Inyuvesi yaseSwatini

University of eSwatini

Final Examination: October, 2021

Faculty of Science & Engineering

Department of Physics

Course Name : $\langle Q | uantum\ Computin | g \rangle$

Course Code : $|PHY606\rangle$

Allocated Time : Three (3) Hours

Instructions:

- This examination has four (4) questions. Answer ALL questions.
- Points for different sections are indicated on the right-hand margin.
- The total number of marks is 100.

This paper has six (6) pages, including this page!

! Do not open this page until permission has been given by the invigilator !

Question 1

Consider an unstructured system consisting of $N = 2^3$ elements. In this case we want to search for the element x_0 denoted by the index 5. Fully work out Grover's search algorithm for x_0 in this case.

[30 marks]

Question 2

Consider the tri-qubit state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

Suppose all three qubits of this state are measured in the $|\pm\rangle$ basis.

(a) (i) What are the possible outcomes of the measurement? With what probabilities do they occur?

[6 marks]

(ii) Suppose the first qubit is measured in the $|\pm\rangle$ basis while the second and third qubits are measured in the $|\pm i\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$ basis. What are the possible outcomes of these measurements? With what probabilities do they occur?

[8 marks]

(iii) Now, consider the quantum state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle,$$

what would be the possible outcomes of measuring this state, with what probabilities?

[4 marks]

(iv) Considering the state in (iii) above, could you think of a computational basis (other than the standard computational basis of course!) that could give us access to some information about the phase θ? Demonstrate that perfoming a measurement in your proposed basis does open a window to accessing some information about the phase, θ.

[6 marks]

Question 3

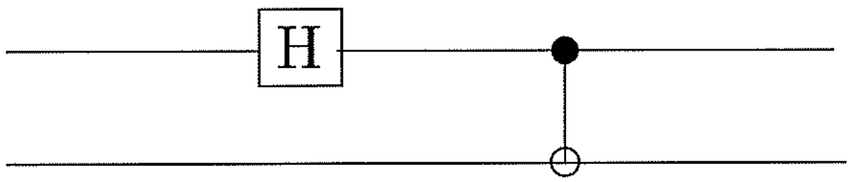

Figure 1:

(a) Consider the circuit in figure (1). Let the input to both "wires" be $|0\rangle$ and $|0\rangle$, i.e, $|0\rangle \otimes |0\rangle$.

(i) Work out the output of this system past the gate denoted by **H**, i.e, what is the state vector of the system after the first gate?

[3.5 marks]

(ii) Work out the output of this system past the second gate, i.e, what is the state vector of the system after the second gate?

[3.5 marks]

(iii) Can you recognize the state you have "created" from starting with just $|0\rangle \otimes |0\rangle$?

[1 mark]

(b) The *popular* state you generated in (a) above has three (3) sisters or brothers or cousins. Propose circuits to generate the other three (3) sister or cousin states and in each case, demonstrate that with your proposed input state vector, you do create the desired output state.

[15 marks]

Question 4

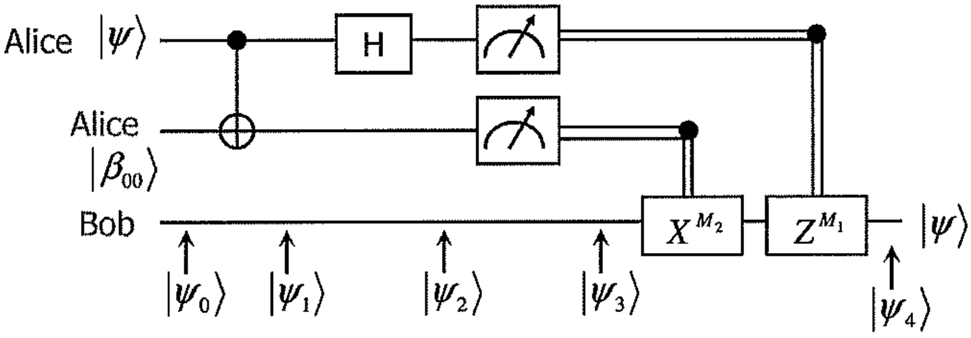

Figure 2:

Consider the most popular hypothetical scientific couple in the North-American context, Alice and Robert. Suppose Alice and Bob share the entangled state (or rather two qubits entangled in the state):

$$\beta_{00} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

At the same time, Alice has her own independent state in a separate qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

Alice wants to send her state, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, to Bob via a teleportation channel. A teleportation circuit, see figure 2, has been proposed. After the teleportation process, Alice will perform a measurement and inform Bob of her results. Based on Alice's measurement, Bob will know how to extract the state sent by Alice. What operation(s), in terms of applying some quantum gates, does Bob have to perform if Alice measures:

(i) $|00\rangle$

(ii) $|10\rangle$

(iii) $|01\rangle$

(iv) $|11\rangle$

[23 marks]

Additional Information

Commonly used operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The *plus-minus* computational basis:

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$$

!!!!!!!!THIS IS THE END OF THE EXAMINATION PAPER!!!!!!!
