UNIVESITY OF SWAZILAND
FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS

Main Examination 2017/2018
COURSE NAME: Electromagnetic Theory
COURSE CODE: PHY332/P331
TIME ALLOWED: 3 hours

ANSWER ANY FOUR QUESTIONS. ALL QUESTIONS CARRY EQUAL (**25**) MARKS

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The exam paper has ten (10) printed pages, including an appendix.

Question 1

(a) A thick wire of radius a carries a constant current I, uniformly distributed over its cros section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in figure 1. Find the magnetic field in the gap at a distance $s < a$ from the axis.

[8 marks]

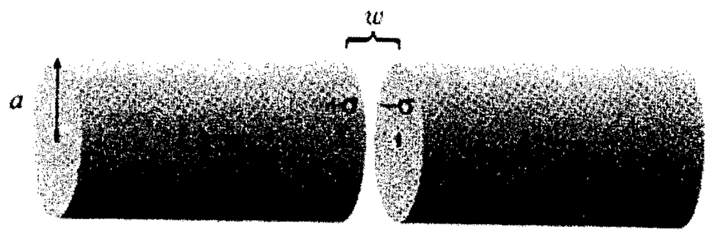

Figure 1:

(b) A solenoid, of radius 8 cm, is driven by an alternating current, so that the field inside is sinusoidal: $\vec{B}(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire, of radius 4 cm and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

[7 marks]

(c) A square loop of wire, with sides of length 2 cm, lies in the first quadrant of the xy-plane. with one corner at the origin. In this region there is a nonuniform time-dependant magnetic field $\vec{B}(y,t) = ky^3t^2\hat{z}$, where k is a constant. Find the emf induced in the loop.

[5 marks]

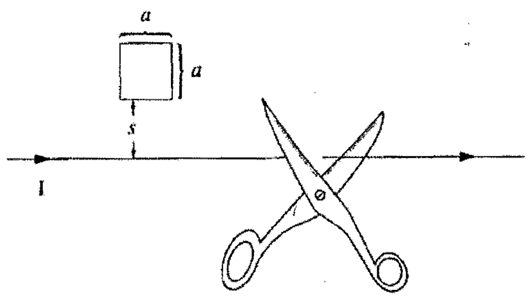

Figure 2:

(d) A square loop, of side $a = 4\ cm$ and $R = ln2\ \Omega$ lies a distance s from a infinite straight wire that carries current I, as shown in figure 2. If the wire is suddenly cut, so that $I \rightarrow 0$, what total charge passes a given point in the loop during the time this current flows?

[5 marks]

Question 2

(a) A conducting sphere of radius a is held at a constant potential V_0 with respect to infinity. At a distance a from its surface ($2a$ from its center) the potential is $\frac{V_0}{2}$. Calculate the potential at a distance $a/2$ from its surface ($1.5a$ from its center).[**6 marks**]

(b) Two point charges are placed in free space at a distance d from each other in free space. One charge is positive and equal to Q, the second is negative and equal to $-Q$ as shown in Figure 3 A. Point P is the center point between the two charges. Now a conductor is brought into the vicinity of the two charges so that the configuration is as in Figure 3 B.

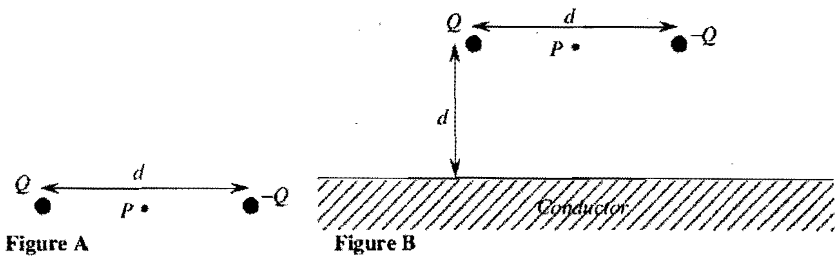

Figure 3:

(i) Find the change in the electric field intensity at point P caused by the conductor.

