UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2015/2016

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE AND LAST PAGE WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit (u) $= 1.6605 \times 10^{-27}$ kg $= 931.5$ MeV$/c^2$

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}

Speed of light (vacuum) $c = 3.0 \times 10^8$ m/s

electron mass $m_e = 9.11 \times 10^{-31}$ kg $= 5.4858 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27}$ kg $= 1.008665$ u $= 939.573$ MeV$/c^2$

proton mass $m_p = 1.6726 \times 10^{-27}$ kg $= 1.0072765$ u $= 938.280$ MeV$/c^2$

$1 year = 3.156 \times 10^7$ s

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2$ fm

fine structure constant, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{1}{137}$

$\hbar c = 197$ MeVfm

The table of nuclear properties is provided in the next page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1: The Shell Model ....

(a) The lowest energy levels in the Shell Model, in order of increasing energy, are $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_j$, ...

i. What are the possible values of j for the $1f$ levels?

ii. What is the value of j for the lowest $1f$ level? Justify your answer.

iii. Determine the spin and parity of the ground state of both the ${}^{40}_{20}\text{Ca}$ and ${}^{41}_{20}\text{Ca}$ nuclides.

iv. In the Shell model, a 'spin-orbit' interaction splits all the energy levels except the 's-type' levels. Why do the s-type levels remain unsplit?

(b) The low-lying energy levels of ${}^{13}C$ are the ground state $\left(\frac{1}{2}^-\right)$; 3.09 MeV $\left(\frac{1}{2}^+\right)$; 3.68 MeV $\left(\frac{3}{2}^-\right)$ and 3.85 MeV $\left(\frac{5}{2}^+\right)$. Interpret these states according to the Shell model.

Question 2: Rutherford Scattering (Classical)..
In the **plum pudding model**, the atom was assumed to be made up of electrons immersed in a šoup' of positive charge.

(a) Assume that the coulomb force between an α-particle and an electron is negligible.

i. Draw a kinematics diagram showing the collision of an α-particle (initially moving at velocity $\vec{v}_0$) with an electron (initially at rest). [Note: proper labels required]

ii. Show that the final velocity of the electron $\vec{v}_e$ is related to the final velocity of the α-particle by

$$v_e^2\left(1-\frac{m_e}{m_\alpha}\right)=2\vec{v}_f\cdot\vec{v}_e,$$

where $\vec{v}_f$ is the final velocity of the α-particle, m_α is its mass and m_e is the mass of the electron.

iii. Using the result in ii), prove that large angle scattering is not possible.

iv. Based on the fact that large angle scattering of an α−particle is not possible if the target is an electron, and that in an experiment, in which atoms were bombarded with α-particles, large angle scattering was observed, suggest improvements to the plum pudding model of the atom.

(b) Relax the assumption that the coulomb interaction is negligible.

i. Draw the kinematics diagram of Rutherford scattering of a projectile of mass m and charge Ze on a stationary target of mass M and charge Ze.

ii. Show that the impact parameter b, is given by

$$b=\frac{zZe^2}{8\pi\epsilon_0}\frac{cot(\theta/2)}{E_{kin}},$$

where θ is the scattering angle and $E_{kin}=\frac{1}{2}mv^2$ is the kinetic energy of the projectile.

Question 3: Radioactivity and Fission ....

(a) A by-product of some fission reactors is ^{239}Pu (plutonium-239), which is an α-emitter with a half life of 24120 years. Consider 1 kg of ^{239}Pu at $t = 0$, [Atomic mass of $^{239}\text{Pu} = 239.052163$ u.

i. What is the number of ^{239}Pu nuclei at $t = 0$?

ii. What is the initial activity?

iii. For how long would you need to store plutonium-239 until it has decayed to a safe activity level of 0.1 Bq?

(b) Radionuclides are useful sources of small amounts of energy in space vehicles, remote communication stations, heart pacemakers, etc. Calculate the initial power available from a gram of ^{210}Po, an α-emitter with an energy of 5.30 MeV and a half life of 138 days. Give your answer in Watts. [Atomic mass of $^{210}_{84}\text{Po} = 209.982848$ u]

(c) In stars that are slightly more massive than the Sun, hydrogen burning is carried out mainly by the CNO cycle, whose first step is $\text{p} + ^{12}_{6}\text{C} \rightarrow ^{13}_{7}\text{N} + \gamma$. Estimate the energy of the γ, assuming the two initial nuclei are essentially at rest. Justify any simplifying assumptions you make. [Atomic masses: $^{1}_{1}H = 1.007825$ u, $^{12}_{6}\text{C} = 12.00000$ u, $^{13}_{7}\text{N} = 13.005739$ u]

(d) Consider the nuclear fission reaction $\text{n} + ^{235}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\text{n}$.

i. Calculate the energy released (in MeV) in the reaction. [Atomic masses: $^{235}_{92}\text{U} = 235.043915$ u, $^{92}_{36}\text{Kr} = 91.8973$ u, $^{141}_{56}\text{Ba} = 140.9139$ u and neutron mass is 1.008665 u]

ii. You wish to run a 1000 MW power reactor using $^{235}_{92}$U fission. How much $^{235}_{92}$U is required for one day's operation?

Question 4: Form Factors ....

(a) The differential cross section for Rutherford scattering is proportional to $\sin^{-4}(\theta/2)$ where θ is the scattering angle. Show that this term leads to an infinite cross section in the limit $\theta \to 0$. Explain why, in reality, experimental differential cross sections remain finite as $\theta \to 0$.

(b) The nuclear electric form factor is

$$F(\vec{q}) = \int \rho_{ch}(\vec{r}) \exp(-i\vec{q} \cdot \vec{r}) d^3\vec{r},$$

where ρ_{ch} is the charge density.

i. In the case of spherical symmetry, we have only the radial dependence. Show that $F(\vec{q})$ becomes

$$F(q^2) = \frac{4\pi}{q} \int \rho_{ch}(r) \sin(qr) r dr$$

ii. Assuming that the nuclear charge density is uniform and that the nucleus is a sphere of radius R, obtain an expression for the form factor of a nucleus.

(c) Show that, for high-energy elastic scattering where the projectile rest mass may be ignored, the magnitude of the momentum transfered q from the incident particle is given by

$$(cq)^2 = 4E^2 \sin^2(\theta/2),$$

where E is the energy of the projectile, and θ the scattering angle.

Question 5: Short Answer Questions ....

(a) Write brief notes on the following instruments

i. Geiger-Muller counter (An Ionization Chamber)

ii. Scintillation detector

(b) Discuss three modes by which a photon can interact with matter.

(c) Discuss the essential features of the strong nuclear force.

(d) Show that the decay $n \rightarrow p + e^-$ cannot conserve angular momentum.

(e) Write short notes on the following.

i. Internal conversion

ii. Bremsstrahlung