UNIVERSITY OF ESWATINI
INYUVESI YASESWATINI

RE-SIT EXAMINATION: JULY, 2021

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF PHYSICS

COURSE NAME : $\langle|NUCLEAR\ PHYSICS|\rangle$

COURSE CODE : $|PHY441\rangle$

ALLOCATED TIME : THREE (3) HOURS

INSTRUCTIONS:

- THIS EXAMINATION HAS FIVE (5) QUESTIONS. ANSWER ANY FOUR (4) QUESTIONS. EACH QUESTION IS WORTH 25 MRAKS
- POINTS FOR DIFFERENT SECTIONS ARE INDICATED ON THE RIGHT-HAND MARGIN
- THE TOTAL NUMBER OF MARKS IS 100.

THE PAPER HAS TEN (10) PAGES, INCLUDING THIS PAGE.

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Question 1

(a) In the nuclear shell model, orbitals are filled in the order:

$$1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, \ldots$$

What is responsible for the splitting between the $p_{3/2}$ and $p_{1/2}$ orbitals?

[3 marks]

(b) What spin-parity and isospin would the shell model predict for the ground states of ${}^{13}_{5}B$, ${}^{13}_{6}C$, and ${}^{13}_{7}N$?

[12 marks]

(c) Order the above isobaric triad according to mass with the lowest mass first. Provide physical arguments to justify your ordering.

[5 marks]

(d) Indicate how you could estimate rather closely the energy difference between the two (2) lowest-mass members of the above triad. What is the mass difference?

[5 marks]

Question 2

(a) Consider the principle of operation of a mass spectrometer:

(i) The velocity selector consists of perpendicular electric (**E**) and magnetic fields ($\mathbf{B}_1$). In terms of the magnitudes of **E** and $\mathbf{B}_1$, find an expression for the velocity of ions that will pass through the velocity selector undeflected.

[4 marks]

(ii) After the velocity selector, ions enter the momentum selector, a segment characterized by a uniform magnetic field. What do you expect to happen to ions (charged particles) passing through a magnetic field?

[2 marks]

(iii) Assuming that the magnetic fields of the velocity selector and the momentum selector have the same magnitude, find, in terms of **E**, and **B**, an expression for the charge-to-mass ratio of ions bent into a circular path of radius r.

[7 marks]

(b) Singly charged chlorine ions are accelerated through a fixed potential difference and then caused to travel in circular paths by means of a uniform field of magnetic induction of 1000 Gauss. What increase in induction is necessary to cause the mass 37 ion to follow the path previously taken by the mass 35 ion?

[12 marks]

Question 3

(a) The nuclear shell model takes leaf from the atomic/spectroscopic model where electrons tend to occupy distinct shells in a Coulombic potential field established by the nucleus. This arrangement, in atoms, leads to the formation of an inert core of filled shells, some valence electrons, and the possibility of an unpaired electron state. Atomic properties are then determined by such valence electrons. In the nuclear shell model, the nucleons generate the same potential they have to arrange themselves in but the model draws so much inspiration from atomic spectroscopy!

(i) Give atleast four (4) pieces of evidence that support a shell structure arrangement of nucleons in the nucleus.

[4 marks]

(ii) What are the so called nuclear *magic numbers* and what is so unique about them? Give the full list of the nuclear magic numbers.

[6 marks]

(iii) Starting from the *Schrödinger* equation for a proton, derive expressions for the energy eigenvalues for the s states. The requirements on the wavefunction is that the radial component should be finite at $r = 0$ and vanish at the boundary ($r = R$) of the nucleus. Useful hint: In spherical coordinates, the Laplacian is expressed as;

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$$

[10 marks]

(iv) Introduction of spin-orbit coupling to the nucleon potential introduces the dependence of the energy eigenvalues on the relative alignment of nucleon spin and orbital angular momentum. What is the effect of the spin-orbit coupling to the $4l + 2$ degenerate energy level $E(n, l)$?

