay to take one chance at this fish pond if you wanted to get your money's worth. [6 marks]

b) A charity affair sells raffle tickets for £5 each. There are 10 prizes worth £25 each and one grand prize worth £100. If 200 tickets are sold, and you bought one of them, what is your expectation?

[6 marks]

c) Suppose a test for detecting a certain rare disease is capable of discovering the disease in 97% of all afflicted individuals. Suppose further that when it is tried on healthy individuals, 5% of them are incorrectly diagnosed as having the disease. Finally, suppose that when it is tried on individuals who have certain other milder diseases, 10% of them are incorrectly diagnosed. It is known that the percentages of individuals of the three types being considered here in the

population at large are 1%, 96% and 3% respectively. Calculate the probability that an individual, selected at random from the population at large and tested for the disease, actually has the disease if the test indicates that he is so afflicted.

[13 marks]

SOME USEFUL FORMULAE

$$1. \hat{\beta}_1 = \frac{\sum x_i y_i}{\sum x_i^2} , \quad \text{var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum x_i^2}$$

$$2. \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} , \quad \text{var}(\hat{\beta}_0) = \frac{\sum x_i^2 \sigma^2}{N \sum x_i^2}$$

$$3. \sum e_i^2 = \sum (Y_i - \hat{Y}_i)^2$$

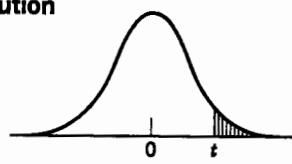
$$4. r^2 = \frac{\sum (\hat{Y}_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2}$$

where: $x_i = X_i - \bar{X}$

and $y_i = Y_i - \bar{Y}$

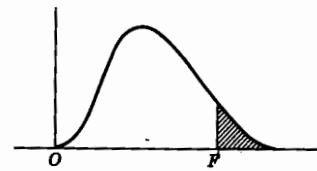
TABLE V. Student's t Distribution

The first column lists the number of degrees of freedom (ν). The headings of the other columns give probabilities (P) for t to exceed the entry value. Use symmetry for negative t values.



$\nu \backslash P$.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

TABLE IX. F Distribution
 5% (Roman Type) and 1% (Boldface Type) Points for the Distribution of F