[9 marks]

(ii) Calculate the change in the potential at point P caused by the conductor.

[3 marks]

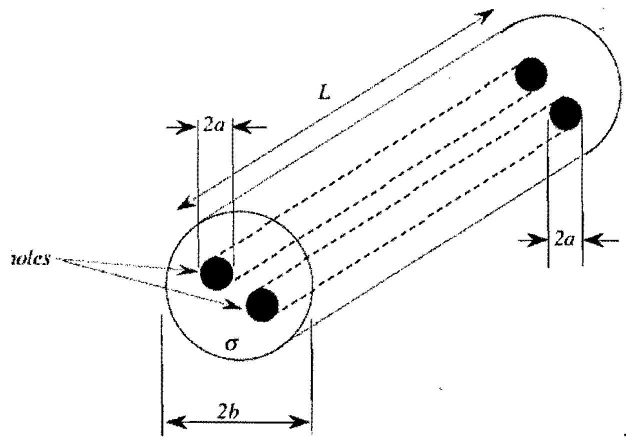

Figure 4:

(c) A solid conductor of radius b and length L has a known conductivity σ. Two holes are now drilled into the conductor, parallel to the axis of the conductor, each hole being of radius a ($a < b/2$) (see Figure 4). Calculate the change in resistance of the device due to the drilling of the holes. The resistance is calculated along the cylinder (i.e. as if the current flows along the cylinder).

[7 marks]

Question 3

(a) A hollow sphere of radius R has a potential $V_0(\theta)$, with azimuthal symmetry, specified on the surface. The solutions to Laplace's equation are

$$V(r,\theta) = \sum_{\ell=0}^{\infty} A_\ell r^\ell P_\ell(\cos\theta), \qquad r \leq R$$

$$V(r,\theta) = \sum_{\ell=0}^{\infty} B_\ell r^{-(\ell+1)} P_\ell(\cos\theta) \qquad r \geq R$$

where $P_\ell(\cos\theta)$ are the well-known Legendre polynomials.

(i). Use the Fourier trick to find expressions for A_ℓ and B_ℓ in terms of $V_0(\theta)$ and the Legendre polynomials.

[10 marks]

(ii). By assuming the potential is continuous on the surface of the sphere, express B_ℓ in terms of A_ℓ.

[5 marks]

(b). An ideal electric dipole lies at the origin with $\hat{\mathbf{p}} = p\hat{z}$. Find $\overrightarrow{E}$ in Cartesian coordinates.

[5 marks]

(c). From $\overrightarrow{E} = -\nabla\Phi$ find a vector expression for $\overrightarrow{E}$ produced by an electric dipole, in terms of $\hat{\mathbf{p}}$ and $\hat{r}$.

[5 marks]

Question 4

(a) Determine the electrostatic field intensity $\vec{E}$ at the point (*given in Cartesian coordinates*) $(1, 1, 0)$ for the following scalar electric potentials

(i) $\Phi = E_0 e^{-x} \sin\left(\frac{\pi y}{4}\right)$.

[3 marks]

(ii) $\Phi = E_0 R \cos(\theta)$.

[3 marks]

(b) A conical surface, with height h has a radius, also equal to the height. The uniform surface of the cone carries a uniform charge σ.

(i) Find the potential, $\Phi(\mathbf{a})$, at the vertex point **a**.

[5 marks]

(ii) Find the potential, $\Phi(\mathbf{b})$, at the centre top **b**.

[5 marks]

(i) Show that $\Phi(\mathbf{b}) - \Phi(\mathbf{a}) = -\frac{\sigma h}{2\epsilon_0}\left[1 - ln(1 + \sqrt{2})\right]$.

[3 marks]

(c) Two infinite grounded metal plates lie parallel to the xz-plane, one at $y = 0$, the other at $y = a$ as ahown in figure 5. The left end, at $x = 0$, is closed off with an infinite strip insulated from the two plates and maintained at a specific potential, $V_0(y)$.

