UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2019

TITLE OF PAPER: STATISTICAL PHYSICS AND THERMODYNAMICS

COURSE NUMBER: PHY461

TIME ALLOWED: THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE

(a) The volume of momentum space from p to $p + dp$ is a spherical shell in 3-D momentum space as shown in Figure 1. The range p to $p + dp$ represents a spherical shell in momentum space of inner radius p and thickness dp with momentum-space volume $4\pi p^2 dp$.

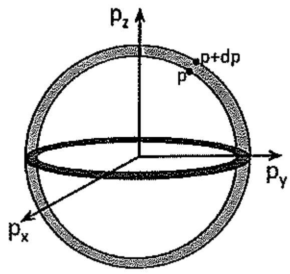

Figure 1

Derive the expression for the volume element in terms of the energy ε. (6 marks)

(b) Briefly explain the following terms associated with a system of particles:

(i) **Macrostate** (2 marks)

(ii) **Microstate** (2 marks)

(iii) **Thermodynamic probability** (2 marks)

(c) Consider a system of 5 classical particles arranged in three energy levels, each split into two compartments.

(i) Write down all the possible macrostates. (2 marks)

(ii) Find the number of microstates corresponding to each of the above macrostates using the formula:

$$W = N! \prod_s \left(\frac{g_s^{n_s}}{n_s!} \right)$$

(5 marks)

(iii) Which macrostate corresponds to an equilibrium state? Support your answer.

(2 marks)

(d) Briefly explain these fundamental postulates of statistical mechanics in terms of macrostates or microstates:

(i) **Ergodicity** (2 marks)

(ii) **Equiprobability** (2 marks)

QUESTION THREE

(a) Briefly explain the following terms:

(i) **Entropy** (2 marks)

(ii) **Gibb's paradox** (3 marks)

(b) Given that the entropy of a system of particles is given by

$$S = k \ \text{In}W$$

where

$$W = N! \prod_s \left(\frac{g_s^{n_s}}{n_s!} \right)$$

Derive the expression for entropy in terms of the partition function Z.

(9 marks)

(c) Given the partition function for a classical perfect gas:

$$Z = \frac{V}{h^3}(2\pi mkT)^{\frac{3}{2}}$$

Show that the ideal gas equation is given by:

$$PV = NkT$$

(4 marks)

(d) Two equal volumes of the same gas, each having entropy 'S' and at the same temperature and pressure, are mixed together.

(i) Compute the total entropy of the mixture and comment on your results. (5 marks)

(ii) Briefly discuss the Sackur-Tetrode formula with reference to your answer in (d)(i) (2 marks)

QUESTION FOUR

(a) Briefly explain the difference between bosons and fermions. (2 marks)

(b) The expression that gives the distribution of bosons over various energy levels for the most probable configuration is:

$$n_s = \frac{g_s}{\frac{1}{A}e^{\frac{\varepsilon_s}{kT}} - 1} \tag{4.1}$$

(i) Derive the expression given by equation 4.1, given that the weight of a system of bosons is given by:

$$W = \prod_s \frac{(g_s + n_s - 1)!}{(g_s - 1)!\, n_s!}$$

(10 marks)

(ii) Recall that:

$$A = \frac{Nh^3}{(2\pi mkT)^{\frac{3}{2}} V}$$

Evaluate A at room temperature for helium gas (^4_2He), which is made up of bosons.

(6 marks)

(iii) Show that the Bose-Einstein distribution function of the form 4.1 reduces to the Maxwell-Boltzmann function using your answer in (b)(ii).

(4 marks)

(c) Give a detailed description of the **Bose-Einstein condensation** phenomenon.

(3 marks)

QUESTION FIVE

(a) The quantum statistical expression that tells us how energy varies with wavelength for a black body is given by:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1}$$

(i) Sketch a graph of $E(\lambda)$ versus λ for three temperatures, such that $T_1 > T_2 > T_3$, according to the expression above.

(4 marks)

(ii) State three important characteristics of the plots in (a)(i).

(3 marks)

(b) Derive the Fermi-Dirac distribution function for a system of fermions given by:

$$n_s = \frac{g_s}{e^{-(\alpha+\beta\varepsilon_s)} + 1}$$

(10 marks)

(c) Given that the density of states for a system of fermions is given by:

$$g(\varepsilon)\, d\varepsilon = \frac{4\pi}{h^3}\, V\, (2m)^{\frac{3}{2}}\, \varepsilon^{\frac{1}{2}}\, d\varepsilon$$

Show that the Fermi energy of such a system is given by:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}$$

(8 marks)

APPENDIX 2

Table of physical constants

QUANTITY	SYMBOL	VALUE
Speed of light	c	3.00×10^{8} m.s^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} J.K^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J.mol^{-1}.K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} J.T^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H.m^{-1}
Stefan constant	σ	5.67×10^{-8} W.m^{-2}.K^{-4}
Atmospheric pressure		1.01×10^{5} N.m^{-2}
Mass of $^{4}_{2}$He atom		6.65×10^{-27} kg
Mass of $^{3}_{2}$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.41 mol^{-1}