Degrees of freedom for denominator (v_2)	Degrees of freedom for numerator (v_1)																							
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
1	161 4052	200 4999	216 5403	225 5625	230 5764	234 5859	237 5928	239 5981	241 6022	242 6056	243 6082	244 6106	245 6142	246 6169	248 6208	249 6234	250 6258	251 6286	252 6302	253 6323	253 6334	254 6352	254 6361	254 6366
2	18.51 98.49	19.00 99.01	19.16 99.17	19.25 99.25	19.30 99.30	19.33 99.33	19.36 99.36	19.38 99.38	19.39 99.40	19.40 99.41	19.42 99.42	19.43 99.43	19.44 99.44	19.45 99.45	19.46 99.46	19.47 99.47	19.48 99.48	19.49 99.49	19.49 99.49	19.49 99.49	19.49 99.49	19.49 99.49	19.50 99.50	19.50 99.50
3	10.13 34.12	9.55 30.81	9.28 29.46	9.12 28.71	9.01 28.24	8.94 27.91	8.88 27.67	8.84 27.49	8.81 27.34	8.78 27.23	8.76 27.13	8.74 27.05	8.71 26.92	8.69 26.83	8.66 26.69	8.64 26.50	8.62 26.41	8.60 26.30	8.58 26.27	8.57 26.23	8.56 26.18	8.54 26.14	8.53 26.12	
4	7.71 21.20	6.94 18.00	6.59 16.69	6.39 15.98	6.26 15.52	6.16 15.21	6.09 14.98	6.04 14.80	6.00 14.66	5.96 14.54	5.93 14.45	5.91 14.37	5.87 14.24	5.84 14.15	5.80 14.02	5.77 13.93	5.74 13.83	5.71 13.74	5.70 13.69	5.68 13.61	5.66 13.57	5.65 13.52	5.64 13.48	5.63 13.46
5	6.61 16.26	5.79 13.27	5.41 12.06	5.19 11.39	5.05 10.67	4.95 10.45	4.88 10.27	4.82 10.15	4.78 10.05	4.74 9.96	4.70 9.89	4.68 9.77	4.64 9.68	4.60 9.55	4.56 9.47	4.53 9.38	4.50 9.29	4.46 9.24	4.42 9.17	4.40 9.13	4.38 9.07	4.37 9.04	4.36 9.02	
6	5.99 13.74	5.14 10.92	4.76 9.78	4.53 9.15	4.39 8.75	4.28 8.47	4.21 8.26	4.15 8.10	4.10 7.98	4.06 7.87	4.03 7.79	4.00 7.72	3.96 7.60	3.92 7.52	3.87 7.39	3.84 7.31	3.81 7.23	3.77 7.14	3.75 7.09	3.72 7.02	3.71 6.99	3.69 6.94	3.68 6.88	
7	5.59 12.25	4.74 9.55	4.35 8.45	4.12 7.85	3.97 7.46	3.87 7.19	3.79 7.00	3.73 6.84	3.68 6.71	3.63 6.62	3.60 6.54	3.57 6.47	3.52 6.35	3.49 6.27	3.44 6.15	3.41 6.07	3.38 5.98	3.34 5.90	3.32 5.85	3.29 5.78	3.28 5.75	3.25 5.70	3.24 5.65	
8	5.32 11.26	4.46 8.65	4.07 7.59	3.84 7.01	3.69 6.63	3.58 6.37	3.50 6.19	3.44 6.03	3.39 5.91	3.34 5.82	3.31 5.74	3.28 5.67	3.23 5.56	3.20 5.48	3.15 5.36	3.12 5.28	3.08 5.20	3.05 5.11	3.03 5.06	3.00 4.96	2.98 4.91	2.96 4.86	2.94 4.86	
9	5.12 10.56	4.26 8.02	3.86 6.99	3.63 6.42	3.48 6.06	3.37 5.80	3.29 5.62	3.23 5.47	3.18 5.35	3.13 5.26	3.10 5.18	3.07 5.11	3.02 5.00	2.98 4.92	2.93 4.80	2.90 4.73	2.86 4.64	2.82 4.56	2.80 4.51	2.77 4.45	2.76 4.36	2.72 4.33	2.71 4.31	

