UNIVERSITY OF ESWATINI FACULTY OF SCIENCE & ENGINEERING DEPARTMENT OF PHYSICS SEMESTER I, MAIN EXAMINATION: 2019/2020

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE CODE: PHY441/P442

TOTAL TIME ALLOWED: THREE(3) HOURS

INSTRUCTIONS:

- THIS EXAMINATION HAS FIVE (5) QUESTIONS. ANSWER ANY FOUR (4).
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE INDICATED IN THE RIGHT-HAND MARGIN.

THE PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Question 1

(a) Atomic electrons are confined by the Coulombic field generated by the nucleus. On the other hand nucleons are under the influence of both the Coulombic and strong forces. Both these forces set up a potential in which particles can "reside" in certain allowed energy states. Sketch the interaction potential for an atomic electron, a bound proton and bound neutron. Use the same diagram for the proton and neutron potentials, then a separate diagram for the electron potential.

[7 marks]

(b) The proton experiences a lowered potential compared to the neutron. Explain why.

[3 marks]

(c) The proton potential also has a different feature compared to that of the neutron, the Coulomb barrier potential. What is the significance of this feature?

[2 marks]

(d) The radial wavefunction for the hydrogenic electron in the 1s orbital is:

$$\psi_{10} = 2\sqrt{\left(\frac{Z^3}{a_0^3}\right)}e^{\frac{-Zr}{a_0}}$$

Calculate the mean radius of the hydrogen 1s orbital.

Hint:

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

[6 marks]

(e) Calculate the most probable radius (r) for an electron occupying the 1s orbital of a hydrogen atom.

[7 marks]

Question 2

- (a) Consider the principle of operation of a mass spectrometer:
 - (i) The velocity selector consists of perpendicular electric (E) and magnetic fields (B). In terms of the magnitudes of E and B, find an expression for the velocity of ions that will pass through the velocity selector undeflected.

[6 marks]

(ii) After the velocity selector, ions enter the momentum selector, a segment characterized by a uniform magnetic field. What do you expect to happen to ions (charged particles) passing through a magnetic field?

[2 marks]

(iii) Assuming that the magnetic fields of the velocity selector and the momentum selector have the same magnitudes, show that the charge-to-mass ratio of ions bent into a circular path of radius r is given by:

$$\frac{q}{m} = \frac{E}{rB^2}$$

[6 marks]

(b) Singly-charged lithium ions are liberated from a heated anode and accelerated by using an electric field generated by a potential difference of 400 V between the anode and cathode. They then pass through a uniform magnetic field perpendicular to their direction of motion. The magnetic flux density is 8 x 10^{-2} Wb/m² and the radii of the paths of the ions are 8.83 and 9.54 cm, respectively. Calculate the mass numbers of the lithium ions (isotopes).

[11 marks]

Question 3

(a) Assume that a given radioactive nuclide has a decay constant λ_1 and it decays into a daughter nuclide which is itself radioactive with decay constant λ_2 . Furthermore, assume that at time t=0 there are N_1 parent nuclei and $N_2=0$ daughter nuclei. Show that at any time t>0 the number of daughter nuclei is:

$$N_2 = \frac{N_1 \lambda_1}{\lambda_1 - \lambda_2} \left(1 - e^{-(\lambda_1 - \lambda_2)t} \right) e^{-\lambda_2 t}.$$

[15 marks]

- (b) Considering the radioactive decay problem above, investigate the limits:
 - (i) $\lambda_2 \gg \lambda_1$

[5 marks]

(ii) $t \to \infty$ for the case $\lambda_2 \gg \lambda_1$.

[5 marks]

Question 4

(a) The masses of ⁶⁴₂₈Ni, ⁶⁴₂₉Cu and ⁶⁴₃₀Zn are: 63.927959 u, 63.929759 u and 63.929145 u, respectively. It follows that ⁶⁴₂₉Cu is radioactive. Detail the various possible decay modes to the ground state of the ⁶⁴₂₉Cu nucleus and give the maximum energy of each component, ignoring the recoil energy. Compute the recoil energy in one of the cases and verify that it was justifiable to ignore it.

[12 marks]

(i) Consider the alpha particle decay $^{230}_{90}Th \rightarrow ^{226}_{88}Ra + \alpha$ and use the following expression to calculate the values of the binding energy B for the two heavy nuclei involved in this process.

$$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{z(z-1)}{A^{13}} - a_a \frac{(N-Z)^2}{A} - a_p A^{\frac{-3}{4}},$$

where values for the constants a_v , a_s , a_c , a_a and a_p are respectively 15.5, 16.8, 0.72, 23.0 and 34.5 MeV. Given that the total binding energy of the alpha particle is 28.3 MeV, find the energy Q released in the decay.

[6 marks]

(ii) The above energy appears as the kinetic energy of the products of the decay. If the original thorium nucleus was at rest, use conservation of momentum and conservation of energy to find the kinetic energy of the daughter nucleus ^{226}Ra .

[4 marks]

(b) Estimate the energy needed to remove one neutron from the nucleus $^{40}_{20}$ Ca.

[3 marks]

Question 5

(a) Using the following relativistic relationship between energy and momentum for the electron wavefunction:

$$E^2 - p^2 c^2 = m^2 c^4.$$

The Dirac equation allows the existence of both negative-energy and positive-energy particles. The negative-energy particles were later concluded to be anti-electrons (later named positrons). It turned out this was actually a universal feature of quantum field theory: every particle has an anti-particle with the same mass but opposite electric charge: electron (e⁻) - positron (e⁺), muon (μ^-) - antimuon (μ^+) , proton(p)-antiproton(\bar{p}), etc:

(i) Even though it is electrically neutral, the neutrino (ν) has the antineutrino $(\bar{\nu})$ for an antiparticle. How does the neutrino differ from the antineutrino? Use a diagram to demonstrate your answer.

[4 marks]

(ii) Some particles are their own anti-particles. Name one such example.

[2 marks]

(b) In explaining the photo-electric effect, Einstein came up with the following relation for the maximum energy an electron can emerge with:

$$E \le h\nu - \Phi,$$

where E is the electron energy.

(i) What was (or rather is) the significance of this explanation by Einstein, particulary about the nature of electromagnetic radiation?

[2 marks]

- (ii) Basing your arguments on the inequality above, how and why would both the number and the energy of the produced electrons be affected if the intensity of the incident radiation would be increased at a fixed frequency? [2 marks]
- (iii) What if we used incident radiation with an increased intensity and a higher frequency, what would happen to the number of produced electrons and their energy? Explain your answer.

[2 marks]

- (c) At the most fundamental level, matter is made out of 3 kinds of elementary particles. Two of these are Fermions and the third class is a set of Bosons.
 - (i) Name all the 3 kinds of particles and give an example for each.

[3 marks]

(ii) For any two examples (of particles) named above, associate them with the force(s) they experience and explain why? For the forces they do not experience, also explain why?

[4 marks]

(d) Feynman diagrams are an essential particle physics language in the sense that they give a pictorial depiction of the interactions underlying each physical process. Draw a fully labelled first order diagram for electron-electron scattering $(e^- + e^- \rightarrow e^- + e^-)$, i.e, Moller scattering.

[3 marks]

(e) Even though the neutron is an electrically neutral particle, it does have a magnetic moment. What does the existence of a neutron magnetic moment mean about the sub-structure of the neutron?

[3 marks]