(i) Use separation of method to find a general solution for the x- and y- components of the potential inside the two plates.

[3 marks]

(ii) After applying the boundary conditions, the general solution can be written as

$$V(x, y) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi x}{a}\right)} \sin\left(\frac{n\pi y}{a}\right). \qquad (1)$$

Employ Fourier's trick to find the coefficients C_n for $V_0(y) = V_0$. Write the expression for $V(x, y)$ for this potential.

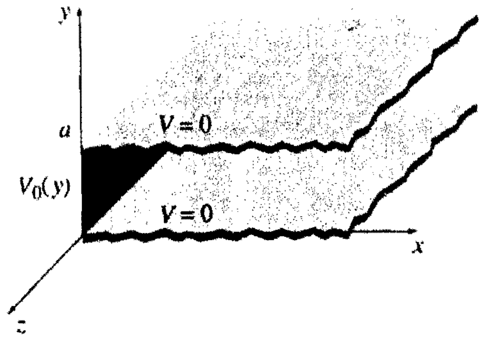

Figure 5:

[3 marks]

Question 5

(a) Considering the differential forms of Maxwell's equations, use Stoke's theorem and Gauss's theorem to derive the integral form of the equations.

[12 marks]

(b) Show that, for electromagnetic waves in a source free region, the homogeneous vector waves equations for $\vec{E}$ and $\vec{H}$ are;

$$\nabla^2 \vec{\mathbf{E}} - \frac{1}{u^2}\frac{\partial^2 \vec{\mathbf{E}}}{\partial t} = 0$$
$$\nabla^2 \vec{\mathbf{H}} - \frac{1}{u^2}\frac{\partial^2 \vec{\mathbf{H}}}{\partial t} = 0$$

where $u = 1/\sqrt{\mu\epsilon}$.

[13 marks]

Appendix

Some useful information:

1(a) dipole potential

$$\Phi_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{r}}}{r^2}$$

1(b) dipole

$$\hat{\mathbf{p}} = i\int_v r\rho(r)d\tau$$

2 for Legendre polynomials

$$\int_0^{\pi} P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta d\theta = 0 \qquad \text{if } \ell' \neq \ell$$

$$= \frac{2}{2\ell+1} \qquad \text{if } \ell' = \ell$$

3 Maxwell's general equations;

$$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$$
$$\nabla \times \overrightarrow{H} = \overrightarrow{J} + \frac{\partial \overrightarrow{D}}{\partial t}$$
$$\nabla \cdot \overrightarrow{D} = \rho$$
$$\nabla \cdot \overrightarrow{B} = 0$$

where $\overrightarrow{D} = \epsilon\overrightarrow{E}$, and $\overrightarrow{H} = \frac{1}{\mu}\overrightarrow{B}$

4 Maxwell's equations in a nonconducting medium;

$$\nabla \times \overrightarrow{E} = -\mu\frac{\partial \overrightarrow{H}}{\partial t}$$
$$\nabla \times \overrightarrow{H} = \epsilon\frac{\partial \overrightarrow{E}}{\partial t}$$
$$\nabla \cdot \overrightarrow{D} = 0$$
$$\nabla \cdot \overrightarrow{B} = 0$$

5

$$\overrightarrow{A} \times (\overrightarrow{B} \times \overrightarrow{C} = \overrightarrow{B}(\overrightarrow{A}\cdot\overrightarrow{C}) - \overrightarrow{C}(\overrightarrow{A}\cdot\overrightarrow{B})$$

6 Permeability of free space $\mu_0 = 1.25663706 \times 10^{-6} (H/m)$.

7 Permittivity of free space $\epsilon_0 = 8.854 \times 10^{-12} (F/m)$.

8 Gauss's theorem:

$$\int_v \nabla \cdot \vec{A} dv = \oint_c \vec{A} \cdot d\vec{s}$$

9 Stoke's theorem:

$$\int_s \nabla \times \vec{A} \cdot d\vec{s} = \oint_c \vec{A} \cdot d\vec{\ell}$$

10

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$