10	4.96 10.04	4.10 7.56	3.71 6.55	3.48 5.99	3.33 5.64	3.22 5.39	3.14 5.21	3.07 5.06	3.02 4.95	2.97 4.85	2.94 4.78	2.86 4.71	2.82 4.60	2.77 4.52	2.74 4.41	2.74 4.33	2.70 4.25	2.67 4.17	2.64 4.12	2.61 4.05	2.59 4.01	2.56 3.96	2.55 3.93	2.54 3.91
11	4.84 9.65	3.98 7.20	3.59 6.22	3.36 5.67	3.20 5.32	3.09 5.07	3.01 4.88	2.95 4.74	2.90 4.63	2.86 4.54	2.82 4.46	2.79 4.40	2.74 4.29	2.70 4.21	2.65 4.10	2.61 4.02	2.57 3.94	2.53 3.86	2.50 3.80	2.47 3.74	2.45 3.70	2.42 3.66	2.41 3.62	2.40 3.60
12	4.75 9.33	3.88 6.93	3.49 5.95	3.26 5.41	3.11 5.06	3.00 4.82	2.92 4.65	2.85 4.50	2.80 4.39	2.76 4.30	2.72 4.22	2.67 4.16	2.63 4.05	2.54 3.98	2.50 3.86	2.50 3.78	2.46 3.70	2.50 3.61	2.46 3.56	2.42 3.49	2.40 3.46	2.36 3.41	2.32 3.38	2.30 3.36
13	4.67 9.07	3.80 6.70	3.41 5.74	3.18 5.20	3.02 4.86	2.92 4.62	2.84 4.44	2.77 4.30	2.72 4.19	2.67 4.02	2.63 3.96	2.60 3.85	2.55 3.78	2.51 3.73	2.46 3.67	2.42 3.59	2.38 3.51	2.32 3.43	2.32 3.37	2.28 3.30	2.26 3.27	2.22 3.21	2.22 3.18	2.21 3.16
14	4.60 8.86	3.73 6.51	3.34 5.56	3.11 5.03	2.96 4.69	2.85 4.46	2.77 4.28	2.70 4.14	2.65 4.03	2.60 3.94	2.56 3.86	2.53 3.80	2.48 3.70	2.44 3.62	2.39 3.51	2.35 3.43	2.31 3.34	2.27 3.26	2.24 3.21	2.21 3.14	2.19 3.11	2.16 3.06	2.14 3.02	2.13 3.00
15	4.54 8.68	3.68 6.36	3.29 5.42	3.06 4.89	2.90 4.56	2.79 4.32	2.70 4.14	2.64 4.00	2.59 3.89	2.55 3.80	2.51 3.73	2.48 3.67	2.43 3.56	2.39 3.48	2.33 3.36	2.29 3.29	2.25 3.20	2.21 3.12	2.18 3.07	2.15 3.00	2.12 2.97	2.10 2.92	2.08 2.89	2.07 2.87
16	4.49 8.53	3.63 6.23	3.24 5.29	3.01 4.77	2.85 4.44	2.74 4.20	2.66 4.03	2.59 3.89	2.54 3.78	2.49 3.69	2.45 3.61	2.42 3.55	2.37 3.45	2.33 3.37	2.28 3.25	2.24 3.18	2.20 3.10	2.16 3.01	2.13 2.96	2.09 2.89	2.07 2.86	2.04 2.80	2.02 2.77	2.01 2.75
17	4.45 8.40	3.59 6.11	3.20 5.18	2.96 4.67	2.81 4.34	2.70 4.10	2.62 3.93	2.55 3.79	2.50 3.69	2.45 3.59	2.41 3.52	2.38 3.45	2.33 3.35	2.29 3.27	2.23 3.16	2.20 3.08	2.15 3.00	2.11 2.92	2.08 2.86	2.04 2.79	2.02 2.76	1.99 2.70	1.97 2.67	1.96 2.65
