UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

RE-SIT /SUPPLEMENTARY EXAMINATION 2018/2019

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: PHY461/P461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

APPENDICES 1 AND 2 CONTAIN DEFINITE INTEGRALS AND PHYSICAL CONSTANTS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) State what is meant by the following terms for a system of particles:

(i) Macrostate. (2 marks)
(ii) Microstate. (2 marks)
(iii) Statistical weight. (2 marks)

(b) A classical system of 5 distinguishable particles are arranged in 3 energy levels.

(i) What are the possible macrostates of the system? (2 marks)

(ii) Use appropriate equations to find the weights of each macrostate and hence the most probable distribution of these particles for the following cases.

1. The energy levels are non-degenerate. (4 marks)
2. The energy levels are degenerate. (4 marks)

(c) (i) Derive expressions for the volume element in phase space in terms of
1. momentum, p and (3 marks)
2. energy, ϵ (4 marks)

(ii) Show that the density of states of a system can be expressed as

$$g(\varepsilon)d\varepsilon = \frac{2\pi}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon.$$

where the symbols have their usual meanings. (2 marks)

Question Two

(a) (i) The partition function Z of a system is defined as $Z = \sum_{s} g_s e^{\beta\varepsilon_s}$ where the symbols have their usual meanings. Show that the partition function of a classical perfect gas,

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

[See appendix for definite integrals] (5 marks)

(ii) Derive the following expressions for the entropy and the total energy of a classical system:

1. $S = Nk\ln Z + \frac{E}{T}$ (5 marks)

2. $E = NkT^2 \frac{\partial}{\partial T}\ln Z$ (4 marks)

(iii) Use the above result to show that the entropy of a classical gas:

$$S = Nk\ln\left[\frac{V}{h^3}(2\pi mkT)^{3/2}\right] + \frac{3}{2}Nk.$$ (5 marks)

(b) Calculate the translational partition function of a hydrogen molecule confined to a volume of 100 cm^3 at 300 K. (6 marks)

[

Question Three

(a) (i) For the most probable configuration, the distribution of classical particles in a system at temperature T can be expressed as

$$n_S = g_S e^{\alpha + \beta \epsilon_s}$$

where the symbols have their usual meanings. Show that the multipliers in the above expression

$$\beta = -\left(\frac{1}{kT}\right)$$ β = - 1/kT (5 marks)

$$\alpha = \ln\left[\frac{Nh^3}{(2\pi mkT)^{3/2} V}\right]$$ (5 marks)

(ii) Find the average energy for the most probable configuration of a classical system with two non-degenerate energy levels having energy ϵ in level 1 and 2ϵ in level 2.

(5 marks)

(b) (i) State the *law of equipartition of energy.* (4 marks)
(ii) assuming that the energy of the particles is purely kinetic, verify the validity of this principle. (6 marks)

Question Four

(a) Explain how classical physics fails to deal with the specific heat of solids. (3 marks)

(b) Set up the partition function of a harmonic oscillator and obtain its mean energy. (7 marks)

(c) (i) State the basic assumptions Einstein made in developing his theory of specific heat of solids (3 marks)

(ii) Obtain an expression for the specific heat of an insulator according to Einstein. (6 marks)

(iii) Does the above theory agree well with experiment at all temperatures? Discuss. (Obtain appropriate expressions to justify your argument). (6 marks)

Question Five

(a) Show that for the most probable configuration the distribution of a system of fermions at temperature T can be represented as:

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} + 1}$$

where the symbols have their usual meanings.

Given: the weight of a system of fermions $W = \prod_S \frac{g_S!}{n_S!(g_S - n_S)!}$

(12 marks)

(b) (i) Given the density of states for a system of fermions,

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

show that the Fermi energy can be expressed as

$$\varepsilon_F = \frac{h^2}{2m}\left[\frac{3N}{8\pi V}\right]^{2/3}$$

(5 marks)

(ii) Show that the average kinetic energy of a particle a fermi gas at 0 K is (3/5) times the Fermi energy. (5 marks)

(c) The Fermi energy of a solid is 8.6 eV. Find the probability of occupation of an electron in an energy level 0.1 eV above the Fermi level at 300 K and at 400 K.

Comment on the result. (2 + 1 marks)

Given: Fermi function $f(\varepsilon) = \frac{1}{e^{(\varepsilon-\varepsilon_F)/kT} + 1}$

Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\, x\, dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\, x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\, x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\, x^5\, dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\, e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\, dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} Js
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan- Boltzmann constant	σ	5.67×10^{-8} Wm^{-2} K^{-4}
Atmospheric pressure		1.01 10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}