UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2018/2019

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: PHY221/P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Fundamental Theorem of Gradients

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: ELECTROSTATICS

Consider the charge configuration in Fig. 1 and answer the following questions.

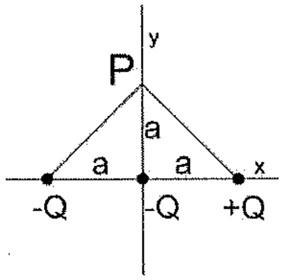

Figure 1: Three charges on a line.

(a) Use the definition of the electrostatic field of a point charge to determine the field at P. (6)

(b) Use the definition of the electric potential of a point charge to determine the electric potential at P. (5)

(c) How much energy would it take to assemble the charge configuration? (10)

(d) How much energy would it take to add a fourth charge $+3Q$ at P? (4)

Question 2: ELECTROSTATICS II..

(a) Consider the line charge configuration shown in Fig. 2. The rod carries a total charge Q that is uniformly distributed.

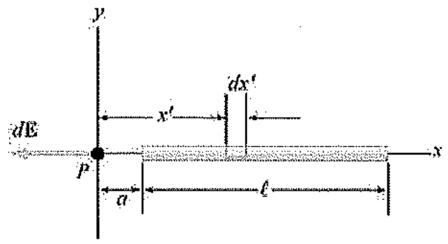

Figure 2: A rod of lenght l carries a uniformly distributed charge Q.

i. Calculate the field at point P. (10)

ii. What is the field in the limit the length of the rod l goes to zero? Is this the expected expression? (5)

(b) When dielectric materials are immersed in an electrostatic field **E**, they get polarized.

i. Describe the process of polarization for materials in which the constituents (i.e atoms and molecules) are not polarized. (5)

ii. Describe the process of polarization for materials in which the constituents (i.e atoms and molecules) are polarized. (5)

Question 3: Magnetostatics..

(a) For charge flowing through a volume, the current density $\mathbf{J}$ is defined as the current per unit area perpendicular to flow. Local charge conservation results in the continuity equation (10)

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0,$$

where ρ is the volume charge density. Use charge conservation and the appropriate definitions to derive the continuity equation.

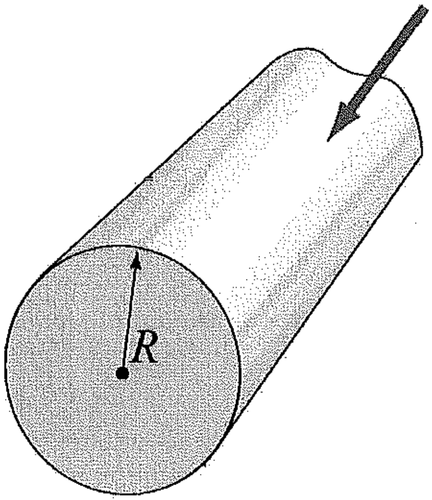

Figure 3: A cylindrical conductor whose current density varies inversely with the distance from its axis.

(b) Consider a cylindrical conductor, shown in Fig. 3 with radius R and nonuniform current density $\mathbf{J} = J_0 \frac{R}{r}$.

i. Determine the magnetic field for points $r < R$. (6)

ii. Determine the magnetic field for points $r > R$. (4)

(c) Show that for a point charge Q moving with a velocity $\mathbf{v}$ inside a magnetic field $\mathbf{B}$, the magnetic force on the charge does no work. (5)

Question 4: Magnetostatics II...

A conductor of lenghth l is in contact with two rails which are connected by a resistor on one end. The conductor is pulled across the rails at constant velocity $\mathbf{v}$ and there is a magnetic field directed into the page as shown in Fig. 4.

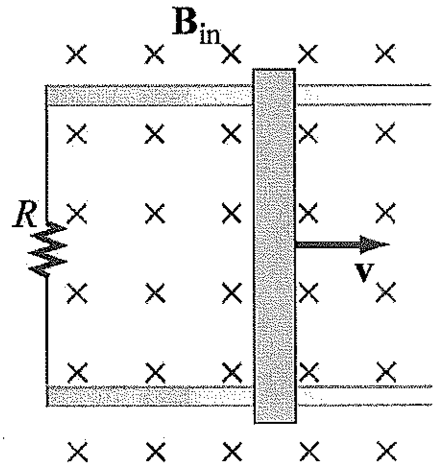

Figure 4: A conductor of length l pulled at constant velocity.

(a) Is the induced current flowing clockwise or anticlockwise? Show how you arrive at the direction. (4)

(b) A current carrying conductor inside a magnetic field experiences a force. In which direction is this force? (4)

(c) What is the magnitude of the induced emf? (8)

(d) What is the magnitude of the induced current? (2)

(e) From the definitions of work and power, show that power is given by $P = \mathbf{F} \cdot \mathbf{v}$. Where all symbols have their usual meaning. (4)

(f) What power must be externally supplied to move the conductor at a constant velocity $\mathbf{v}$? (3)

Question 5: Electrodynamics and Alternating Current Circuits ....

The circuit shown in Fig. 5 contains a generator which provides a sinusoidal varying emf $\epsilon(t) = \epsilon_0 \sin \omega t$, a resistor R, and a blackbox which contains either an inductor or a capacitor but not both. The amplitude of the driving emf is $\epsilon_0 = 100\sqrt{2}V$ and the angular frequency ω is 10 rad/sec. We measure the current in the circuit and find that it is given as a function of time by $I(t) = 10\sin(\omega t + \pi/4)$ A

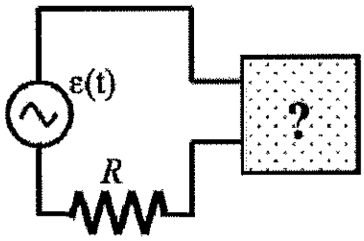

Figure 5: A circuit containing a generator, a resistor and an unknown component.

(a) Does this current lead or lag the voltage $\epsilon(t) = \epsilon_0 \sin \omega t$? (3)

(b) What is the unknown circuit element, a capacitor or an inductor? (3)

(c) What is the numerical value of the resistance R? Indicate units. (6)

(d) What is the numerical value of the capacitance or inductance in the black box? Indicate units. (6)

(e) What is the time averaged power dissipated in the circuit? (7)

Note: $\int_0^T \sin(\omega t - \phi)\sin(\omega t)dt = \frac{1}{2}\cos\phi$