18	4.41 8.28	3.55 6.01	3.16 5.09	2.93 4.58	2.77 4.25	2.66 4.01	2.58 3.85	2.51 3.71	2.46 3.60	2.41 3.51	2.37 3.44	2.34 3.37	2.29 3.27	2.25 3.19	2.19 3.07	2.15 3.00	2.11 2.91	2.07 2.83	2.04 2.78	2.00 2.71	1.98 2.68	1.96 2.62	1.93 2.57	1.92 2.55
19	4.38 8.18	3.52 5.93	3.13 5.01	2.90 4.50	2.74 4.17	2.63 3.94	2.55 3.77	2.48 3.63	2.43 3.52	2.38 3.43	2.34 3.36	2.31 3.30	2.26 3.19	2.21 3.12	2.15 3.00	2.11 2.92	2.07 2.84	2.02 2.76	2.00 2.70	1.96 2.63	1.92 2.56	1.90 2.53	1.87 2.47	1.85 2.44
20	4.35 8.10	3.49 5.85	3.10 4.94	2.87 4.43	2.71 4.10	2.60 3.87	2.52 3.71	2.45 3.56	2.40 3.45	2.35 3.37	2.31 3.30	2.28 3.13	2.23 3.05	2.18 2.94	2.12 2.86	2.08 2.77	2.04 2.69	1.99 2.63	1.92 2.56	1.90 2.53	1.87 2.47	1.85 2.44	1.84 2.42	1.82 2.40
21	4.32 8.02	3.47 5.78	3.07 4.87	2.84 4.37	2.68 4.04	2.57 3.81	2.49 3.65	2.42 3.51	2.37 3.40	2.32 3.31	2.28 3.17	2.22 3.07	2.15 2.99	2.09 2.88	2.05 2.80	2.00 2.72	1.96 2.63	1.89 2.58	1.87 2.51	1.84 2.47	1.82 2.42	1.81 2.38	1.80 2.36	1.79 2.34
22	4.30 7.94	3.44 5.72	3.05 4.82	2.82 4.31	2.66 3.99	2.55 3.76	2.47 3.59	2.40 3.45	2.35 3.35	2.30 3.26	2.26 3.18	2.23 3.12	2.18 3.02	2.13 2.94	2.07 2.83	1.98 2.75	1.96 2.67	1.93 2.58	1.91 2.53	1.87 2.46	1.84 2.42	1.81 2.37	1.78 2.33	1.76 2.31
23	4.28 7.88	3.42 5.66	3.03 4.76	2.80 4.26	2.64 3.94	2.53 3.71	2.45 3.54	2.38 3.31	2.32 3.14	2.28 3.03	2.24 2.97	2.20 2.89	2.14 2.87	2.10 2.88	2.04 2.78	2.00 2.70	1.96 2.62	1.88 2.53	1.84 2.48	1.81 2.41	1.77 2.37	1.76 2.32	1.74 2.26	1.72 2.22
24	4.26 7.82	3.40 5.61	3.01 4.72	2.78 4.22	2.62 3.90	2.51 3.67	2.43 3.50	2.36 3.36	2.30 3.25	2.26 3.17	2.22 3.09	2.18 3.03	2.09 2.93	2.05 2.85	2.02 2.74	1.98 2.66	1.94 2.58	1.89 2.49	1.86 2.44	1.82 2.36	1.80 2.33	1.76 2.27	1.74 2.23	1.73 2.21
25	4.24 7.77	3.38 5.57	2.99 4.68	2.76 4.18	2.60 3.86	2.49 3.63	2.41 3.46	2.34 3.32	2.28 3.21	2.24 3.13	2.20 3.05	2.16 2.99	2.11 2.89	2.06 2.81	2.00 2.70	1.96 2.62	1.92 2.54	1.87 2.45	1.84 2.40	1.80 2.32	1.77 2.29	1.74 2.23	1.72 2.19	1.71 2.17