[2 marks]

(v) What are the possible total angular momentum (j) values for f-states?

[3 marks]

Question 4

(a) Based on just binding energy arguments, mathematically argue that the electron cannot exist inside the nucleus. As an example, consider an element with $A = 124$ and $Z = A/2$.

[5 marks]

(b) Discuss what physical quantities are measured in the following experiments and how these quantities are related to the radius of the nucleus:

(i) Elastic electron scattering

[10 marks]

(ii) Ground state energies of the isotopic spin multiplet

[10 marks]

Question 5

(a) Given that the normalized wave function

$$\psi(r) = \frac{1}{r}\left(\frac{\alpha}{2\pi}\right)^{\frac{1}{2}} e^{-\alpha r}$$

with $\frac{1}{\alpha} = 4.3$ fm, is a useful approximation to describe the ground state of the deuteron, find the root mean square separation of the neutron and the proton in the deuteron nucleus. You may find the following integration recipe useful:

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

[8 marks]

(b) A narrow beam of singly charged ^{10}B and ^{11}B ions of energy 5 keV pass through a slit of width 1 mm into a uniform magnetic field of 1500 Gauss and after a deviation of 180^0 the ions are recorded on a photographic plate.

(i) What is the spatial separation of the images?

[10 marks]

(ii) What is the mass resolution ($\delta = \frac{M}{\Delta M}$) of the system?

[7 marks]

!THIS IS THE END OF THE QUESTION PART OF THE EXAMINATION PAPER, SOME USEFUL INFORMATION AND DATA FOLLOWS ON THE NEXT THREE (3) PAGES!

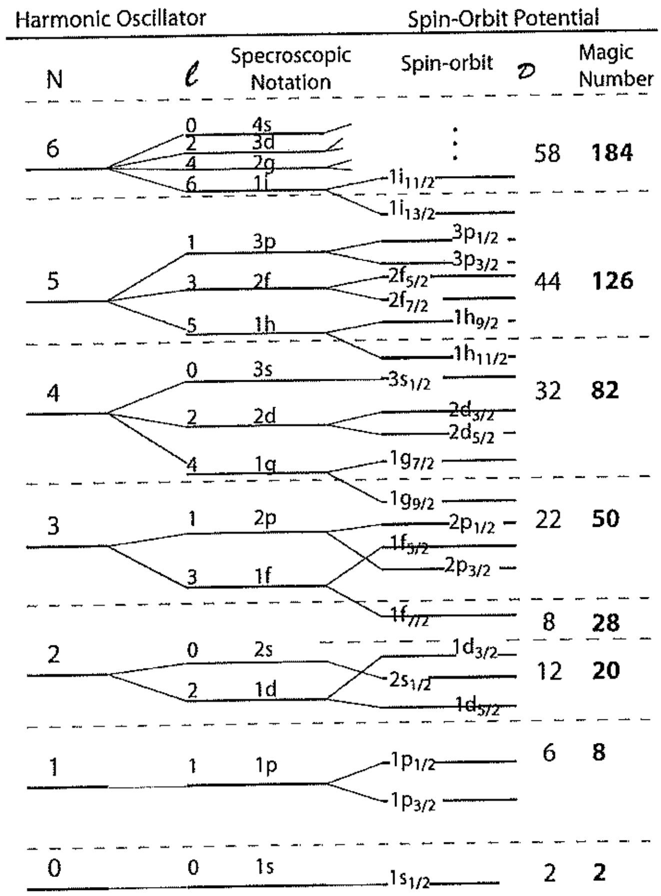

Figure 1: The shell model prediction of the magic numbers.

Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27}$ kg $= 931.5$ MeV$/c^2$

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg $= 1.008665$ u $= 939.573$ MeV$/c^2$

proton mass $m_p = 1.6726 \times 10^{-27}$ kg $= 1.0072765$ u $= 938.280$ MeV$/c^2$

1year $= 3.156 \times 10^7$ s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%