TABLE IX. (Continued)

Degrees of freedom for denominator (v_2)	Degrees of freedom for numerator (v_1)																								
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞	
26	4.22 7.72	3.37 5.53	2.89 4.64	2.74 4.14	2.59 3.82	2.47 3.59	2.39 3.42	2.32 3.29	2.27 3.17	2.22 3.09	2.18 3.02	2.15 2.96	2.10 2.86	2.05 2.77	1.99 2.66	1.95 2.58	1.90 2.50	1.85 2.41	1.82 2.36	1.78 2.28	1.76 2.25	1.72 2.21	1.70 2.19	1.69 2.15	
27	4.21 7.68	3.35 5.49	2.96 4.60	2.73 4.11	2.57 3.79	2.46 3.56	2.37 3.39	2.30 3.26	2.25 3.14	2.20 3.06	2.16 2.98	2.13 2.93	2.08 2.83	2.03 2.74	1.97 2.63	1.93 2.55	1.88 2.47	1.84 2.38	1.80 2.33	1.76 2.25	1.74 2.21	1.71 2.16	1.68 2.12	1.67 2.10	
28	4.20 7.64	3.34 5.45	2.95 4.57	2.71 4.07	2.56 3.76	2.44 3.53	2.36 3.36	2.29 3.23	2.34 3.11	2.19 3.03	2.15 2.95	2.12 2.90	2.06 2.88	2.02 2.71	1.96 2.60	1.91 2.52	1.87 2.44	1.81 2.35	1.78 2.30	1.75 2.22	1.72 2.18	1.69 2.13	1.67 2.09	1.65 2.06	
29	4.18 7.60	3.33 5.52	2.93 4.54	2.70 4.04	2.54 3.73	2.43 3.50	2.35 3.33	2.28 3.20	2.22 3.08	2.18 3.00	2.14 2.92	2.10 2.87	2.05 2.77	2.00 2.68	1.94 2.57	1.90 2.49	1.85 2.41	1.80 2.32	1.77 2.27	1.73 2.19	1.71 2.15	1.68 2.10	1.66 2.06	1.65 2.03	
30	4.17 7.56	3.32 5.39	2.92 4.51	2.69 4.02	2.53 3.70	2.42 3.47	2.34 3.30	2.27 3.17	2.21 3.06	2.16 2.98	2.12 2.84	2.09 2.74	2.04 2.66	1.99 2.55	1.93 2.47	1.89 2.38	1.84 2.29	1.79 2.24	1.76 2.21	1.72 2.16	1.69 2.13	1.66 2.07	1.64 2.03	1.62 2.01	
32	4.15 7.50	3.30 5.34	2.90 4.46	2.67 3.97	2.51 3.66	2.40 3.42	2.32 3.25	2.25 3.12	2.19 3.01	2.14 2.94	2.10 2.86	2.07 2.80	2.02 2.70	1.97 2.62	1.91 2.51	1.86 2.42	1.82 2.34	1.76 2.25	1.74 2.20	1.69 2.12	1.67 2.08	1.64 2.02	1.61 1.98	1.59 1.96	
34	4.13 7.44	3.28 5.29	2.88 4.42	2.65 3.93	2.49 3.61	2.38 3.38	2.30 3.21	2.23 3.08	2.17 2.97	2.12 2.89	2.08 2.82	2.05 2.76	2.00 2.56	1.95 2.58	1.89 2.47	1.84 2.38	1.80 2.30	1.74 2.21	1.71 2.15	1.67 2.08	1.64 2.04	1.61 1.98	1.59 1.94	1.57 1.91	
36	4.11 7.39	3.26 5.25	2.86 4.38	2.63 3.89	2.48 3.58	2.36 3.35	2.28 3.18	2.21 3.04	2.15 2.94	2.10 2.86	2.06 2.78	2.03 2.72	1.98 2.62	1.93 2.54	1.89 2.43	1.87 2.35	1.82 2.26	1.78 2.17	1.72 2.12	1.69 2.04	1.65 2.00	1.62 1.94	1.59 1.90	1.56 1.87	
38	4.10 7.35	3.25 5.21	2.85 4.34	2.62 3.86	2.46 3.54	2.35 3.32	2.26 3.15	2.19 3.02	2.14 2.91	2.09 2.82	2.05 2.75	2.02 2.69	1.96 2.59	1.92 2.51	1.85 2.40	1.80 2.32	1.76 2.22	1.71 2.14	1.67 2.08	1.63 2.00	1.60 1.97	1.57 1.90	1.54 1.86	1.53 1.84	
40	4.08 7.31	3.23 5.18	2.84 4.31	2.61 3.83	2.45 3.51	2.34 3.29	2.25 3.12	2.18 2.99	2.12 2.88	2.07 2.80	2.04 2.73	2.00 2.66	1.95 2.56	1.90 2.49	1.84 2.37	1.79 2.29	1.74 2.20	1.69 2.11	1.66 2.05	1.61 1.97	1.59 1.94	1.55 1.88	1.53 1.81		
42	4.07 7.27	3.22 5.15	2.83 4.29	2.59 3.80	2.44 3.49	2.32 3.26	2.24 3.10	2.17 2.96	2.11 2.86	2.06 2.77	2.02 2.70	1.99 2.64	1.94 2.54	1.89 2.46	1.82 2.35	1.78 2.26	1.73 2.08	1.68 2.02	1.64 1.94	1.60 1.91	1.57 1.85	1.51 1.78			
44	4.06 7.24	3.21 5.12	2.82 4.26	2.58 3.78	2.43 3.46	2.31 3.24	2.23 3.07	2.16 2.94	2.10 2.84	2.05 2.75	2.01 2.68	1.98 2.62	1.92 2.52	1.88 2.44	1.81 2.32	1.76 2.24	1.72 2.15	1.66 2.06	1.63 2.00	1.58 1.92	1.56 1.88	1.52 1.78	1.48 1.75		
46	4.05 7.21	3.20 5.10	2.81 4.24	2.57 3.76	2.42 3.44	2.30 3.22	2.22 3.05	2.14 2.92	2.09 2.82	2.04 2.73	2.00 2.66	1.97 2.60	1.91 2.50	1.87 2.42	1.80 2.30	1.75 2.22	1.71 2.13	1.65 2.04	1.62 1.98	1.57 1.90	1.54 1.86	1.48 1.76	1.46 1.72		
48	4.04 7.19	3.19 5.08	2.80 4.22	2.56 3.74	2.41 3.42	2.30 3.20	2.21 3.04	2.14 2.90	2.08 2.80	2.03 2.71	1.99 2.64	1.96 2.58	1.90 2.48	1.86 2.40	1.79 2.28	1.74 2.20	1.70 2.11	1.64 2.02	1.61 1.96	1.56 1.88	1.53 1.84	1.50 1.73	1.47 1.70		

50	4.03 7.17	3.18 5.06	2.79 4.20	2.56 3.72	2.40 3.41	2.29 3.18	2.20 3.02	2.13 2.88	2.07 2.78	2.02 2.70	1.98 2.62	1.95 2.56	1.90 2.46	1.85 2.39	1.78 2.26	1.74 2.18	1.69 2.10	1.63 2.00	1.60 1.94	1.55 1.86	1.52 1.82	1.48 1.76	1.46 1.71	1.44 1.68
55	4.02 7.12	3.17 5.01	2.78 4.16	2.54 3.68	2.38 3.37	2.27 3.18	2.18 2.98	2.11 2.85	2.05 2.75	2.00 2.66	1.97 2.59	1.93 2.53	1.88 2.43	1.83 2.35	1.76 2.23	1.72 2.15	1.67 2.06	1.61 1.96	1.58 1.90	1.52 1.82	1.50 1.82	1.46 1.71	1.43 1.64	
60	4.00 7.08	3.15 4.98	2.76 4.13	2.52 3.65	2.37 3.34	2.25 3.12	2.17 2.92	2.10 2.82	2.04 2.72	1.99 2.63	1.95 2.56	1.92 2.50	1.86 2.40	1.81 2.32	1.75 2.20	1.70 2.12	1.65 2.03	1.59 1.93	1.56 1.87	1.50 1.79	1.48 1.68	1.44 1.63	1.39 1.60	
65	3.99 7.04	3.14 4.95	2.75 4.10	2.51 3.62	2.36 3.31	2.24 3.09	2.15 2.93	2.08 2.70	2.02 2.61	1.98 2.54	1.94 2.47	1.90 2.37	1.85 2.30	1.78 2.18	1.73 2.09	1.68 2.00	1.63 1.90	1.57 1.84	1.54 1.76	1.49 1.64	1.46 1.64	1.39 1.56		
70	3.98 7.01	3.13 4.92	2.74 4.08	2.50 3.60	2.35 3.29	2.32 3.07	2.14 2.91	2.07 2.77	2.01 2.67	1.97 2.59	1.93 2.51	1.89 2.45	1.84 2.35	1.79 2.28	1.72 2.15	1.67 2.07	1.62 1.98	1.56 1.88	1.53 1.82	1.47 1.74	1.45 1.69	1.40 1.63	1.35 1.53	
80	3.96 6.96	3.11 4.88	2.72 4.04	2.48 3.56	2.33 3.25	2.21 3.04	2.12 2.87	2.05 2.74	1.99 2.64	1.95 2.55	1.91 2.48	1.88 2.41	1.82 2.32	1.77 2.24	1.70 2.11	1.65 2.03	1.60 1.94	1.54 1.84	1.45 1.76	1.42 1.64	1.38 1.57	1.32 1.52		
100	3.94 6.90	3.09 4.82	2.70 3.98	2.46 3.51	2.30 3.20	2.19 2.99	2.10 2.82	2.03 2.69	1.97 2.59	1.92 2.51	1.88 2.43	1.85 2.36	1.79 2.26	1.75 2.19	1.68 2.06	1.63 1.98	1.57 1.89	1.51 1.79	1.48 1.73	1.42 1.54	1.39 1.51	1.35 1.43		
125	3.92 6.84	3.07 4.78	2.68 3.94	2.44 3.47	2.29 3.17	2.17 2.95	2.08 2.79	2.01 2.65	1.95 2.56	1.86 2.47	1.83 2.40	1.77 2.33	1.72 2.23	1.65 2.03	1.60 1.94	1.55 1.85	1.49 1.75	1.45 1.68	1.39 1.59	1.36 1.54	1.36 1.46	1.27 1.37		
150	3.91 6.81	3.06 4.75	2.67 3.91	2.43 3.44	2.27 3.13	2.16 2.92	2.07 2.76	2.00 2.62	1.94 2.53	1.89 2.44	1.85 2.37	1.82 2.30	1.76 2.20	1.71 2.12	1.64 2.00	1.59 1.91	1.54 1.83	1.47 1.72	1.44 1.66	1.37 1.56	1.34 1.51	1.25 1.33		
200	3.89 6.76	3.04 4.71	2.65 3.88	2.41 3.41	2.26 3.11	2.14 2.90	2.05 2.73	1.98 2.60	1.92 2.50	1.87 2.41	1.83 2.34	1.78 2.28	1.74 2.17	1.69 2.09	1.62 1.97	1.57 1.88	1.51 1.79	1.45 1.62	1.45 1.62	1.35 1.53	1.32 1.33			
400	3.86 6.70	3.02 4.66	2.62 3.83	2.39 3.36	2.23 3.06	2.12 2.85	2.03 2.69	1.96 2.55	1.90 2.46	1.85 2.37	1.81 2.29	1.76 2.23	1.72 2.12	1.67 2.04	1.60 1.92	1.54 1.84	1.49 1.74	1.42 1.64	1.38 1.57	1.32 1.47	1.28 1.42	1.16 1.19		
1000	3.85 6.66	3.00 4.62	2.61 3.80	2.38 3.34	2.22 3.04	2.10 2.82	2.02 2.66	1.95 2.53	1.89 2.43	1.84 2.34	1.80 2.26	1.76 2.20	1.70 2.09	1.65 2.01	1.58 1.89	1.53 1.81	1.47 1.71	1.41 1.61	1.36 1.54	1.30 1.44	1.26 1.38	1.19 1.28	1.13 1.19	
	3.84 6.64	2.99 4.60	2.60 3.78	2.37 3.32	2.21 3.02	2.09 2.80	2.01 2.64	1.94 2.51	1.88 2.41	1.83 2.32	1.79 2.24	1.75 2.18	1.69 2.07	1.64 1.99	1.57 1.87	1.52 1.79	1.46 1.69	1.40 1.59	1.35 1.52	1.28 1.41	1.24 1.36	1.17 1.25	1.11 1.00	